BSS2, Lecture #2

Exercises

15 September 1999

Exercise 1 Let $W \subseteq U$, and let ϕ be a potential on U. The expression $\sum_{U \setminus W} \phi$ denotes the *(sum-)margin* of ϕ on W. Sometimes the domain of potentials are indicated explicitly: $\phi_U \equiv \phi$ and $\phi_W = \sum_{U \setminus W} \phi_U$. The marginal potential ϕ_W is defined by, for $x \in \mathcal{W} = \mathrm{Sp}(W)$,

$$\phi_W(x) = \left(\sum_{U \setminus W} \phi_U \right)(x) = \sum_{z \in \mathcal{U} \setminus \mathcal{W}} \phi_U(z.x),$$

where $z.x$ is the element in $\mathcal{U} = \mathrm{Sp}(U)$ with projections x to $\mathcal{W}$ and z to $\mathcal{U} \setminus \mathcal{W} = \mathrm{Sp}(U \setminus W)$. Max-marginalization is defined similarly:

$$\phi_W^{\max}(x) = \left(\mathop{\mathsf{M}}_{U \setminus W} \phi_U \right)(x) = \max_{z \in \mathcal{U} \setminus \mathcal{W}} \phi_U(z.x).$$

Let $U = \{A, B, C, D\}$, $W = \{A, C\}$, and let ϕ_U be given by the table below.

		c_1	c_1	c_2	c_2
		d_1	d_2	d_1	d_2
a_1	b_1	0.0957	0.0672	0.0341	0.0513
a_1	b_2	0.1021	0.0162	0.0634	0.1287
a_2	b_1	0.0174	0.1297	0.0040	0.1089
a_2	b_2	0.0624	0.0776	0.0307	0.0107

(a) Compute ϕ_W and $\phi_W^{\max}$.

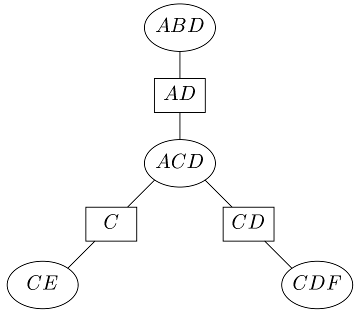
(b) Compute $x_U^{\max} = \arg\max_{x \in \mathcal{U}} \phi_U$ and $(x_A^{\max}, x_B^{\max}, x_C^{\max}, x_D^{\max})$.

(c) Indicate the table for the evidence function $\mathcal{E}_D$ (defined on $\mathcal{D}$), representing the evidence "D is in state d_1".

(d) Compute $\phi_U * \mathcal{E}_D$.

(e) Compute the normalization constant, μ, in $P(U \mid e) = \mu * \phi_U * \mathcal{E}_D$.

Exercise 2 Calculate the number of arithmetic operations by a call to Collect Evidence in clique ABD in the junction tree below, assuming sum-marginalization and binary variables.



Exercise 3 Let $\mathcal{T}$ be a consistent junction tree with cliques $C_1 = \{A, B\}$ and $C_2 = \{B, C\}$, and clique tables given as follows.

	b_1	b_2
a_1	0.07	0.37
a_2	0.28	0.28

	b_1	b_2
c_1	0.13	0.24
c_2	0.22	0.41

(a) Compute $t_{C_1}^{\max}$ and $t_{C_2}^{\max}$ (i.e., the max-margins obtained through max-propagation).

(b) From max-equilibrium, perform sum-propagation. Compare the resulting tables with the ones above.

Exercise 4 Let g be a probability function over V, let $(x_1, \ldots, x_n)$ be an arbitrary but fixed ordering of $\mathrm{Sp}(V)$, and let L be a uniformly distributed random variable on the interval $]0; 1]$. On the basis of a simulated value l for L (generated by a (pseudo) random function), we can then determine a simulated value $x \in \mathrm{Sp}(V)$ for V (i.e., x is drawn with probability $g(x)$) as follows.

(1) Let $i = 1$ and sum $= 0$.

(2) While sum $< l$ do

  (a) sum := sum $+ g(x_i)$

  (b) $i := i + 1$.

(3) Let $x = x_{i-1}$.

Simulation in a junction tree, $\mathcal{T}$, is performed through outward propagation from a root clique, C^*. Let C be a clique in the tree with neighbour C' such that C' is located on the path from C to C^* and let $S = C \cap C'$. When configuration x_S (i.e., values) for variables S has been selected a configuration for $C \setminus S$ is found using the above algorithm with the modification that L must be uniformly distributed on $]0;\sigma]$, where $\sigma = \sum_{C\setminus S} f_C(x_S)$ and f is a probability function factorizing on $\mathcal{T}$.

Now, let $\mathcal{T}$ consist of two cliques $C_1 = \{X, Y, Z\}$ and $C_2 = \{Y, Z, W\}$ interconnected by separator S. Let f_{C_1} and f_{C_2} be given as follows.

f_{C_1}	y_1		y_2	
	z_1	z_2	z_1	z_2
x_1	0.048218	0.128582	0.063861	0.099339
x_2	0.140174	0.021026	0.037200	0.111600
x_3	0.117000	0.065000	0.048000	0.120000

f_{C_2}	z_1		z_2	
	w_1	w_2	w_1	w_2
y_1	0.133609	0.171783	0.067065	0.147543
y_2	0.087683	0.061378	0.141831	0.189108

Let $((x_1, y_1, z_1), (x_1, y_1, z_2), (x_1, y_2, z_1), (x_1, y_2, z_2), (x_2, y_1, z_1), (x_2, y_1, z_2), (x_2, y_2, z_1), (x_2, y_2, z_2), (x_3, y_1, z_1), (x_3, y_1, z_2), (x_3, y_2, z_1), (x_3, y_2, z_2))$ and $((y_1, z_1, w_1), (y_1, z_1, w_2), (y_1, z_2, w_1), (y_1, z_2, w_2), (y_2, z_1, w_1), (y_2, z_1, w_2), (y_2, z_2, w_1), (y_2, z_2, w_2))$ be orderings of, respectively, $\mathrm{Sp}(C_1)$ and $\mathrm{Sp}(C_2)$.

(a) Find the simulated values for X, Y, Z and W through "random propagation" from C_1 to C_2, using these orderings and the "random" numbers 0.67 and 0.22 (in that order) in the above algorithm.

(b) Hugin requires the junction tree to be in sum-equilibrium before random propagation is initiated. What do you think might be the reason for that?

Exercise 5 Prim's algorithm for constructing a maximal spanning tree (e.g. a junction tree over a junction graph) is given as follows.

(1) Start with a tree, $\mathcal{T}$, containing a single node.

(2) Successively, add a node, V, to $\mathcal{T}$ whose intersection with a node of $\mathcal{T}$ is maximal.

(3) Link V to that node.

Show that Prim's algorithm always yields a maximum spanning tree.