Semantics and Verification 2006

Lecture 2

- informal introduction to CCS
- syntax of CCS
- semantics of CCS

CCS Basics (Sequential Fragment)

- *Nil* (or 0) process (the only atomic process)
- action prefixing ($a.P$)
- names and recursive definitions ($\stackrel{\mathrm{def}}{=}$)
- nondeterministic choice ($+$)

This is Enough to Describe Sequential Processes

Any finite LTS can be (up to isomorphism) described by using the operations above.

CCS Basics (Parallelism and Renaming)

- parallel composition ($|$)
  (synchronous communication between two components = handshake synchronization)
- restriction ($P \smallsetminus L$)
- relabelling ($P[f]$)

Definition of CCS (channels, actions, process names)

Let

- $\mathcal{A}$ be a set of channel names (e.g. *tea*, *coffee* are channel names)
- $\mathcal{L} = \mathcal{A} \cup \overline{\mathcal{A}}$ be a set of labels where
  - $\overline{\mathcal{A}} = \{\overline{a} \mid a \in \mathcal{A}\}$
    ($\mathcal{A}$ are called names and $\overline{\mathcal{A}}$ are called co-names)
  - by convention $\overline{\overline{a}} = a$
- $Act = \mathcal{L} \cup \{\tau\}$ is the set of actions where
  - τ is the internal or silent action

  (e.g. τ, *tea*, $\overline{coffee}$ are actions)
- $\mathcal{K}$ is a set of process names (constants) (e.g. CM).

Definition of CCS (expressions)

$P :=$	K	$\mid$	process constants ($K \in \mathcal{K}$)
	$\alpha.P$	$\mid$	prefixing ($\alpha \in Act$)
	$\sum_{i \in I} P_i$	$\mid$	summation (I is an arbitrary index set)
	$P_1 \vert P_2$	$\mid$	parallel composition
	$P \setminus L$	$\mid$	restriction ($L \subseteq \mathcal{A}$)
	$P[f]$	$\mid$	relabelling ($f : Act \rightarrow Act$) such that

- $f(\tau) = \tau$
- $f(\bar{a}) = \overline{f(a)}$

The set of all terms generated by the abstract syntax is called CCS process expressions (and denoted by $\mathcal{P}$).

Notation

$P_1 + P_2 = \sum_{i \in \{1,2\}} P_i$ $Nil = 0 = \sum_{i \in \emptyset} P_i$

Precedence

Precedence

1. restriction and relabelling (tightest binding)
2. action prefixing
3. parallel composition
4. summation

Example: $R + a.P | b.Q \smallsetminus L$ means $R + \big((a.P)|(b.(Q \smallsetminus L))\big)$.

Definition of CCS (defining equations)

CCS program

A collection of defining equations of the form

$$K \stackrel{\mathrm{def}}{=} P$$

where $K \in \mathcal{K}$ is a process constant and $P \in \mathcal{P}$ is a CCS process expression.

- Only one defining equation per process constant.
- Recursion is allowed: e.g. $A \stackrel{\mathrm{def}}{=} \overline{a}.A \mid A$.

Semantics of CCS

Syntax		Semantics
CCS (collection of defining equations)	$\longrightarrow$	LTS (labelled transition systems)

HOW?

Structural Operational Semantics for CCS

Structural Operational Semantics (SOS) – G. Plotkin 1981

Small-step operational semantics where the behaviour of a system is inferred using syntax driven rules.

Given a collection of CCS defining equations, we define the following LTS $(Proc, Act, \{\stackrel{a}{\longrightarrow} \mid a \in Act\})$:

- $Proc = \mathcal{P}$ (the set of all CCS process expressions)
- $Act = \mathcal{L} \cup \{\tau\}$ (the set of all CCS actions including τ)
- transition relation is given by SOS rules of the form:

$$\text{RULE} \quad \frac{\textit{premises}}{\textit{conclusion}} \quad \textit{conditions}$$

SOS rules for CCS ($\alpha \in Act$, $a \in \mathcal{L}$)

ACT $$\frac{}{\alpha.P \xrightarrow{\alpha} P}$$

SUM$_j$ $$\frac{P_j \xrightarrow{\alpha} P'_j}{\sum_{i \in I} P_i \xrightarrow{\alpha} P'_j} \quad j \in I$$

COM1 $$\frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q}$$

COM2 $$\frac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'}$$

COM3 $$\frac{P \xrightarrow{a} P' \quad Q \xrightarrow{\overline{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

RES $$\frac{P \xrightarrow{\alpha} P'}{P \smallsetminus L \xrightarrow{\alpha} P' \smallsetminus L} \quad \alpha, \overline{\alpha} \notin L$$

REL $$\frac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$$

CON $$\frac{P \xrightarrow{\alpha} P'}{K \xrightarrow{\alpha} P'} \quad K \stackrel{\mathrm{def}}{=} P$$

Deriving Transitions in CCS

Let $A \stackrel{\text{def}}{=} a.A$. Then

$$\big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a] \stackrel{c}{\longrightarrow} \big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a].$$

$$\text{REL}\ \dfrac{\text{COM1}\ \dfrac{\text{COM1}\ \dfrac{\text{CON}\ \dfrac{\text{ACT}\ \dfrac{}{a.A \stackrel{a}{\longrightarrow} A}}{A \stackrel{a}{\longrightarrow} A}\ A \stackrel{\text{def}}{=} a.A}{A \,|\, \overline{a}.Nil \stackrel{a}{\longrightarrow} A \,|\, \overline{a}.Nil}}{(A \,|\, \overline{a}.Nil) \,|\, b.Nil \stackrel{a}{\longrightarrow} (A \,|\, \overline{a}.Nil) \,|\, b.Nil}}{\big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a] \stackrel{c}{\longrightarrow} \big((A \,|\, \overline{a}.Nil) \,|\, b.Nil\big)[c/a]}$$

LTS of the Process $a.Nil \mid \overline{a}.Nil$