Optimally Resilient Strategies in Pushdown Safety Games

Joint work with Daniel Neider (MPI-SWS) and Patrick Totzke (Liverpool)
Artwork by Paulina Zimmermann

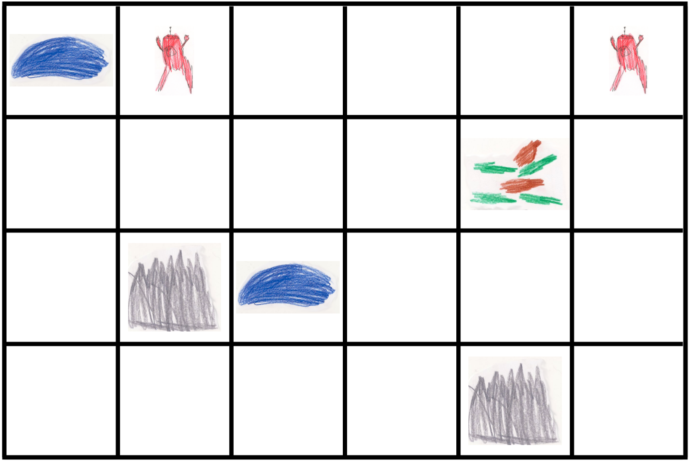
Martin Zimmermann

University of Liverpool

August 2020
MFCS 2020

Reactive Synthesis

Automatically generate correct-by-construction systems.



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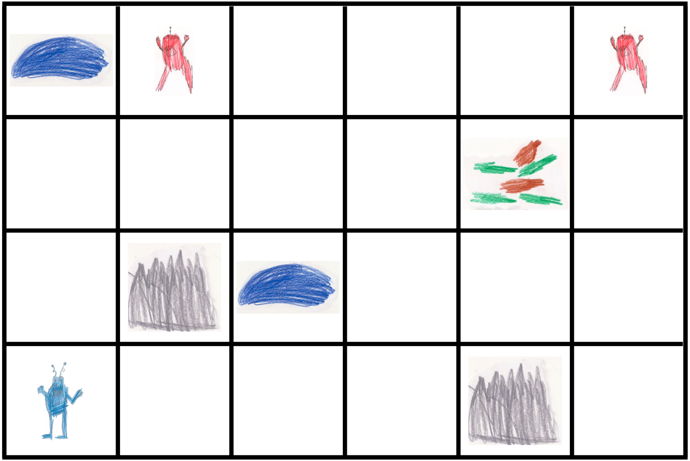

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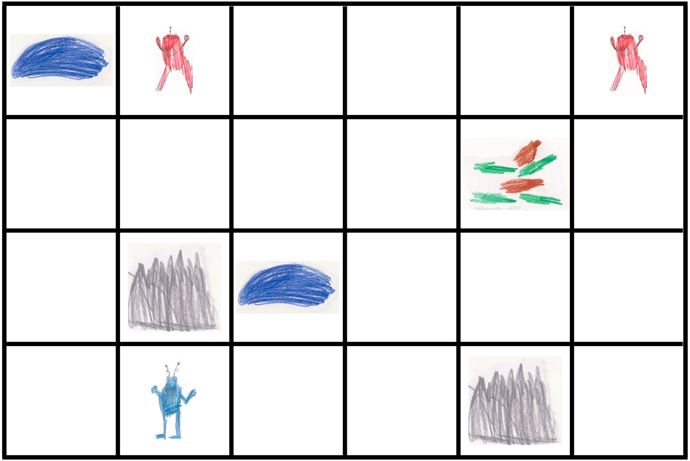

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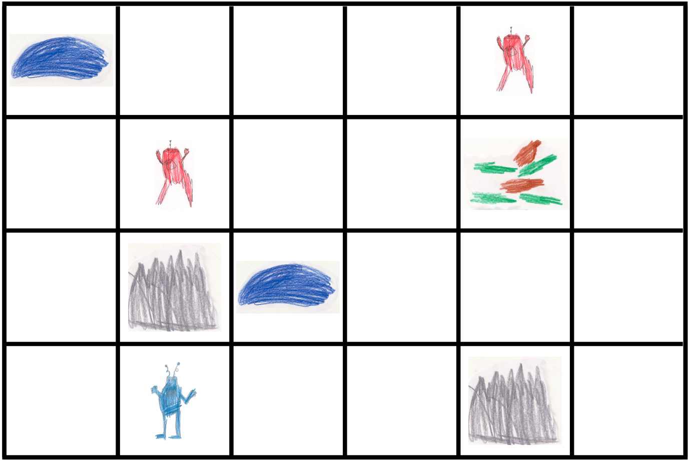

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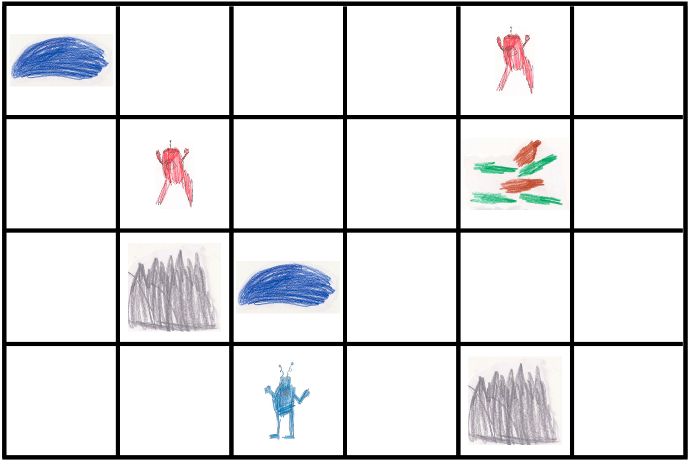

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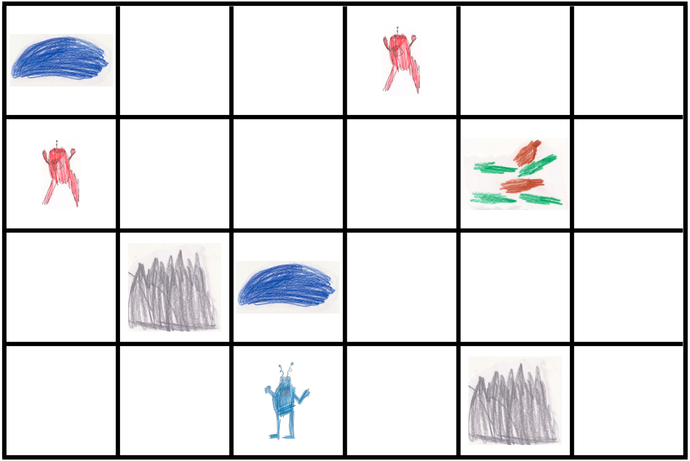

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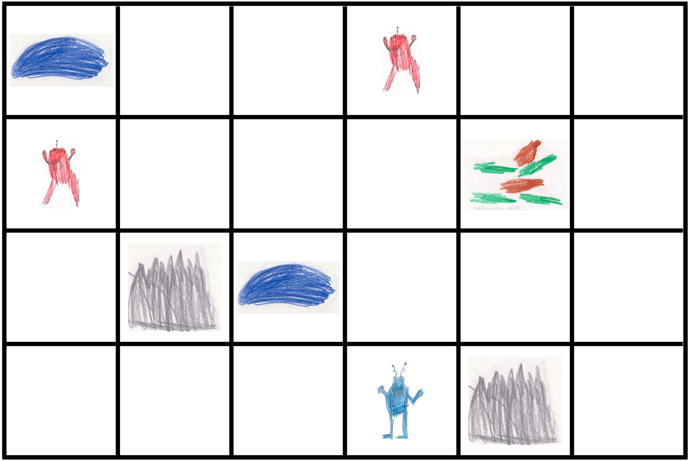

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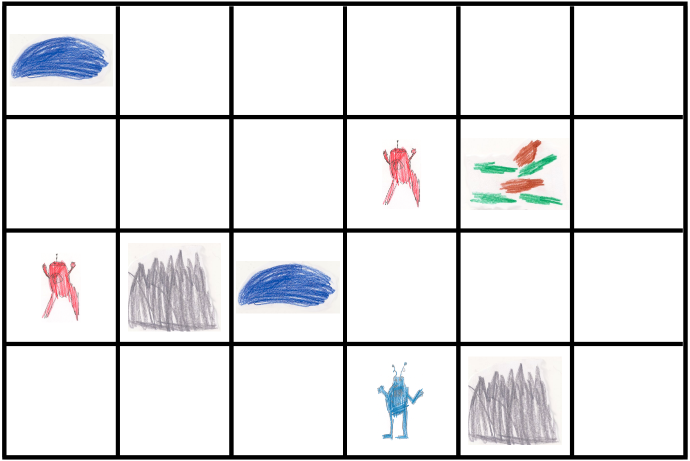

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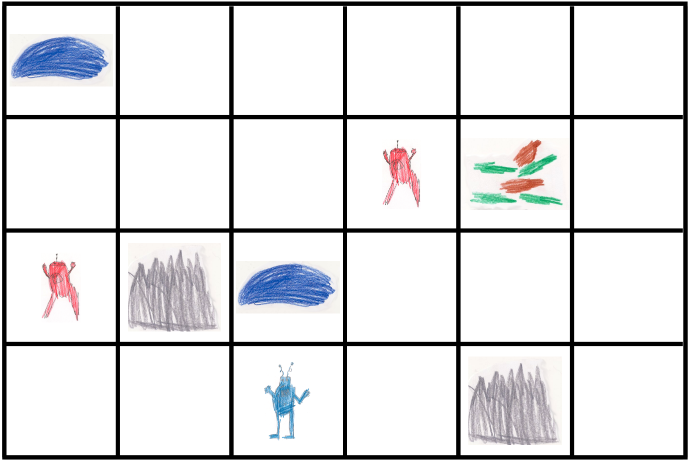

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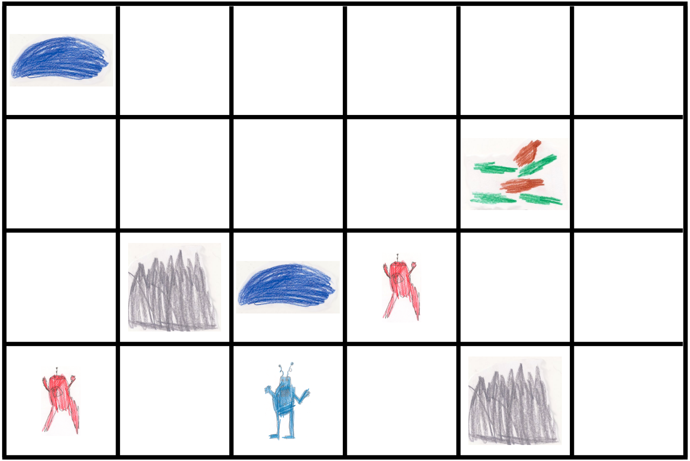

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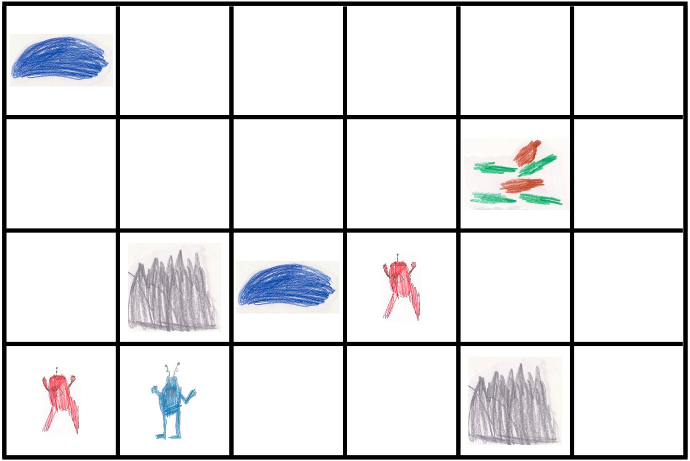

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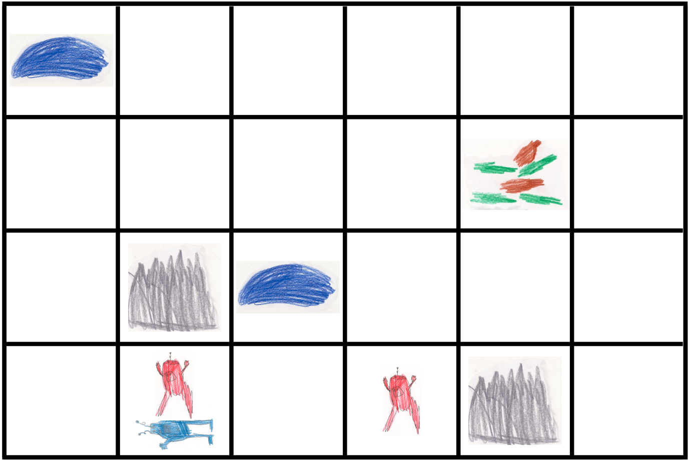
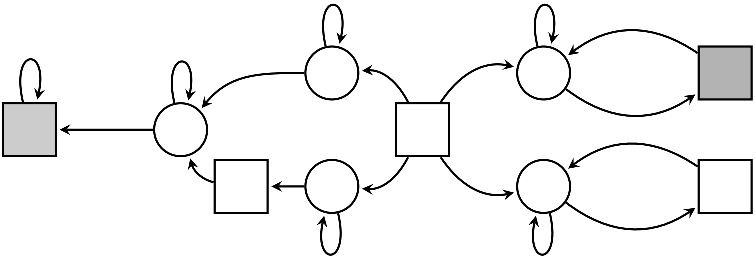


Reactive Synthesis

Automatically generate correct-by-construction systems.

- Model the interaction between a system and its environment by an infinite-duration zero-sum game on graph. The winning condition captures a specification of the system.
- A winning strategy for the system player corresponds to an implementation satisfying the system specification.



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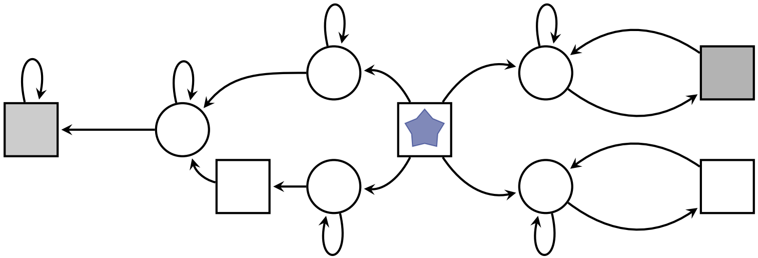

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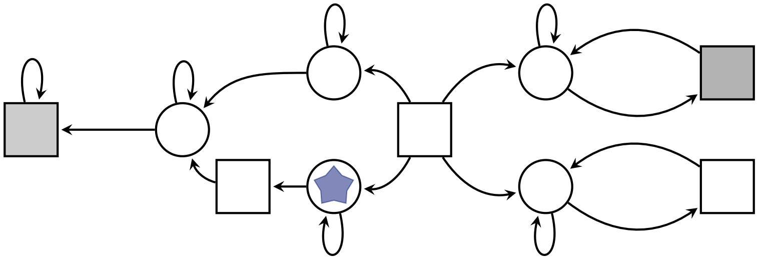

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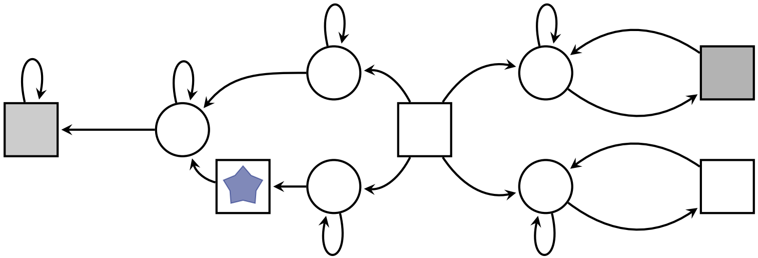

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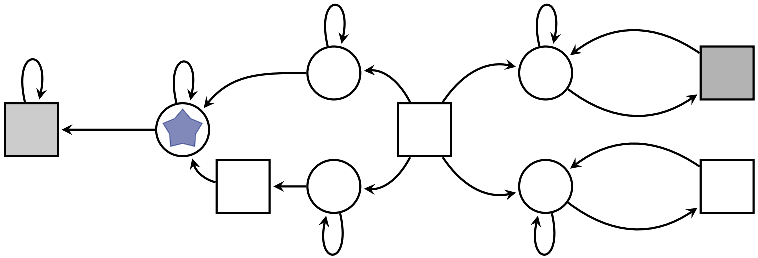

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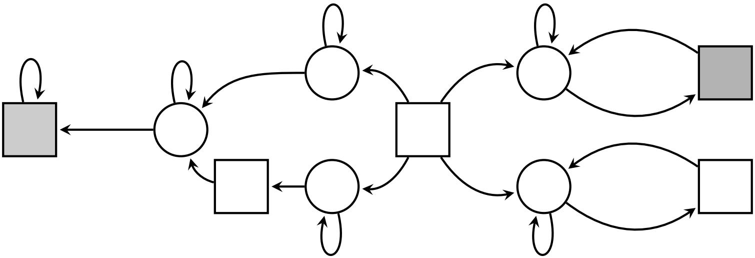

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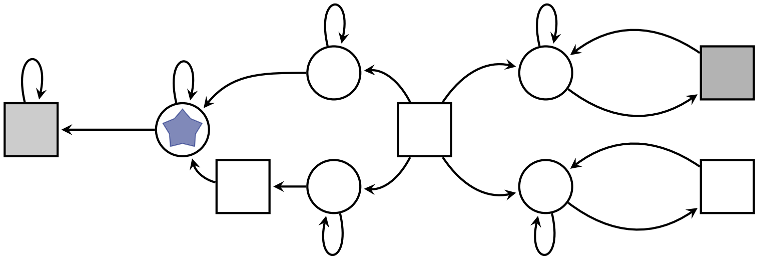

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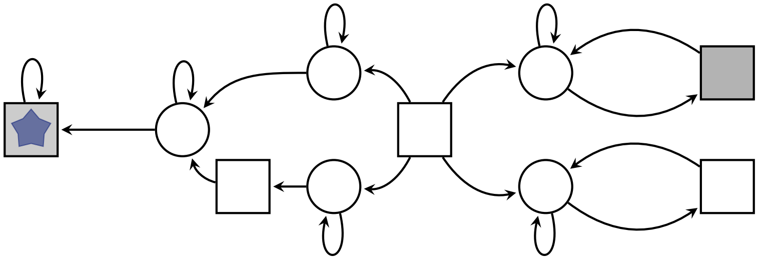
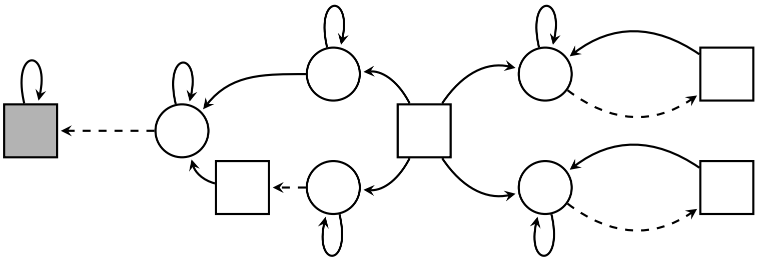


Resilience in Safety Games

Dallal, Tabuada and Neider: Add disturbances edges to model non-antagonistic external influences.



Question: How many disturbances make the system player lose?

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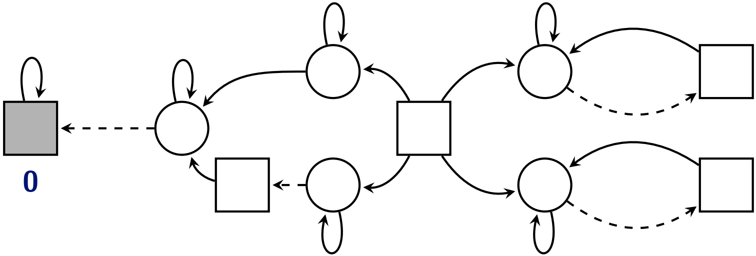

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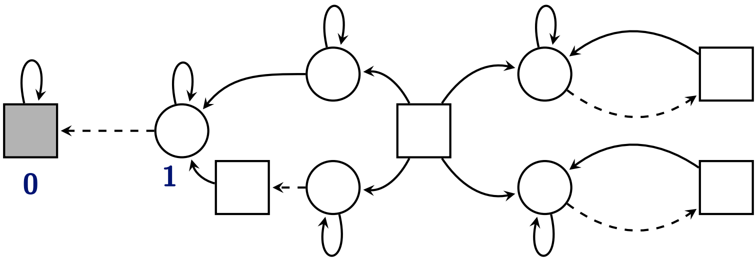

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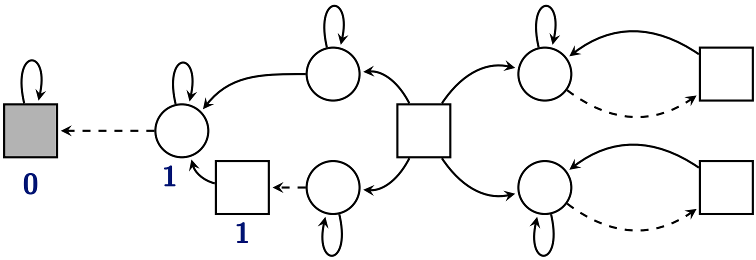

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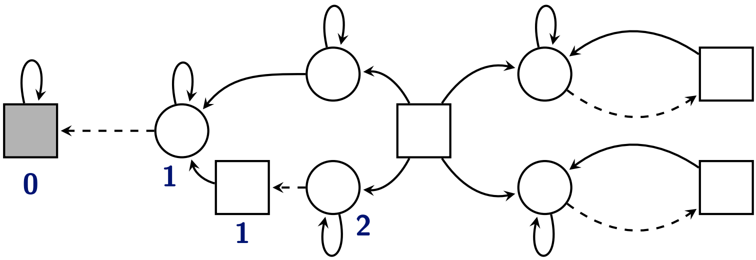

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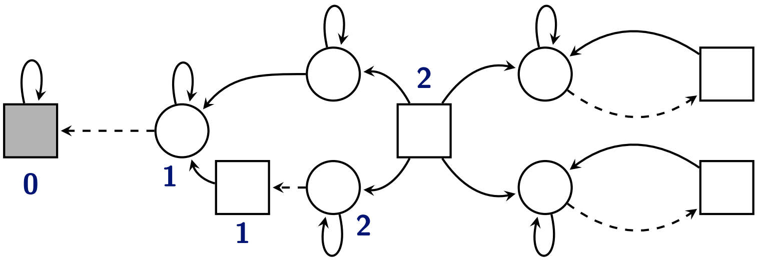

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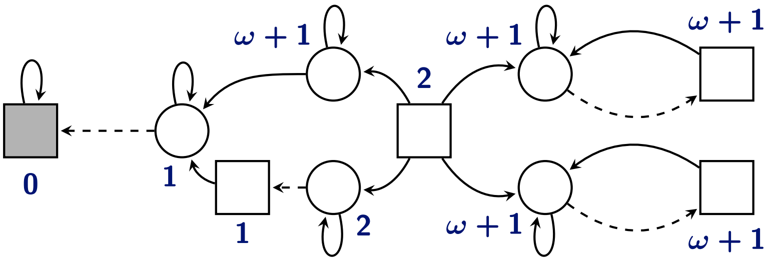

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Resilience in Safety Games

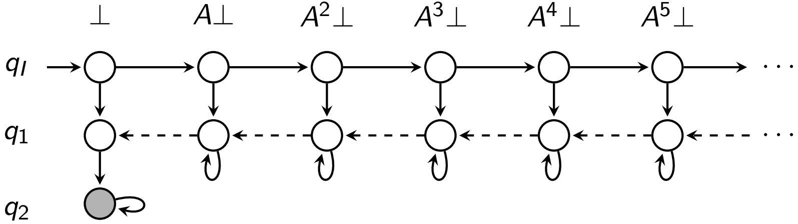
Dallal, Tabuada and Neider: Add disturbances edges to model non-antagonistic external influences.

Theorem (Dallal, Neider & Tabuada, 2016)

- *A safety game with n vertices has resilience values in $\{0, \cdots, n-1\} \cup \{\omega + 1\}$.*
- *The resilience values and an optimally resilient strategy can be computed in polynomial time.*

Systems with Infinite State Space

- Pushdown graphs are configuration graphs of pushdown automata.
- One-counter automata are pushdown automata with a single stack symbol (that can still test the stack for emptiness).



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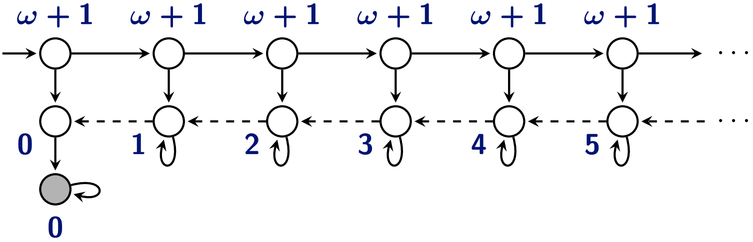

Building Blocks for an Algorithm

Theorem

- *A pushdown safety game has resilience values in* $\{0, 1, 2, \cdots\} \cup \{\omega + 1\}$.
- *An optimally resilient strategy always exists.*

Lemma

The following problem is in $\textsc{ExpTime}$*: "Given a pushdown safety game* $\mathcal{G}$ *with initial vertex* v_I*, is* $r(v_I) = \omega + 1$*?".*

Note

$\textsc{PSpace}$ for one-counter safety games.

Building Blocks for an Algorithm

Theorem

- *A pushdown safety game has resilience values in* $\{0, 1, 2, \cdots\} \cup \{\omega + 1\}$.
- *An optimally resilient strategy always exists.*

Lemma

The following problem is in $2\textsc{ExpTime}$*: "Given a pushdown safety game* $\mathcal{G}$ *with initial vertex* v_I *and* $k \in \omega$ *(encoded in binary), is* $r(v_I) = k$*?".*

Note

$\textsc{ExpSpace}$ for one-counter safety games.

A Naive Algorithm

```
if r(v_I) = ω + 1 then
    return ω + 1
k = 0
while true do
    if r(v_I) = k then
        return k
    else
        k = k+1
```

- The algorithm terminates, as the only possible resilience values are $\omega + 1$ or some $k \in \omega$.
- To obtain an upper bound on the running time, we need an upper bound on the resilience value of the initial vertex.

Upper Bounds on Resilience Values

Note that resilience values can be unbounded. Nevertheless, we can bound the resilience value of the initial vertex.

For a pushdown automaton $\mathcal{P}$ with n states and s stack symbols, define

$$b(\mathcal{P}) = n \cdot h(\mathcal{P}) \cdot s^{h(\mathcal{P})}$$

with

$$h(\mathcal{P}) = n \cdot s \cdot 2^{n+1} + 1$$

Lemma

Let $\mathcal{G}$ be a pushdown safety game with initial vertex v_I. If $r(v_I) \neq \omega + 1$, then $r(v_I) < b(\mathcal{P})$, where $\mathcal{P}$ is the automaton underlying $\mathcal{G}$.

An Improved Algorithm

if $r(v_I) = \omega + 1$ **then**
  return $\omega + 1$
for $k = 0$ **to** $b(\mathcal{P})$ **do**
  if $r(v_I) = k$ **then**
    return k

An Improved Algorithm

```
if r(v_I) = ω + 1 then
    return ω + 1
for k = 0 to b(P) do
    if r(v_I) = k then
        return k
```

Theorem

The following problem can be solved in triply-exponential time: "Given a pushdown safety game $\mathcal{G}$ with initial vertex v_I, determine the resilience value of v_I". If yes, an $r(v_I)$-resilient strategy from v_I can be computed in triply-exponential time.

An Improved Algorithm

```
if r(v_I) = ω + 1 then
    return ω + 1
for k = 0 to b(P) do
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        return k
```

Theorem

The following problem can be solved in polynomial space: "Given a one-counter safety game $\mathcal{G}$ with initial vertex v_I, determine the resilience value of v_I".

Note

No strategy computed.

Conclusion

Also in the paper/arXiv version:

1. An outlook on resilient strategies in pushdown reachability games (new resilience values appear).
2. A new result on optimal strategies in one-counter reachability games (without disturbance edges).
3. Lower bounds on computational complexity and on the resilience value of the initial vertex.

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Also in the paper/arXiv version:

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3. Lower bounds on computational complexity and on the resilience value of the initial vertex.

Open problems:

1. Extension to more expressive winning conditions.
2. Better complexity bounds for pushdown safety games via saturation.
3. Computing optimally resilient strategies for one-counter safety games in polynomial space.

Thank you for watching.

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