Tutorial 1

Exercise 1

What are all the possible values that can be stored in the variable x after the execution of the following parallel program?

$$\texttt{x:=10; } \big(\texttt{(x:=x*2; x:=x-11; x:=x+2) } \| \texttt{ x:=x-5}\big)$$

Exercise 2

Let R be a binary relation on a set A. Let us define the binary relation

$$E \stackrel{\text{def}}{=} \{(x,x) \mid x \in A\}.$$

It is trivially true that $R \cup E$ is a reflexive relation.

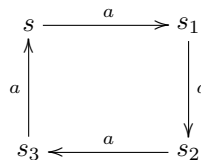- Argue that $R \cup E$ is a reflexive closure of R.

Exercise 3

Let R be a binary relation on a set A. Let us define the binary relation

$$R^{-1} \stackrel{\text{def}}{=} \{(y,x) \mid (x,y) \in R\}.$$

- Argue that $R \cup R^{-1}$ is a symmetric relation.
- Argue that $R \cup R^{-1}$ is a symmetric closure of R.

Exercise 4*

Let us consider the following labelled transition system.

- Define the labelled transition system as a triple $(Proc, Act, \{\stackrel{a}{\longrightarrow} \mid a \in Act\})$.
- What is the reflexive closure of the binary relation $\stackrel{a}{\longrightarrow}$? (A drawing is fine.)
- What is the symmetric closure of the binary relation $\stackrel{a}{\longrightarrow}$? (A drawing is fine.)
- What is the transitive closure of the binary relation $\stackrel{a}{\longrightarrow}$? (A drawing is fine.)

Example 5

Let us consider the following CCS definition of a coffee machine.

$$\text{CM} \stackrel{\text{def}}{=} coin.\overline{coffee}.\text{CM}$$

- Give a CCS process which describes a coffee machine that may behave like CM but may also steal the money it receives and fail at any time.

Example 6

Assume a given labelled transition system $T = (Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ such that the sets $Proc$ and Act are finite.

- Does it imply that $\xrightarrow{a}$ is also a finite set? Why?

- Draw an example of an LTS with four states and two actions.

- How can your example be described by a sequential fragment of CCS (with Nil, action prefixing, nondeterminism and recursive definitions of names)?

- Show that in general any finite LTS T can be described by using only a sequential fragment of CCS.