

Semantics and Verification 2005

Lecture 1

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Focus of the Course

- Study of mathematical models for the formal description and analysis of programs.
- Particular focus on parallel and reactive systems.
- Verification tools and implementation techniques underlying them.

Overview of the Course

- Transition systems and CCS.
- Strong and weak bisimilarity, bisimulation games.
- Hennessy-Milner logic and bisimulation.
- Tarski's fixed-point theorem.
- Hennessy-Milner logic with recursively defined formulae.
- Timed automata and their semantics.
- Binary decision diagrams and their use in verification.
- Two mini projects.

Mini Projects

- Verification of a communication protocol in CWB.
- Verification of an algorithm for mutual exclusion in UPPAAL.
- Pensum dispensation.

Lectures

- Two guest lectures (G. Behrmann, K. G. Larsen).
- Ask questions.
- **Take your own notes.**
- Read the recommended literature as soon as possible after the lecture.

Tutorials

- Regularly before each lecture.
- Supervised peer learning.
- Two classrooms, work in groups of 2 or 3 people.
- **Print out the exercise list**, bring literature and your notes.
- Feedback from teaching assistant on your request.
- **Star exercises (*)** (part of the exam).

Exam

- Individual and oral.
- Preparation time (star exercises).
- Pensum dispensation.

Literature

- On-line literature.
- Compendiums (2004 + 2005, 141 kr).
- **Best Reader Competition** with award!

Hints

- Check regularly the course web-page.
- Anonymous feedback form on the course web-page.
- Attend and actively participate during tutorials.
- Take your own notes.

Aims of the Course

Present a general theory of reactive systems and its applications.

- Design.
 - Specification.
 - Verification (possibly automatic and compositional).
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- 1 Give the students practice in modelling parallel systems in a formal framework.
 - 2 Give the students skills in analyzing behaviours of reactive systems.
 - 3 Introduce algorithms and tools based on the modelling formalisms.

Classical View

Characterization of a Classical Program

Program transforms an input into an output.

- Denotational semantics:
a meaning of a program is a partial function

$$states \mapsto states$$

- **Nontermination is bad!**
- In case of termination, the result is unique.

Is this all we need?

Reactive systems

What about:

- Operating systems?
- Communication protocols?
- Control programs?
- Mobile phones?
- Vending machines?

Reactive systems

Characterization of a Reactive System

Reactive System is a system that computes by reacting to stimuli from its environment.

Key Issues:

- communication and interaction
- parallelism

Nontermination is good!

The result (if any) does not have to be unique.

Analysis of Reactive Systems

Questions

- How can we develop (design) a system that "works"?
- How do we analyze (verify) such a system?

Fact of Life

Even short parallel programs may be hard to analyze.

The Need for a Theory

Conclusion

We need formal/systematic methods (tools), otherwise ...

- Intel's Pentium-II bug in floating-point division unit
- Ariane-5 crash due to a conversion of 64-bit real to 16-bit integer
- Mars Pathfinder
- ...

Classical vs. Reactive Computing

	Classical	Reactive/Parallel
interaction	no	yes
nontermination	undesirable	often desirable
unique result	yes	no
semantics	$states \hookrightarrow states$?

How to Model Reactive Systems

Question

What is the most abstract view of a reactive system (process)?

Answer

A process performs an action and becomes another process.

Labelled Transition System

Definition

A **labelled transition system** (LTS) is a triple $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ where

- $Proc$ is a set of **states** (or **processes**),
- Act is a set of **labels** (or **actions**), and
- for every $a \in Act$, $\xrightarrow{a} \subseteq Proc \times Proc$ is a binary relation on states called the **transition relation**.

We will use the infix notation $s \xrightarrow{a} s'$ meaning that $(s, s') \in \xrightarrow{a}$.

Sometimes we distinguish the **initial** (or **start**) state.

Sequencing, Nondeterminism and Parallelism

LTS explicitly focuses on **interaction**.

LTS can also describe:

- sequencing $(a; b)$
- choice (nondeterminism) $(a + b)$
- limited notion of parallelism (by using interleaving) $(a \parallel b)$

Binary Relations

Definition

A binary relation R on a set A is a subset of $A \times A$.

$$R \subseteq A \times A$$

Sometimes we write $x R y$ instead of $(x, y) \in R$.

Properties

- R is **reflexive** if $(x, x) \in R$ for all $x \in A$
- R is **symmetric** if $(x, y) \in R$ implies that $(y, x) \in R$ for all $x, y \in A$
- R is **transitive** if $(x, y) \in R$ and $(y, z) \in R$ implies that $(x, z) \in R$ for all $x, y, z \in A$

Closures

Let R , R' and R'' be binary relations on a set A .

Reflexive Closure

R' is the **reflexive closure** of R if and only if

- 1 $R \subseteq R'$,
- 2 R' is reflexive, and
- 3 R' is the *smallest* relation that satisfies the two conditions above, i.e., for any relation R'' :
if $R \subseteq R''$ and R'' is reflexive, then $R' \subseteq R''$.

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Closures

Let R , R' and R'' be binary relations on a set A .

Transitive Closure

R' is the **transitive closure** of R if and only if

- 1 $R \subseteq R'$,
- 2 R' is transitive, and
- 3 R' is the *smallest* relation that satisfies the two conditions above, i.e., for any relation R'' :
if $R \subseteq R''$ and R'' is transitive, then $R' \subseteq R''$.

Labelled Transition Systems – Notation

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS.

- we extend \xrightarrow{a} to the elements of Act^*
- $\longrightarrow = \bigcup_{a \in Act} \xrightarrow{a}$
- \longrightarrow^* is the reflexive and transitive closure of \longrightarrow
- $s \xrightarrow{a}$ and $s \not\xrightarrow{a}$
- reachable states

How to Describe LTS?

Syntax

unknown entity

→

Semantics

known entity

programming language

→

what (denotational) or
how (operational) it computes

???

→

Labelled Transition Systems

CCS

Calculus of Communicating Systems

CCS

Process algebra called “Calculus of Communicating Systems”.

Insight of Robin Milner (1989)

Concurrent (parallel) processes have an algebraic structure.

$$\boxed{P_1} \text{ op } \boxed{P_2} \Rightarrow \boxed{P_1 \text{ op } P_2}$$

Process Algebra

Basic Principle

- 1 Define a few **atomic processes** (modelling the simplest process behaviour).
- 2 Define compositionally **new operations** (building more complex process behaviour from simple ones).

Example

- 1 atomic instruction: assignment (e.g. $x:=2$ and $x:=x+2$)
- 2 new operators:
 - sequential composition ($P_1; P_2$)
 - parallel composition ($P_1 \parallel P_2$)

Now e.g. $(x:=1 \parallel x:=2); x:=x+2; (x:=x-1 \parallel x:=x+5)$ is a process.

CCS Basics (Sequential Fragment)

- *Nil* (or 0) process (the only atomic process)
- action prefixing ($a.P$)
- names and recursive definitions ($\stackrel{\text{def}}{=}$)
- nondeterministic choice ($+$)

This is Enough to Describe Sequential Processes

Any finite LTS can be (up to isomorphism) described by using the operations above.