A Bit of Nondeterminism Makes Pushdown Automata Expressive and Succinct

Joint work with Shibashis Guha, Ismaël Jecker, and Karoliina Lehtinen

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Highlights Conference

$$\{a^j \$ b^k \mid k = j \text{ or } k = 2j\}$$

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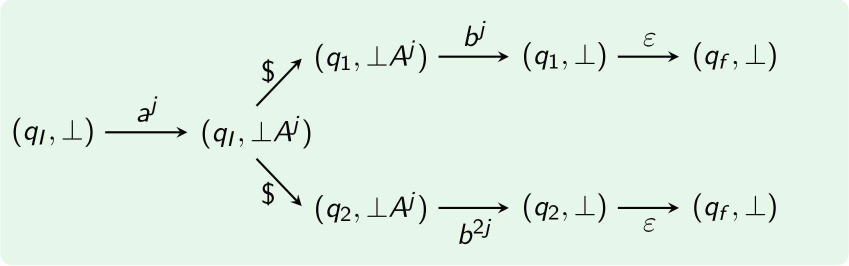
$$(q_I, \bot) \xrightarrow{a^j} (q_I, \bot A^j)$$

$\{a^j\$b^k \mid k = j \text{ or } k = 2j\}$

$$(q_I, \bot) \xrightarrow{a^j} (q_I, \bot A^j) \xrightarrow{\$} (q_1, \bot A^j) \xrightarrow{b^j} (q_1, \bot) \xrightarrow{\varepsilon} (q_f, \bot)$$

$\{a^j\$b^k \mid k = j \text{ or } k = 2j\}$



$$\{a^i\$b^j\$c^k \mid k \leq i \text{ or } k \leq j\}$$

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$$(q_I, \bot) \xrightarrow{a^i\$b^j} (q_I, \bot A^i\$B^j)$$

$\{a^i\$b^j\$c^k \mid k \leq i \text{ or } k \leq j\}$

$$(q_I, \perp) \xrightarrow{a^i\$b^j} (q_I, \perp A^i\$B^j) \xrightarrow{\$} (q_1, \perp A^i\$B^j) \xrightarrow{c^k} (q_1, A^j\$B^{j-k})$$

$\{a^i\$b^j\$c^k \mid k \le i \text{ or } k \le j\}$

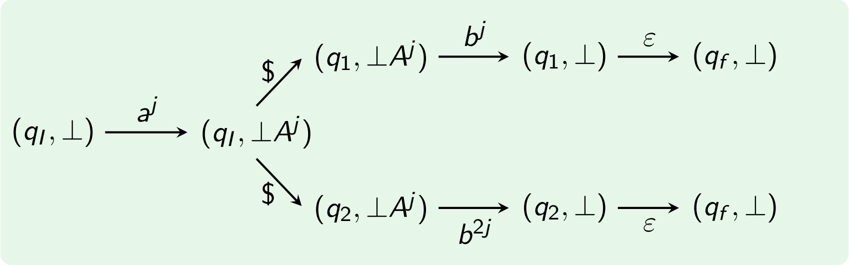
$$(q_1, \bot A^i\$B^j) \xrightarrow{c^k} (q_1, A^j\$B^{j-k})$$

$$(q_I, \bot) \xrightarrow{a^i\$b^j} (q_I, \bot A^i\$B^j)$$

$$(q_I, \bot A^i\$B^j) \xrightarrow{\$} (q_1, \bot A^i\$B^j)$$

$$(q_I, \bot A^i\$B^j) \xrightarrow{\$} (q_2, \bot A^i\$B^j) \xrightarrow[\varepsilon]{} (q_2', \bot A^i) \xrightarrow[c^k]{} (q_2', \bot A^{i-k})$$

$\{a^j \$ b^k \mid k = j \text{ or } k = 2j\}$



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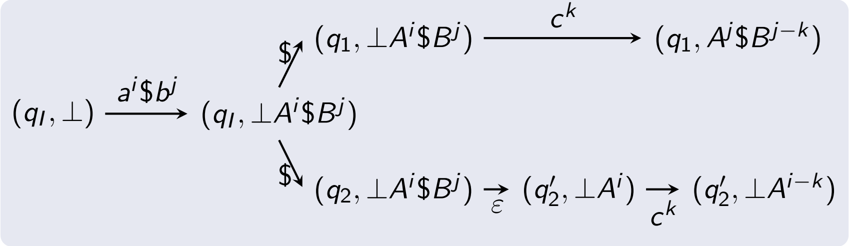

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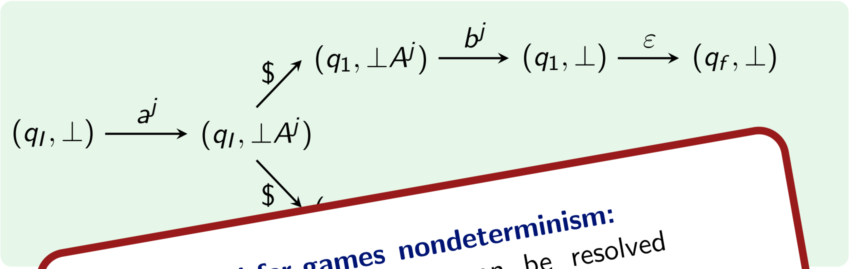

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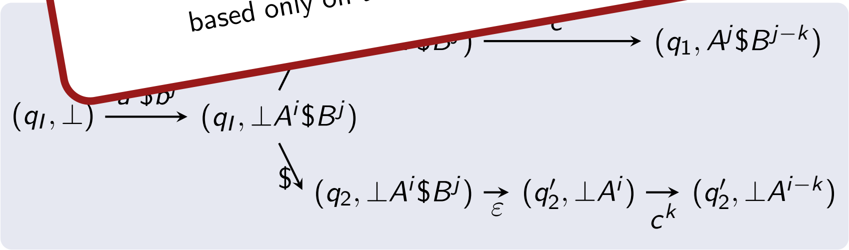

Good-for-games nondeterminism:
Nondeterminism that can be resolved based only on the input processed so far.

- $\{a^i\$b^j\$c^k \mid k \leq i \text{ or } k \leq j\}$ is good-for-games.
- $\{a^j\$b^k \mid k = j \text{ or } k = 2j\}$ is not good-for-games.

- $\{a^i \$ b^j \$ c^k \mid k \leq i \text{ or } k \leq j\}$ is good-for-games.
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Theorem

DCFL $\subsetneq$ *GFG-CFL* $\subsetneq$ *CFL.*

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Why should we care?

1. GFG-PDA can be exponentially more succinct than DPDA (even true for visibly PDA).
2. Solving games and universality are decidable for GFG-PDA.
3. Many open problems.

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In the full paper: closure properties, comparison to unambiguous CFL, complexity of resolving GFG nondeterminism, etc.