

Semantics and Verification 2006

Lecture 3

- value passing CCS
- behavioural equivalences
- strong bisimilarity and bisimulation games
- properties of strong bisimilarity

Value Passing CCS

Main Idea

Handshake synchronization is extended with the possibility to exchange integer values.

$$\begin{aligned} & \overline{\text{pay}(6)}.Nil \mid \text{pay}(x).\overline{\text{save}(x/2)}.Nil \mid \text{Bank}(100) \\ & \quad \downarrow \tau \\ & Nil \mid \overline{\text{save}(3)}.Nil \mid \text{Bank}(100) \\ & \quad \downarrow \tau \\ & Nil \mid Nil \mid \text{Bank}(103) \end{aligned}$$

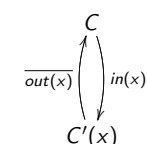
Parametrized Process Constants

For example: $\text{Bank}(\text{total}) \stackrel{\text{def}}{=} \text{save}(x).\text{Bank}(\text{total} + x).$

Translation of Value Passing CCS to Standard CCS

Value Passing CCS

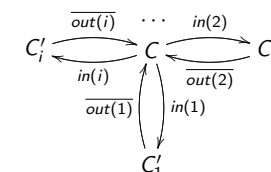
$$\begin{aligned} C & \stackrel{\text{def}}{=} \text{in}(x).C'(x) \\ C'(x) & \stackrel{\text{def}}{=} \overline{\text{out}(x)}.C \end{aligned}$$



symbolic LTS

Standard CCS

$$\begin{aligned} C & \stackrel{\text{def}}{=} \sum_{i \in \mathbb{N}} \text{in}(i).C'_i \\ C'_i & \stackrel{\text{def}}{=} \overline{\text{out}(i)}.C \end{aligned}$$



infinite LTS

CCS Has Full Turing Power

Fact

CCS can simulate a computation of any Turing machine.

Remark

Hence CCS is as expressive as any other programming language but its use is to rather **describe** the behaviour of reactive systems than to perform specific calculations.

Behavioural Equivalence

Implementation

$$\begin{aligned} CM & \stackrel{\text{def}}{=} \text{coin}.\overline{\text{coffee}}.CM \\ CS & \stackrel{\text{def}}{=} \text{pub}.\overline{\text{coin}}.\text{coffee}.CS \\ Uni & \stackrel{\text{def}}{=} (CM \mid CS) \setminus \{\text{coin}, \text{coffee}\} \end{aligned}$$

Specification

$$Spec \stackrel{\text{def}}{=} \overline{\text{pub}}.Spec$$

Question

Are the processes *Uni* and *Spec* behaviorally equivalent?

$$Uni \equiv Spec$$

Goals

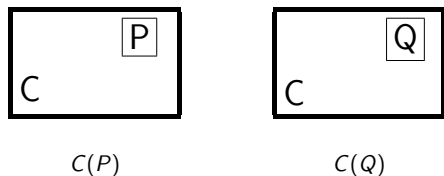
What should a reasonable behavioural equivalence satisfy?

- abstract from states (consider only the behaviour – actions)
- abstract from nondeterminism
- abstract from internal behaviour

What else should a reasonable behavioural equivalence satisfy?

- **reflexivity** $P \equiv P$ for any process P
- **transitivity** $Spec_0 \equiv Spec_1 \equiv Spec_2 \equiv \dots \equiv Impl$ gives that $Spec_0 \equiv Impl$
- **symmetry** $P \equiv Q$ iff $Q \equiv P$

Congruence



Congruence Property

$P \equiv Q$ implies that $C(P) \equiv C(Q)$

Trace Equivalence

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS.

Trace Set for $s \in Proc$

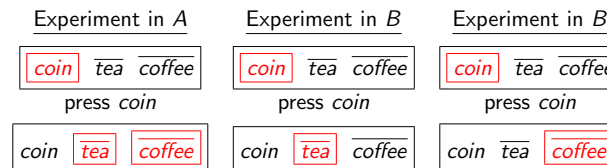
$$Traces(s) = \{w \in Act^* \mid \exists s' \in Proc. s \xrightarrow{w} s'\}$$

Let $s \in Proc$ and $t \in Proc$.

Trace Equivalence

We say that s and t are **trace equivalent** ($s \equiv_t t$) if and only if $Traces(s) = Traces(t)$

Black-Box Experiments



Main Idea

Two processes are behaviorally equivalent if and only if an **external observer** cannot tell them apart.

Strong Bisimilarity

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS.

Strong Bisimulation

A binary relation $R \subseteq Proc \times Proc$ is a **strong bisimulation** iff whenever $(s, t) \in R$ then for each $a \in Act$:

- if $s \xrightarrow{a} s'$ then $t \xrightarrow{a} t'$ for some t' such that $(s', t') \in R$
- if $t \xrightarrow{a} t'$ then $s \xrightarrow{a} s'$ for some s' such that $(s', t') \in R$.

Strong Bisimilarity

Two processes $p_1, p_2 \in Proc$ are **strongly bisimilar** ($p_1 \sim p_2$) if and only if there exists a strong bisimulation R such that $(p_1, p_2) \in R$.

$$\sim = \cup \{R \mid R \text{ is a strong bisimulation}\}$$

Basic Properties of Strong Bisimilarity

Theorem

\sim is an equivalence (reflexive, symmetric and transitive)

Theorem

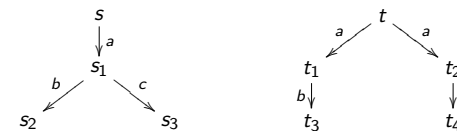
\sim is the largest strong bisimulation

Theorem

$s \sim t$ if and only if for each $a \in Act$:

- if $s \xrightarrow{a} s'$ then $t \xrightarrow{a} t'$ for some t' such that $s' \sim t'$
- if $t \xrightarrow{a} t'$ then $s \xrightarrow{a} s'$ for some s' such that $s' \sim t'$.

How to Show Nonbisimilarity?



To prove that $s \not\sim t$:

- Enumerate **all binary relations** and show that none of them at the same time contains (s, t) and is a strong bisimulation. (Expensive: $2^{|Proc|^2}$ relations.)
- Make certain **observations** which will enable to disqualify many bisimulation candidates in one step.
- Use **game characterization** of strong bisimilarity.

Strong Bisimulation Game

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS and $s, t \in Proc$.

We define a two-player game of an 'attacker' and a 'defender' starting from s and t .

- The game is played in **rounds** and configurations of the game are pairs of states from $Proc \times Proc$.
- In every round exactly one configuration is called **current**. Initially the configuration (s, t) is the current one.

Intuition

The defender wants to show that s and t are strongly bisimilar while the attacker aims to prove the opposite.

Rules of the Bisimulation Games

Game Rules

In each round the players change the current configuration as follows:

- 1 the attacker chooses one of the processes in the current configuration and makes an \xrightarrow{a} -move for some $a \in Act$, and
- 2 the defender must respond by making an \xrightarrow{a} -move in the other process under the same action a .

The newly reached pair of processes becomes the current configuration. The game then continues by another round.

Result of the Game

- If one player cannot move, the other player wins.
- If the game is infinite, the defender wins.

Game Characterization of Strong Bisimilarity

Theorem

- States s and t are strongly bisimilar if and only if the defender has a **universal** winning strategy starting from the configuration (s, t) .
- States s and t are not strongly bisimilar if and only if the attacker has a **universal** winning strategy starting from the configuration (s, t) .

Remark

Bisimulation game can be used to prove both bisimilarity and nonbisimilarity of two processes. It very often provides elegant arguments for the negative case.

Strong Bisimilarity is a Congruence for CCS Operations

Theorem

Let P and Q be CCS processes such that $P \sim Q$. Then

- $\alpha.P \sim \alpha.Q$ for each action $\alpha \in Act$
- $P + R \sim Q + R$ and $R + P \sim R + Q$ for each CCS process R
- $P \mid R \sim Q \mid R$ and $R \mid P \sim R \mid Q$ for each CCS process R
- $P[f] \sim Q[f]$ for each relabelling function f
- $P \setminus L \sim Q \setminus L$ for each set of labels L .

Other Properties of Strong Bisimilarity

Following Properties Hold for any CCS Processes P, Q and R

- $P + Q \sim Q + P$
- $P \mid Q \sim Q \mid P$
- $P + Nil \sim P$
- $P \mid Nil \sim P$
- $(P + Q) + R \sim P + (Q + R)$
- $(P \mid Q) \mid R \sim P \mid (Q \mid R)$