Monitoring Real-Time Systems Under Parametric Delay

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Joint work with

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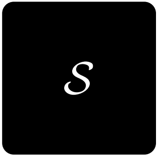
standing on the shoulders of giants:

Andreas Bauer

Martin Leucker

Christian Schallhart

Motivation



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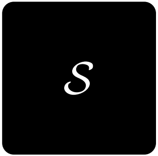

$\{\texttt{init}\}$

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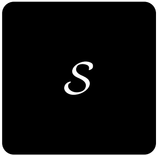

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Motivation

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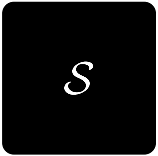

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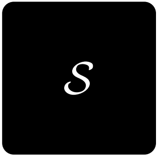

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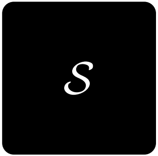

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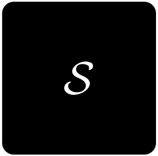
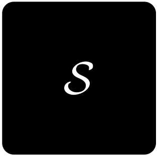


No `grnt` before the first `req`.



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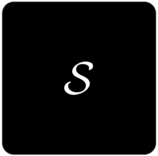

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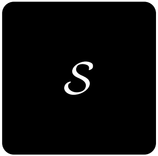

Every `req` is followed by a `grnt` within 3 steps.

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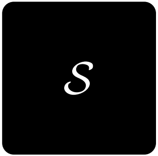

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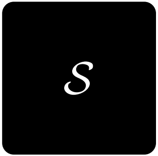

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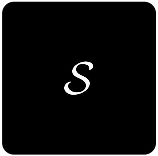

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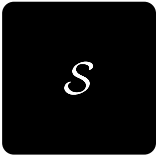

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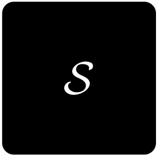

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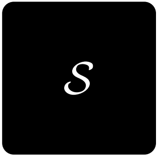

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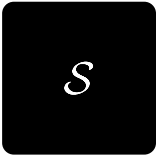

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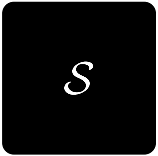

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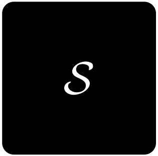

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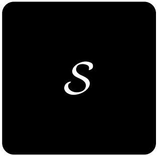

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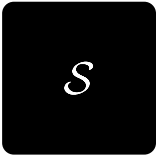

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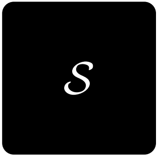

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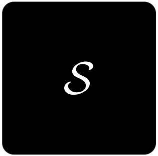

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Monitoring in a Nutshell

1. Observe (finite) executions of a system..
2. and make verdicts about the satisfaction or violation of a property about infinite executions:
   - ✔ if the property is already satisfied.
   - ✘ if the property is already violated.
   - **?** otherwise.

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Advantages

- Lightweight
- Blackbox
- Online
- Violations detected as soon as possible (enables quick mitigation)

Outline

1. Monitoring in Discrete Time
2. Monitoring in Real-Time
3. Monitoring under Delay in Real-Time
4. Conclusion

LTL in One Slide

Syntax

$$\varphi ::= a \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{X}\,\varphi \mid \varphi\,\mathsf{U}\,\varphi \qquad \text{where } a \in \mathrm{AP}$$

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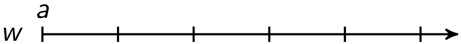
Semantics

- $w \models a$:



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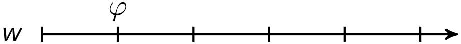

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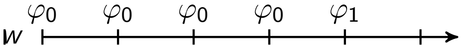

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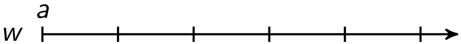

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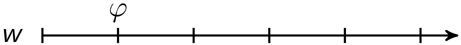

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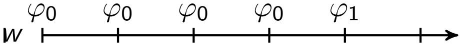

Syntactic Sugar

- $\mathsf{F}\,\psi = \mathsf{tt}\,\mathsf{U}\,\psi$
- $\mathsf{G}\,\psi = \neg\,\mathsf{F}\,\neg\psi$

Examples

- No grnt before the first req.

$$((\neg \mathtt{grnt})\,\mathsf{U}\,\mathtt{req}) \vee \mathsf{G}\,\neg\mathtt{req}$$

Examples

- No $\texttt{grnt}$ before the first $\texttt{req}$.

$$((\neg \texttt{grnt}) \, \mathsf{U} \, \texttt{req}) \vee \mathsf{G} \, \neg \texttt{req}$$

- Every $\texttt{req}$ is followed by a $\texttt{grnt}$ within 3 steps.

$$\mathsf{G}(\texttt{req} \rightarrow \texttt{grnt} \vee \mathsf{X} \, \texttt{grnt} \vee \mathsf{X} \, \mathsf{X} \, \texttt{grnt} \vee \mathsf{X} \, \mathsf{X} \, \mathsf{X} \, \texttt{grnt})$$

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- Every $\texttt{req}$ is eventually followed by a $\texttt{grnt}$.

$$\mathsf{G}(\texttt{req} \rightarrow \mathsf{F}\,\texttt{grnt})$$

Monitoring in Discrete Time

Let $\varphi \subseteq \Sigma^\omega$ be a property. The monitoring function

$$\mathcal{M}_\varphi \colon \Sigma^* \to \{✔, ✘, \textbf{?}\}$$

is defined as

$$\mathcal{M}_\varphi(w) = \begin{cases} ✔ & \text{if } w \cdot \mu \in \varphi \text{ for all } \mu \in \Sigma^\omega, \\ ✘ & \text{if } w \cdot \mu \notin \varphi \text{ for all } \mu \in \Sigma^\omega, \\ \textbf{?} & \text{otherwise.} \end{cases}$$

Examples

- Let $\varphi_1 = (\neg \mathtt{grnt}) \, \mathsf{U} \, \mathtt{req}((\neg \mathtt{grnt}) \, \mathsf{U} \, \mathtt{req}) \vee \mathsf{G} \, \neg \mathtt{req}$
  - $\mathcal{M}_{\varphi_1}(\emptyset) =$ **?**

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  - $\mathcal{M}_{\varphi_2}(\{\mathtt{req}\}\emptyset\{\mathtt{req}\}\{\mathtt{grnt}\}) =$ **?**
- Let $\varphi_3 = \mathsf{G}(\mathtt{req} \rightarrow \mathsf{F}\,\mathtt{grnt})$
  - $\mathcal{M}_{\varphi_3}(w) =$ **?** for all w

Büchi Automata

For algorithmic purposes, we compile LTL-defined properties into Büchi automata processing infinite words.

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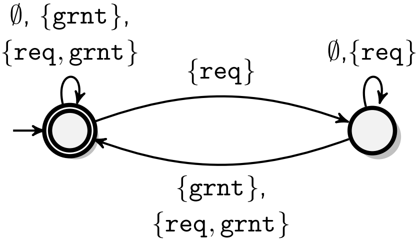
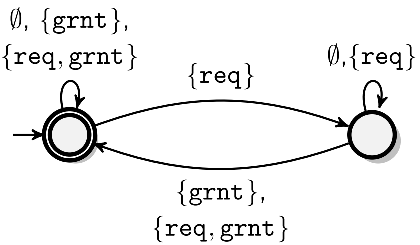
Example

A Büchi automaton $\mathcal{B}_\varphi$ for $\varphi = \mathsf{G}(\mathtt{req} \rightarrow \mathsf{F}\,\mathtt{grnt})$:



Towards an Algorithm



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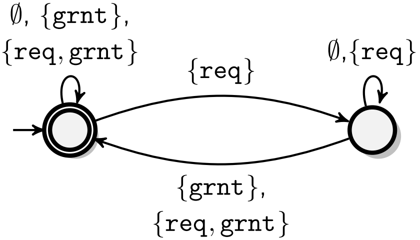

- A state q of $\mathcal{A}_\varphi$ has a nonempty language, if an accepting runs starts in q.

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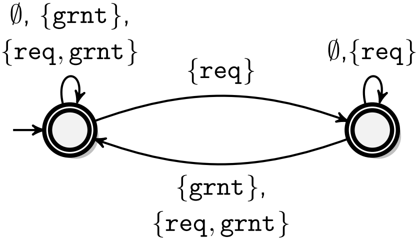

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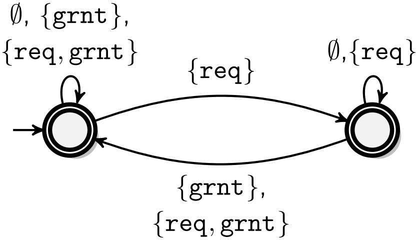
Towards an Algorithm



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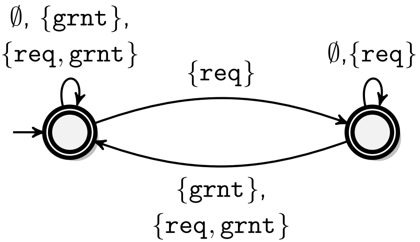

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- Then: $w \in L(\mathcal{N}_\varphi)$ iff there exists $\mu \in \Sigma^\omega$ such that $w \cdot \mu \in \varphi$.
- Equivalently: $w \notin L(\mathcal{N}_\varphi)$ iff $\mathcal{M}_\varphi(w) = $ ✘.

Complete Construction

φ

Complete Construction

$$\varphi \xrightarrow{\text{exp}} \mathcal{B}_{\varphi}$$

Complete Construction

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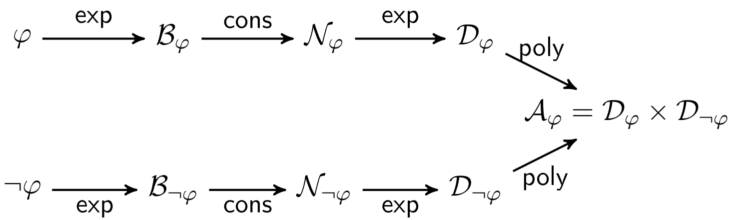

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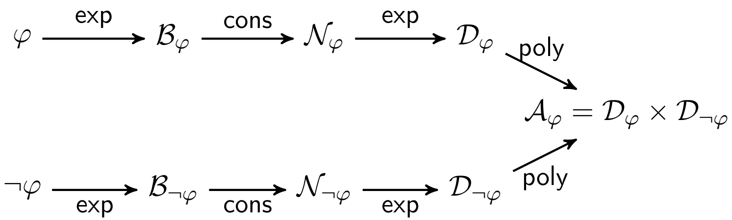
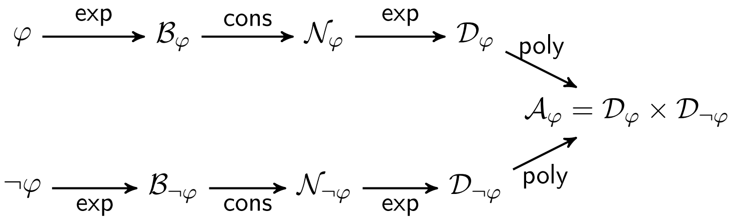


Theorem (Bauer, Leucker, Schallhart '06)

Given an LTL property φ, one can effectively construct a deterministic automaton with output implementing $\mathcal{M}_\varphi$, which has doubly-exponential size.

On-the-fly Monitoring



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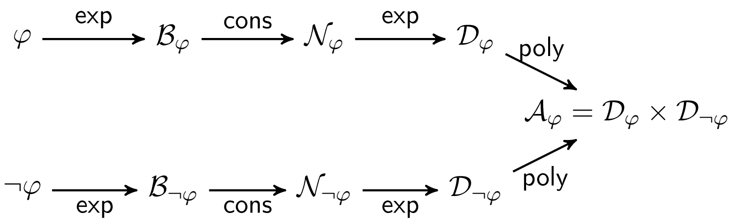

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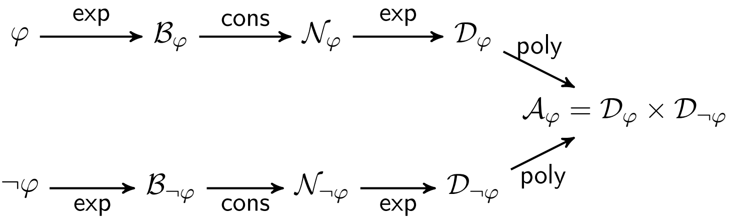

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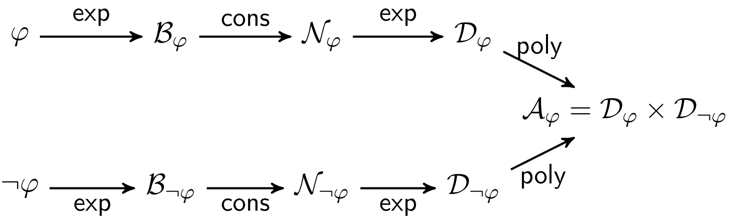

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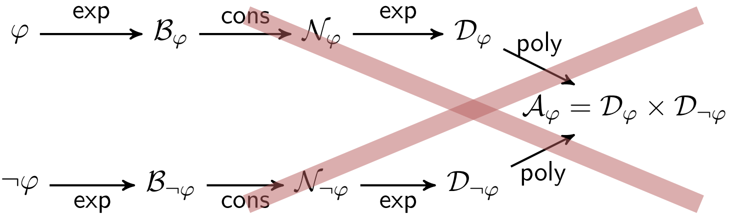

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Outline

1. Monitoring in Discrete Time

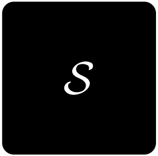
2. Monitoring in Real-Time

3. Monitoring under Delay in Real-Time

4. Conclusion

Motivation

Many systems and their specifications are real-time, not discrete-time!



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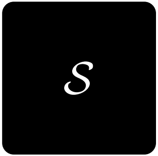

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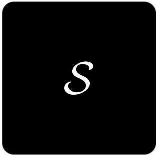

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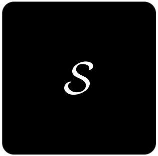

$(\texttt{init}, 0)$ $(\texttt{req}, 2)$ $(\texttt{req}, 3.7)$

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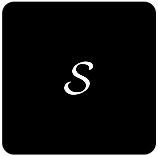

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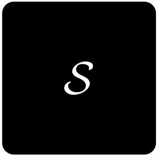

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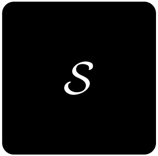

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$\mathcal{S}$

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Monitoring of TLTL properties using event-clock automata and regions.

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Bauer, Leucker, Schallhart '06:
Monitoring of TLTL properties using event-clock automata and regions.

Grosen, Kauffman, Larsen, Z. '22:
Monitoring of MITL properties using timed Büchi automata and zones.

Timed Words

- A (point-based) timed word over an alphabet Σ is a sequence

$$(\sigma_0, t_0)(\sigma_1, t_1)(\sigma_2, t_2)\cdots$$

  such that

  - $\sigma_i \in \Sigma$ for all i,
  - $t_i \in \mathbb{R}_{\geq 0}$ for all i,
  - $t_0 \leq t_1 \leq t_2 \leq \cdots$, and
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- Let $w = (\sigma_0, t_0)\cdots(\sigma_n, t_n)$, $w' = (\sigma_0, t_0)(\sigma_1, t_1)(\sigma_2, t_2)\cdots$, and $t \geq \mathrm{dur}(w)$. Then,

$$w \cdot_t w' = (\sigma_0, t_0)\cdots(\sigma_n, t_n)(\sigma_0, t_0 + t)(\sigma_1, t_1 + t)(\sigma_2, t_2 + t)\cdots$$

MITL in One Slide

Syntax

$$\varphi ::= a \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{X}_I\, \varphi \mid \varphi\, \mathsf{U}_I\, \varphi$$

where

- $a \in \mathrm{AP}$ and
- $I \subseteq \mathbb{R}_{\geq 0}$ is a nonempty interval with endpoints in $\mathbb{N} \cup \{\infty\}$.

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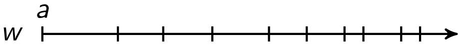
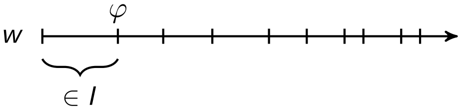
Semantics

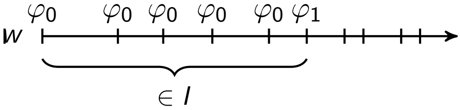
- $w \models a$:



- $w \models \mathsf{X}_I\, \varphi$:



- $w \models \varphi_0\, \mathsf{U}_I\, \varphi_1$:



Monitoring in Real Time

- Every $\mathtt{req}$ is followed by a $\mathtt{grnt}$ within 3 units of time.

$$\mathsf{G}_{[0,\infty)}(\mathtt{req} \rightarrow \mathsf{F}_{[0,3]}\,\mathtt{grnt})$$

Monitoring in Real Time

- Every `req` is followed by a `grnt` within 3 units of time.

$$G_{[0,\infty)}(\mathtt{req} \rightarrow F_{[0,3]}\,\mathtt{grnt})$$

- There is an a in the first 10 units of time and no b in the first 20 units of time.

$$F_{[0,10]}\, a \wedge G_{[0,20]}\, \neg b$$

Monitoring in Real-Time

Let $\varphi \subseteq T\Sigma^\omega$ be a property. The monitoring function

$$\mathcal{M}_\varphi \colon T\Sigma^* \to \{✔, ✘, ?\}$$

is defined as

$$\mathcal{M}_\varphi(w) = \begin{cases} ✔ & \text{if } w \cdot_{\mathrm{dur}(w)} \mu \in \varphi \text{ for all } \mu \in T\Sigma^\omega, \\ ✘ & \text{if } w \cdot_{\mathrm{dur}(w)} \mu \notin \varphi \text{ for all } \mu \in T\Sigma^\omega, \\ ? & \text{otherwise.} \end{cases}$$

We Can Do Better

Consider the property

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- $\mathcal{M}_\varphi((a,4)) =$ **?**
- $\mathcal{M}_\varphi((a,4)(a,100)) =$ ✔
- But we can give the verdict ✔ much earlier, i.e., when no further event has been observed for more than 16 units of time after the initial a at time 4.

Monitoring in Real-Time, Take 2

Let $\varphi \subseteq T\Sigma^\omega$ be a property. The monitoring function

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if $t \geq \mathrm{dur}(w)$, otherwise it is undefined.

Examples

Again, consider the property

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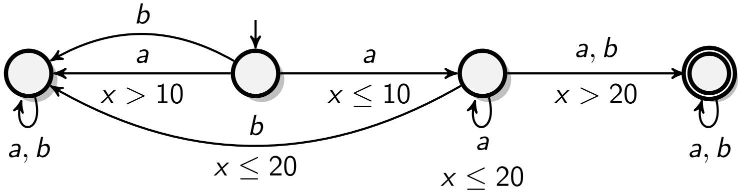
- $\mathcal{M}_\varphi((a,4),20+\varepsilon) =$ ✔ for all $\varepsilon > 0$

Timed Büchi Automata

Again, we compile MITL formulas into automata, here timed Büchi automata.

Example

A timed Büchi automaton for $F_{[0,10]}\, a \wedge G_{[0,20]}\, \neg b$:



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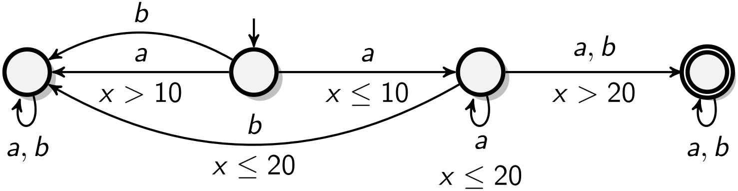

Note

States of a timed Büchi automaton are pairs of locations and clock valuations, functions mapping the clocks to $\mathbb{R}_{\geq 0}$.

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- *Otherwise, $\mathcal{M}_\varphi(w, t) =$ **?**.*

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  where x_1, x_2 are clocks, $\sim \in \{<, \leq, =, \geq, >\}$, and $n \in \mathbb{Q}_{\geq 0}$.

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- A zone describes a convex set of clock valuations.
- A symbolic state is a pair of a location and a zone.

Zone-based Algorithms

There are zone-based algorithms computing

- the set of nonempty language states of a timed Büchi automaton, and
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Thus, $\mathcal{M}_{\varphi}$ can be effectively computed for MITL properties φ.

MoniTAal

```
tmgr@tmgr-pc:~/repos/MoniTAal/build-bundle/src/monitaal-bin$ ./MoniTAal-bin -p p
ositive ~/monitoring/simple-gearcontroller.xml -n negative ~/monitoring/simple-g
earcontroller.xml
Interactive monitor (respond with "q" to quit)
Next event: @3000 GearReq
Next event: @3900 GearChanged
Next event: @4200 GearReq
Next event: @5200 GearChanged
Monitoring ended, verdict is: NEGATIVE
Monitored 4 events
```

https://github.com/DEIS-Tools/MoniTAal

Outline

1. Monitoring in Discrete Time

2. Monitoring in Real-Time

3. **Monitoring under Delay in Real-Time**

4. Conclusion

Motivation

So far we have assumed that the monitor has direct access to the observations. This is not always realistic!

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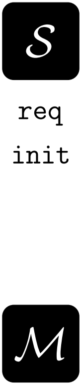

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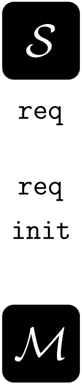

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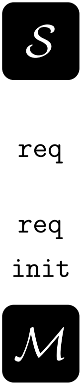

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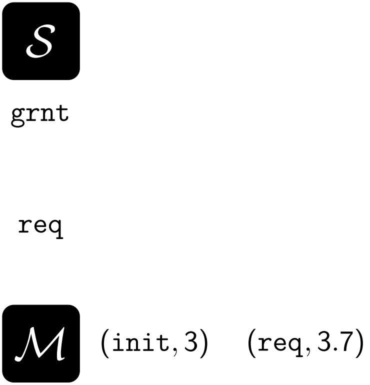

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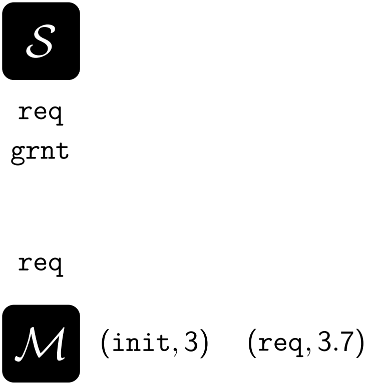

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Delay = Latency + Jitter

Consider a setting where the observations arrive at the monitor via a channel with some delay, and timestamps are assigned at arrival at the monitor.

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Often, bounds are known on these quantities: A delay set has the form

$$\mathcal{D} = \{(\delta, \varepsilon) \mid \delta, \varepsilon \in \mathbb{R}_{\geq 0}\}$$

Intuition

Consider the property

$$\mathsf{F}_{[0,10]}\, a \wedge \mathsf{G}_{[0,20]}\, \neg b$$

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$$(a, 17.3)(b, 27.1)$$

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Thus, we can conclude, even under unknown latency, that the property is violated.

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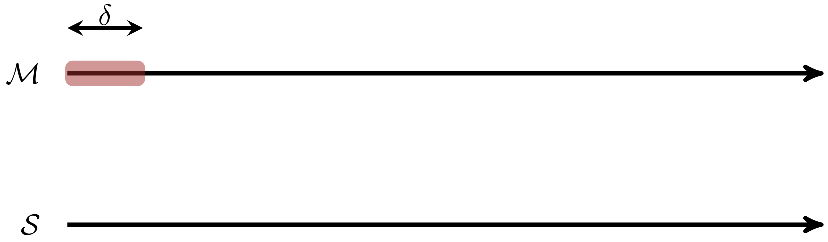

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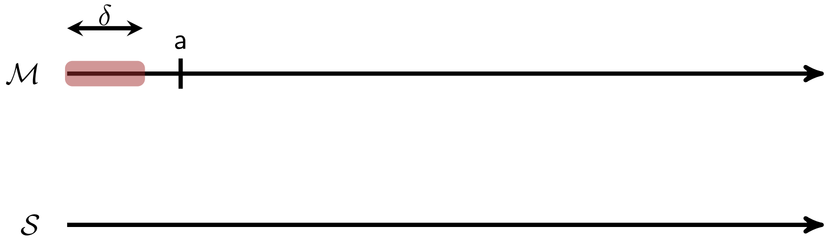

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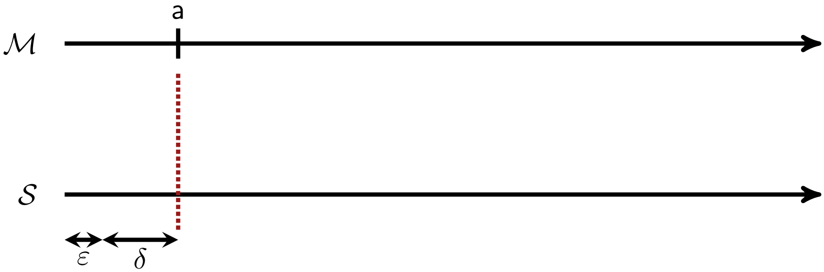

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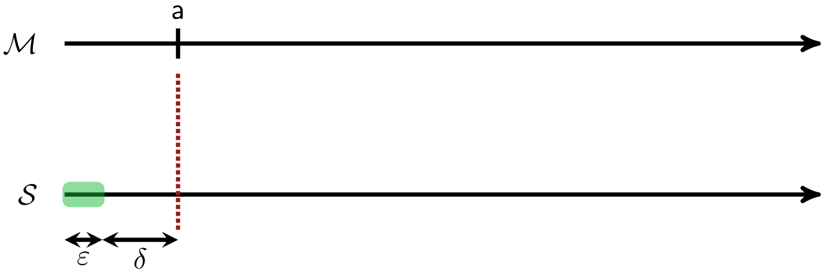

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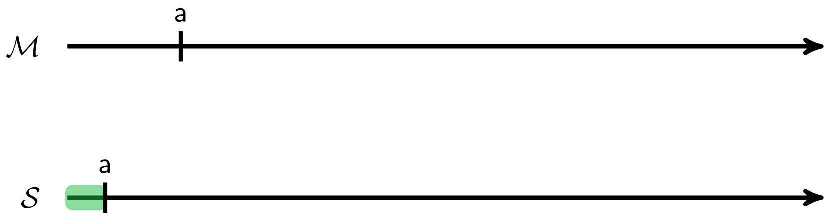

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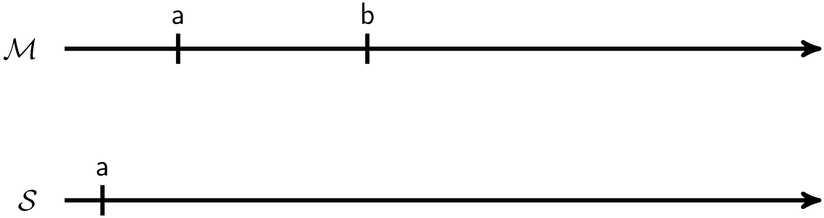

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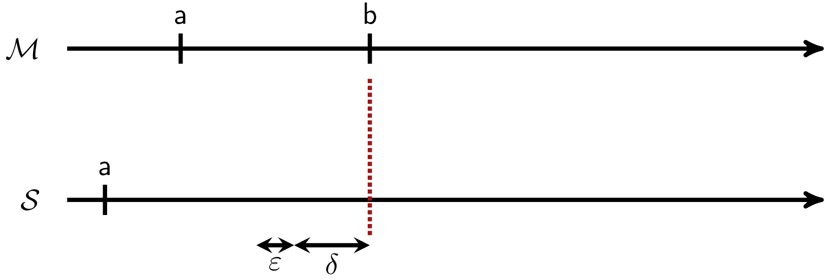

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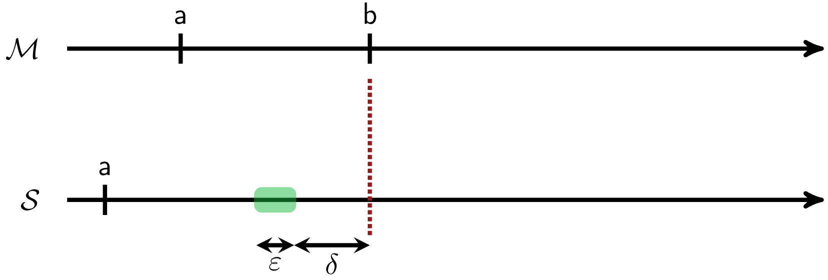

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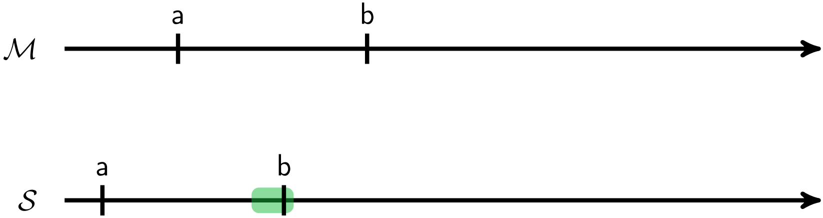

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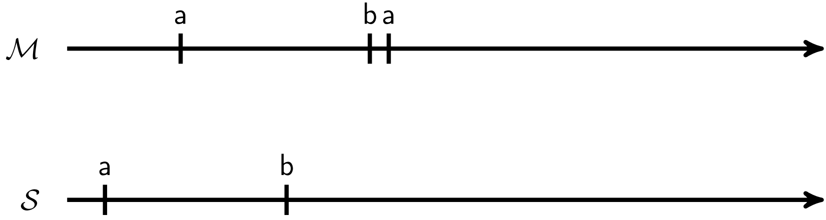

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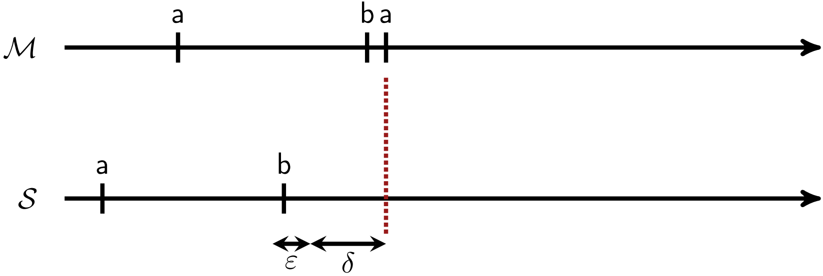

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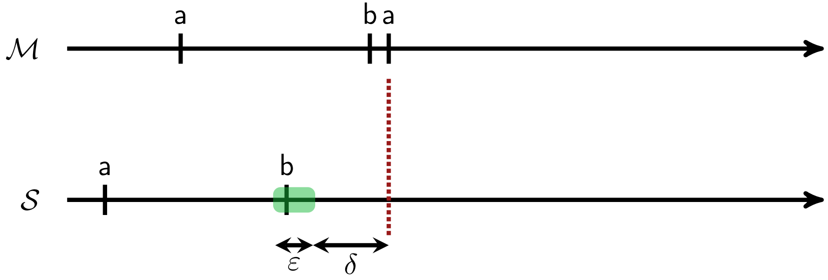

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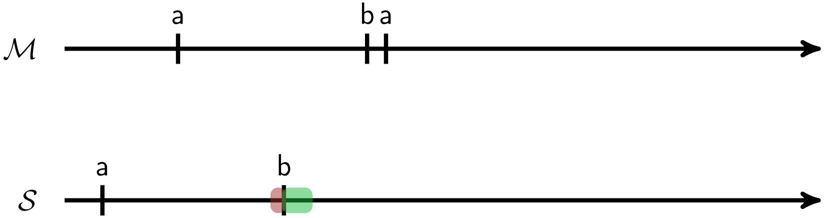

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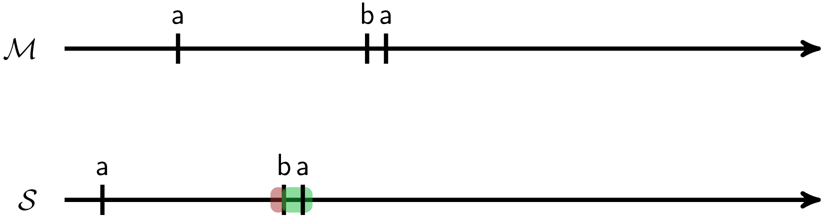

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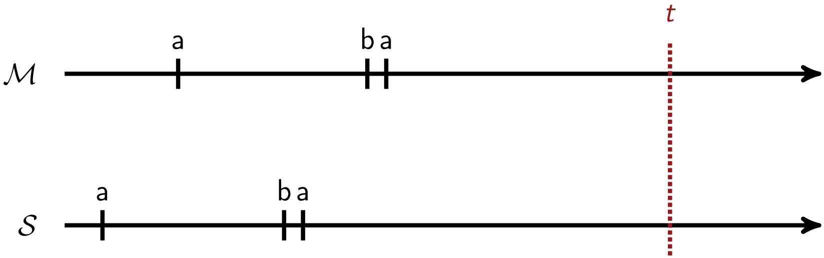

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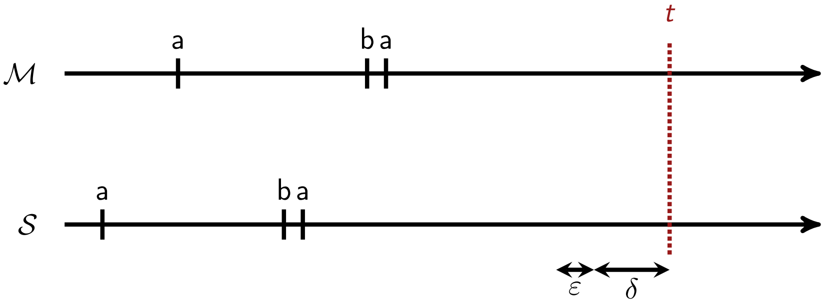

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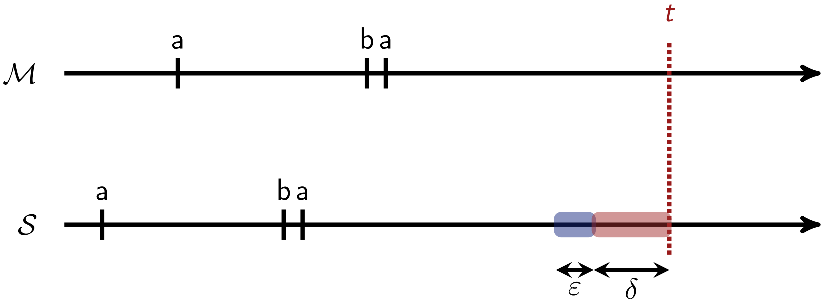

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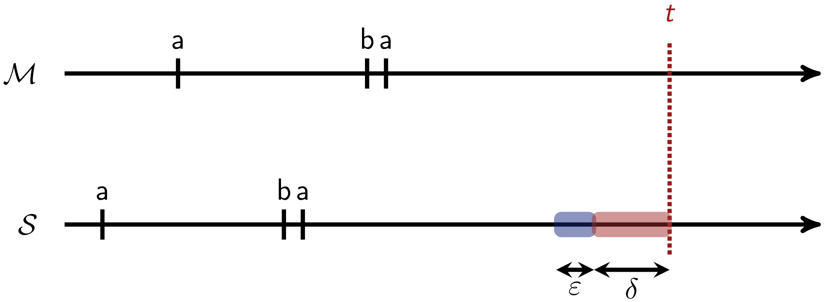

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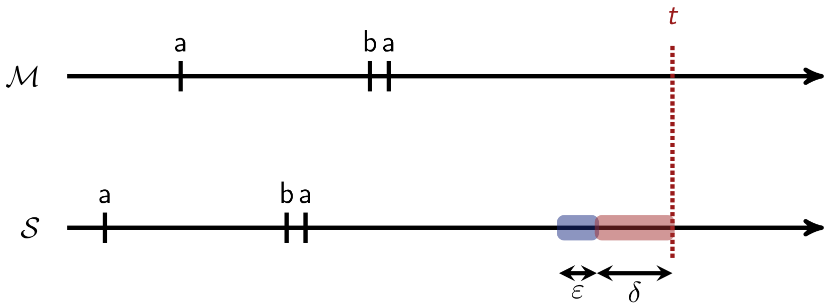

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- So, a ground truth for an observation at time t has duration $t - (\delta + \varepsilon)$.

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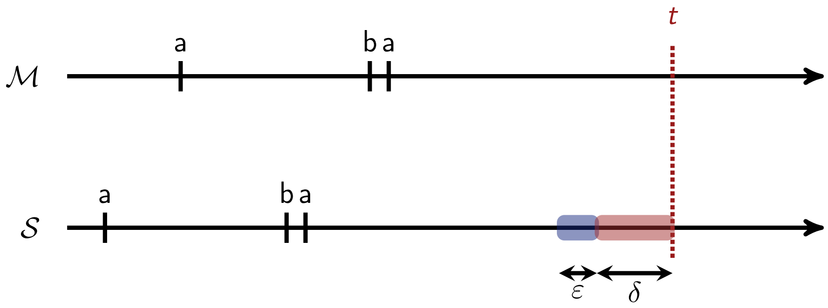

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- So, a ground truth for an observation at time t has duration $t - (\delta + \varepsilon)$.
- Disclaimer: the full definition is more complicated, as we disallow overtaking!

Monitoring under Delay

Let $\varphi \subseteq T\Sigma^\omega$ be a property and $\mathcal{D}$ a delay set. The monitoring function

$$\mathcal{M}_\varphi^\mathcal{D} \colon T\Sigma^* \times \mathbb{R}_{\geq 0} \to \{✔, ✘, ?\}$$

is defined as

$$\mathcal{M}_\varphi^\mathcal{D}(w, t) = \begin{cases} ✔ & \text{if } w^* \cdot_{t-(\delta+\varepsilon)} \mu \in \varphi \text{ for all } (\delta, \varepsilon) \in \mathcal{D}, \\ & \text{all } w^* \in \mathrm{GT}_\mathcal{D}(w, t), \text{ and all } \mu \in T\Sigma^\omega, \\ ✘ & \text{if } w^* \cdot_{t-(\delta+\varepsilon)} \mu \notin \varphi \text{ for all } (\delta, \varepsilon) \in \mathcal{D}, \\ & \text{all } w^* \in \mathrm{GT}_\mathcal{D}(w, t), \text{ and all } \mu \in T\Sigma^\omega, \\ ? & \text{otherwise,} \end{cases}$$

if w is a timed word compatible with some $(\delta, \varepsilon) \in \mathcal{D}$ and if $t \geq \mathrm{dur}(w)$, otherwise it is undefined.

Monotonicity

Lemma

Let $\varphi \subseteq T\Sigma^\omega$, let $\mathcal{D} \subseteq \mathcal{D}'$ be delay sets, let w be compatible with some $(\delta, \varepsilon) \in \mathcal{D}$, and let $t \geq \mathrm{dur}(w)$. Then,

- $\mathcal{M}_\varphi^{\mathcal{D}'}(w, t) = ✔$ *implies* $\mathcal{M}_\varphi^{\mathcal{D}}(w, t) = ✔$*, and*
- $\mathcal{M}_\varphi^{\mathcal{D}'}(w, t) = ✘$ *implies* $\mathcal{M}_\varphi^{\mathcal{D}}(w, t) = ✘$*.*

Consistent Delays

$$\Delta_{\mathcal{D}}(\varphi, w, t) = \{(\delta, \varepsilon) \in \mathcal{D} \mid \exists w^* \in \mathrm{GT}_{(\delta,\varepsilon)}(w, t) \\ \exists \mu \in T\Sigma^\omega \text{ s.t. } w^* \cdot_{t-(\delta+\varepsilon)} \mu \in \varphi\}.$$

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Lemma

Given $\varphi \subseteq T\Sigma^\omega$, a set $\mathcal{D}$ of delays, w compatible with some $(\delta, \varepsilon) \in \mathcal{D}$, and $t \geq \mathrm{dur}(w)$, we have

1. $\Delta_{\mathcal{D}}(\varphi, w, t) = \emptyset$ *if and only if* $\mathcal{M}_\varphi^{\mathcal{D}}(w, t) =$ ✘, *and*
2. $\Delta_{\mathcal{D}}(\overline{\varphi}, w, t) = \emptyset$ *if and only if* $\mathcal{M}_\varphi^{\mathcal{D}}(w, t) =$ ✔.

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Lemma

Let $(w_1, t_1) \sqsubseteq (w_2, t_2)$, $t_1 \geq \mathrm{dur}(w_1)$, $t_2 \geq \mathrm{dur}(w_2)$, and $t_2 \geq t_1$. Then, $\Delta_{\mathcal{D}}(\varphi, w_1, t_1) \supseteq \Delta_{\mathcal{D}}(\varphi, w_2, t_2)$.

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Lemma

Let $\varphi \subseteq T\Sigma^\omega$, $\mathcal{D}$ be a set of delays, and $w = (\sigma_0, t_0), \ldots, (\sigma_m, t_m)$ nonempty and compatible with some $(\delta, \varepsilon) \in \mathcal{D}$. Then, for all $t \geq \mathrm{dur}(w)$,

1. *$\Delta_{\mathcal{D}}(\varphi, w, t) \subsetneq \{(\delta, \varepsilon) \in \mathcal{D} \mid \delta \leq t_0\}$ implies there is no $w' \in T\Sigma^*$ such that $\mathcal{M}_\varphi^{\mathcal{D}}(w \cdot_t w', t') = \checkmark$ for any $t' \geq t + \mathrm{dur}(w')$, and*
2. *$\Delta_{\mathcal{D}}(\overline{\varphi}, w, t) \subsetneq \{(\delta, \varepsilon) \in \mathcal{D} \mid \delta \leq t_0\}$ implies there is no $w' \in T\Sigma^*$ such that $\mathcal{M}_\varphi^{\mathcal{D}}(w \cdot_t w', t') = \times$ for any $t' \geq t + \mathrm{dur}(w')$.*

Delayed Monitoring via Automata

Let $\mathcal{R}_{\mathcal{B},\mathcal{D}}(w, t)$ be the set of states of $\mathcal{A}$ reachable by processing a $w^* \in \mathrm{GT}_{\mathcal{D}}(w, t)$ and then delaying $t - (\delta + \varepsilon) - \mathrm{dur}(w^*)$.

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Lemma (Grosen, Fränzle, Larsen, Z. '24)

- $\mathcal{R}_{\mathcal{B}_{\neg\varphi},\mathcal{D}}(w,t)$ *does not contain a state with nonempty language iff* $\mathcal{M}_{\varphi}(w,t)^{\mathcal{D}} =$ ✔.
- $\mathcal{R}_{\mathcal{B}_{\varphi},\mathcal{D}}(w,t)$ *does not contain a state with nonempty language iff* $\mathcal{M}_{\varphi}(w,t)^{\mathcal{D}} =$ ✘.
- *Otherwise,* $\mathcal{M}_{\varphi}^{\mathcal{D}}(w,t) =$ **?**.

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- *Otherwise, $\mathcal{M}_{\varphi}^{\mathcal{D}}(w, t) =$ **?**.*

Lemma (Grosen, Fränzle, Larsen, Z. '24)

Fix $\mathcal{D} = \{(\delta, \varepsilon) \mid \delta \in [\ell, u]\}$ for some $l, u, \varepsilon \in \mathbb{Q}_{\geq 0}$. There is a zone-based algorithm computing $\mathcal{R}_{\mathcal{B},\mathcal{D}}(w, t)$ for a given timed Büchi automaton $\mathcal{B}$, given finite timed word w, and $t \geq \mathrm{dur}(w)$.

Computing Consistent Delays

Recall

$$\Delta_{\mathcal{D}}(\varphi, w, t) = \{(\delta, \varepsilon) \in \mathcal{D} \mid \exists w^* \in \mathrm{GT}_{(\delta,\varepsilon)}(w, t) \\ \exists \mu \in T\Sigma^\omega \text{ s.t. } w^* \cdot_{t-(\delta+\varepsilon)} \mu \in \varphi\}.$$

- Our algorithm extends zones by clocks encoding the latency values that are consistent with the observation processed so far.

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- Our algorithm extends zones by clocks encoding the latency values that are consistent with the observation processed so far.
- Hence, we are computing $\Delta_{\mathcal{D}}(\varphi, w, t)$ on-the-fly (recall that ε is fixed).

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We have presented a comprehensive framework for monitoring real-time systems using zone-based algorithms for timed automata.

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- All these have been implemented in the tool MoniTAal.

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- **Friday @CONCUR**: Monitorability of real-time properties.