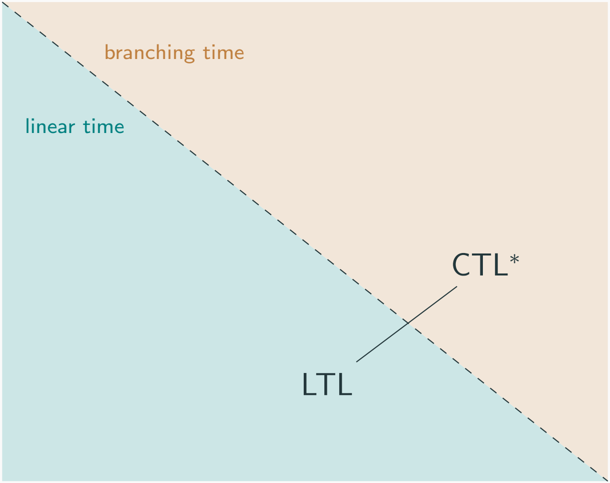
How undecidable are HyperLTL and HyperCTL*?

Marie Fortin, Louwe Kuijer, Patrick Totzke, Martin Zimmermann

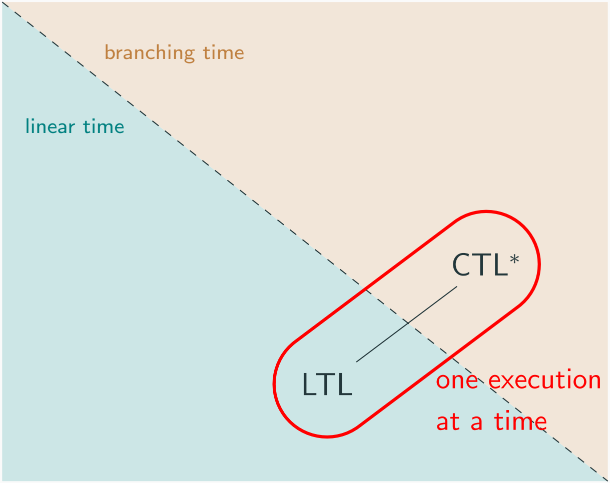
Highlights of Logic, Games and Automata 2021

Based on "HyperLTL Satisfiability Is Σ_1^1-Complete, HyperCTL* Satisfiability Is Σ_1^2-Complete [MFCS 2021]"

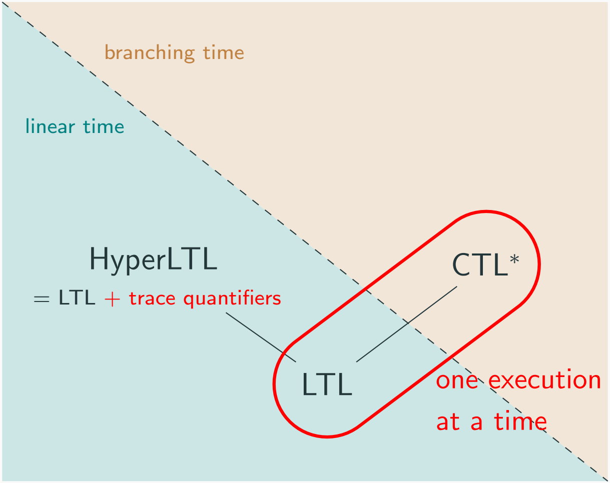
HyperLTL and HyperCTL*



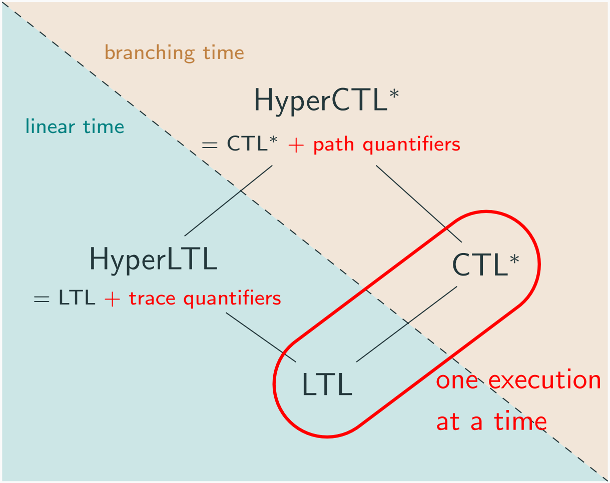
HyperLTL and HyperCTL*



HyperLTL and HyperCTL*



HyperLTL and HyperCTL*



Example

$$\forall \pi . \forall \pi' .\ \mathsf{G}(\texttt{in\_public}_{\pi} \leftrightarrow \texttt{in\_public}_{\pi'}) \\ \rightarrow \mathsf{G}(\texttt{out\_public}_{\pi} \leftrightarrow \texttt{out\_public}_{\pi'})$$

"Any two traces with the same public input have the same public output"

Example

$$\forall\pi.\forall\pi'.\ \mathsf{G}(\texttt{in\_public}_{\pi} \leftrightarrow \texttt{in\_public}_{\pi'}) \rightarrow \mathsf{G}(\texttt{out\_public}_{\pi} \leftrightarrow \texttt{out\_public}_{\pi'})$$

"Any two traces with the same public input have the same public output"

HyperLTL vs. HyperCTL*

- HyperLTL: prenex normal form, models = sets of traces
- HyperCTL*: no prenex normal form, models = transition systems/computation trees

Undecidability

HyperLTL and HyperCTL* model-checking problems are decidable ...

... but their satisfiability problems are undecidable.

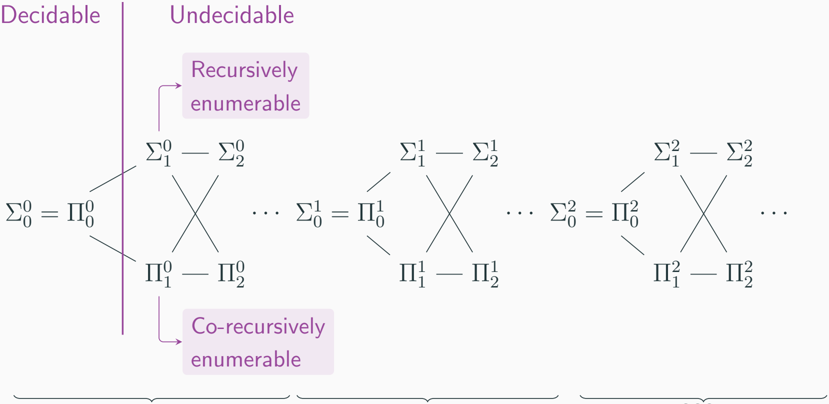
Undecidability

HyperLTL and HyperCTL* model-checking problems are decidable . . .

. . . but their satisfiability problems are undecidable.

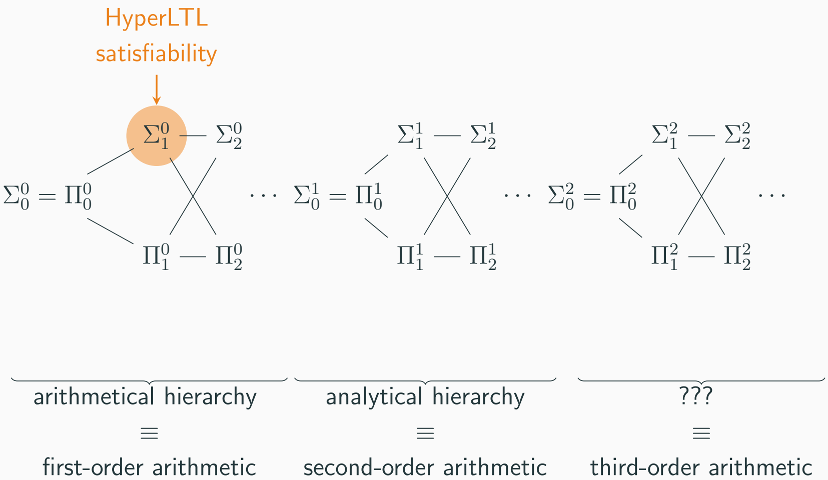
How undecidable is HyperLTL or HyperCTL* satisfiability?

Levels of Undecidability



arithmetical hierarchy	analytical hierarchy	???
$\equiv$	$\equiv$	$\equiv$
first-order arithmetic	second-order arithmetic	third-order arithmetic

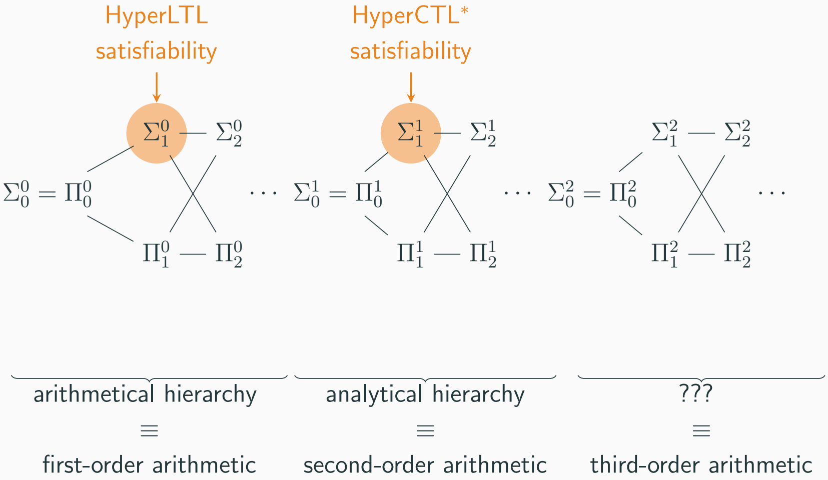
Levels of Undecidability



Theorem

HyperLTL satisfiability is Σ_1^0-hard. [Finkbeiner, Hahn 2016]

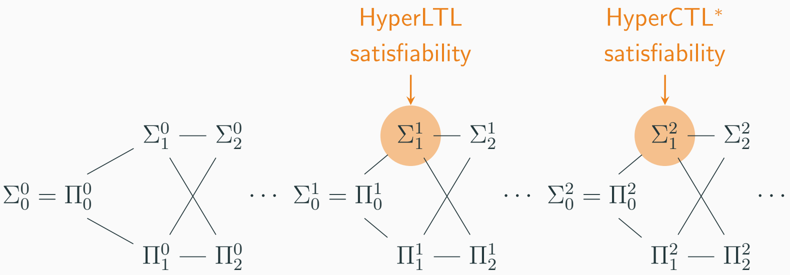
Levels of Undecidability



Theorem

HyperCTL* satisfiability is Σ_1^1-hard. [Clarkson et al. 2014]

Levels of Undecidability



arithmetical hierarchy	analytical hierarchy	???
$\equiv$	$\equiv$	$\equiv$
first-order arithmetic	second-order arithmetic	third-order arithmetic

Main Results

Theorem

HyperLTL satisfiability is Σ_1^1-complete.

- Satisfiable HyperLTL formulas have countable models, some have only infinite models. [Finkbeiner, Z. 2017]

Main Results

Theorem

HyperLTL satisfiability is Σ_1^1-complete.

- Satisfiable HyperLTL formulas have countable models, some have only infinite models. [Finkbeiner, Z. 2017]

Theorem

HyperCTL* satisfiability is Σ_1^2-complete.

- Satisfiable HyperCTL* formulas have models of cardinality $\mathfrak{c} = |2^{\mathbb{N}}|$, some have only uncountable models.