Semantics and Verification 2005

Lecture 15

- round-up of the course
- information about the exam
- selection of star exercises

Reactive systems

Characterization of a Reactive System

Reactive System is a system that computes by reacting to stimuli from its environment.

Key Issues:

- parallelism
- communication and interaction

Nontermination is good!

The result (if any) does not have to be unique.

Classical vs. Reactive Computing

	Classical	Reactive/Parallel
interaction	no	yes
nontermination	undesirable	often desirable
unique result	yes	no
semantics	$states \hookrightarrow states$	LTS

Calculus of Communicating Systems

CCS

Process algebra called "Calculus of Communicating Systems".

Insight of Robin Milner (1989)

Concurrent (parallel) processes have an algebraic structure.

$$\boxed{P_1}\ op\ \boxed{P_2} \Rightarrow \boxed{P_1\ op\ P_2}$$

Process Algebras

Basic Principle

1. Define a few atomic processes (modelling the simplest process behaviour).
2. Define compositionally new operations (building more complex process behaviour from simple ones).

Example

1. atomic instruction: assignment (e.g. x:=2 and x:=x+2)
2. new operators:
   - sequential composition ($P_1; P_2$)
   - parallel composition ($P_1 \mid P_2$)

Usually given by abstract syntax:

$$P, P_1, P_2 \ ::= \ x := e \ \mid \ P_1; P_2 \ \mid \ P_1 | P_2$$

where x ranges over variables and e over arithmetical expressions.

Syntax of CCS

Process expressions:

$P :=$	K	$\mid$	process constants ($K \in \mathcal{K}$)
	$\alpha.P$	$\mid$	prefixing ($\alpha \in Act$)
	$\sum_{i \in I} P_i$	$\mid$	summation (I is an arbitrary index set)
	$P_1 \mid P_2$	$\mid$	parallel composition
	$P \smallsetminus L$	$\mid$	restriction ($L \subseteq \mathcal{A}$)
	$P[f]$	$\mid$	relabelling ($f : Act \to Act$) such that • $f(\tau) = \tau$ • $f(\bar{a}) = \overline{f(a)}$

$$P_1 + P_2 = \sum_{i \in \{1,2\}} P_i \qquad\qquad Nil = 0 = \sum_{i \in \emptyset} P_i$$

CCS program

A collection of defining equations of the form $K \stackrel{\mathrm{def}}{=} P$ where $K \in \mathcal{K}$ is a process constant and P is a process expression.

Semantics of CCS — SOS rules ($\alpha \in Act$, $a \in \mathcal{L}$)

$$\text{ACT} \quad \frac{}{\alpha.P \xrightarrow{\alpha} P} \qquad\qquad \text{SUM}_j \quad \frac{P_j \xrightarrow{\alpha} P'_j}{\sum_{i \in I} P_i \xrightarrow{\alpha} P'_j} \quad j \in I$$

$$\text{COM1} \quad \frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q} \qquad\qquad \text{COM2} \quad \frac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'}$$

$$\text{COM3} \quad \frac{P \xrightarrow{a} P' \qquad Q \xrightarrow{\overline{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

$$\text{RES} \quad \frac{P \xrightarrow{\alpha} P'}{P \setminus L \xrightarrow{\alpha} P' \setminus L} \quad \alpha, \overline{\alpha} \notin L \qquad\qquad \text{REL} \quad \frac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$$

$$\text{CON} \quad \frac{P \xrightarrow{\alpha} P'}{K \xrightarrow{\alpha} P'} \quad K \stackrel{\text{def}}{=} P$$

Verification Approaches

Let *Impl* be an implementation of a system (e.g. in CCS syntax).

Equivalence Checking Approach

$$\mathit{Impl} \equiv \mathit{Spec}$$

- *Spec* is a full specification of the intended behaviour
- Example: $s \sim t$ or $s \approx t$

Model Checking Approach

$$\mathit{Impl} \models \mathit{Property}$$

- *Property* is a partial specification of the intended behaviour
- Example: $s \models \langle a \rangle ([b]\mathit{ff} \land \langle a \rangle \mathit{tt})$

Relationship between Equivalence and Model Checking

- Equivalence checking and model checking are complementary approaches.
- They are strongly connected, however.

Theorem (Hennessy-Milner)

Let us consider an image-finite LTS. Then

$$p \sim q$$

if and only if

for every HM formula F (even with recursion):

$$(p \models F \iff q \models F).$$

Introducing Time Features

In many applications, we would like to explicitly model real-time in our models.

Timed (labelled) transition system

Timed LTS is an ordinary LTS where actions are of the form $Act = L \cup \mathbb{R}^{\geq 0}$.

- $s \xrightarrow{a} s'$ for $a \in L$ are discrete transitions
- $s \xrightarrow{d} s'$ for $d \in \mathbb{R}^{\geq 0}$ are time-elapsing (delay) transitions

Timed and Untimed Bisimilarity

Let s and t be two states in timed LTS.

Timed Bisimilarity (= Strong Bisimilarity)

We say that s and t are timed bisimilar iff $s \sim t$.

Remark: all transitions are considered as visible transitions.

Untimed Bisimilarity

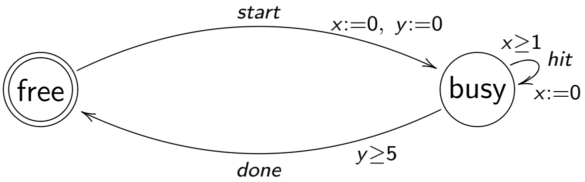
We say that s and t are untimed bisimilar iff $s \sim t$ in a modified transition system where every transition of the form $\xrightarrow{d}$ for $d \in \mathbb{R}^{\geq 0}$ is replaced by a transition $\xrightarrow{\epsilon}$ for a new (single) action ϵ.

Remark:

- $\xrightarrow{a}$ for $a \in L$ are treated as visible transitions, while
- $\xrightarrow{d}$ for $d \in \mathbb{R}^{\geq 0}$ all look the same (action ϵ).

Timed Automata — a Way to Define Timed LTS

- Nondeterministic finite-state automata with additional time features (clocks).
- Clocks can be tested against constants or compared to each other (pairwise).
- Executing a transition can reset selected clocks.



Region Graph — a Verification Technique for TA

We introduce an equivalence on clock valuations ($v \equiv v'$) with finitely many equivalence classes.

state (ℓ, v) $\rightsquigarrow$ symbolic state $(\ell, [v])$

Region Graph Preserves Untimed Bisimilarity

For every location ℓ and any two valuations v and v' from the same symbolic state ($v \equiv v'$) it holds that (ℓ, v) and (ℓ, v') are untimed bisimilar.

Reduction of Timed Automata Reachability to Region Graphs

$(\ell_0, v_0) \longrightarrow^* (\ell, v)$ in a timed automaton if and only if $(\ell_0, [v_0]) \Longrightarrow^* (\ell, [v])$ in its (finite) region graph.

Compact Representation of State-Spaces in the Memory

Boolean Functions (where $\mathbb{B} = \{0, 1\}$)

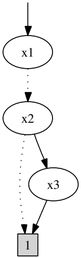
$$f : \mathbb{B}^n \longrightarrow \mathbb{B}$$

Boolean Expressions

$$t, t_1, t_2 ::= 0 \mid 1 \mid x \mid \neg t \mid t_1 \wedge t_2 \mid t_1 \vee t_2 \mid t_1 \Rightarrow t_2 \mid t_1 \Leftrightarrow t_2$$

Boolean expression:

$\neg x_1 \wedge (x_2 \Rightarrow (x_1 \vee x_3))$



Reduced and Ordered Binary Decision Diagram (ROBDD)

Logical operations on ROBDDs can be done efficiently!

The End

The course is over now!

Information about the Exam

- Oral exam with preparation time, passed/failed.
- Preparation time (20 minutes) for solving a randomly selected star exercise.
- Examination time (20 minutes):
  - presentation of the star exercise (necessary condition for passing)
  - presentation of your randomly selected exam question
  - answering questions
  - evaluation
- 8 exam questions (with possible pensum dispensation).
- For a detailed summary of the reading material check the lectures plan.

Exam Questions

1. Transition systems and CCS.
2. Strong and weak bisimilarity, bisimulation games.
3. Hennessy-Milner logic and bisimulation.
4. Tarski's fixed-point theorem and Hennessy-Milner logic with one recursive formulae.
5. Alternating bit protocol and its modelling and verification using CWB. (Possible pensum dispensation.)
6. Timed automata, networks of timed automata and their semantics.
7. Gossiping girls problem and its modelling and verification using UPPAAL. (Possible pensum dispensation.)
8. Binary decision diagrams and their use in verification.

Further details are on the web-page. Check whether you are on the list of students with pensum dispensation before going to the exam!

How to Prepare for the Exam?

- Read the recommended material.
- Try to understand all topics equally well (remember you pick up two random topics out of 6).
- Go through all tutorial exercises and try to solve them. (Make sure that you can solve all star exercises fast!)
- Go through the slides to see whether you didn't miss anything.
- Make a summary for each question on a few A4 papers (you can take them at exam).
- Prepare a strategy how to present each question.

Further Tips

- It does not matter if you make a small error in a star exercise (as long as you understand what you are doing).
- Present a solution to the star exercise quickly (max 5 minutes).
- Start your presentation by writing a road-map (max 4 items).
- Plan your presentation to take about 10 minutes:
  - give a good overview
  - do not start with technical details
  - use the blackboard
  - use examples (be creative)
  - say only things that you know are correct
  - be ready to answer supplementary questions
  - tell a story (covering a sufficient part of the exam question)

Examples of Star Exercises — CCS

- By using SOS rules for CCS prove the existence of the following transition (assume that $A \stackrel{\text{def}}{=} a.A$):

$$((A \,|\, \overline{a}.Nil) + A) \setminus \{a\} \xrightarrow{\tau} (A \,|\, Nil) \setminus \{a\}$$

- Draw the LTS generated by the following CCS expression:

$$(\overline{a}.Nil \,|\, a.Nil) + b.Nil$$

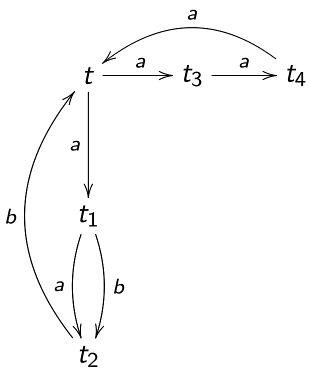
Examples of Star Exercises — Bisimilarity

Determine whether the following two CCS expressions

$$a.(b.Nil + c.Nil) \qquad \text{and} \qquad a.(b.Nil + \tau.c.Nil)$$

are:

- strongly bisimilar?
- weakly bisimilar?



Determine whether

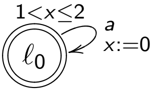
- $t \models [a](\langle b \rangle tt \vee [a][b]ff)$
- $t \models X$ where $X \overset{\max}{=} \langle a \rangle tt \wedge [Act]X$

Find a distinguishing formulae for the CCS expressions:

$a.a.Nil + a.b.Nil$ and $a.(a.Nil + b.Nil)$.

Examples of Star Exercises — TA

Draw a region graph of the following timed automaton:



Examples of Star Exercises — ROBDD

Construct ROBDD for the following boolean expression:

$$x_1 \wedge (\neg x_2 \vee x_1 \vee x_2) \wedge x_3$$

such that $x_1 < x_2 < x_3$.