Prophecies all the Way: Game-based Model-Checking for HyperQPTL beyond ∀*∃*

Joint work with Sarah Winter (IRIF, Paris)

Martin Zimmermann

Aalborg University

CONCUR 2025, Aarhus, Denmark

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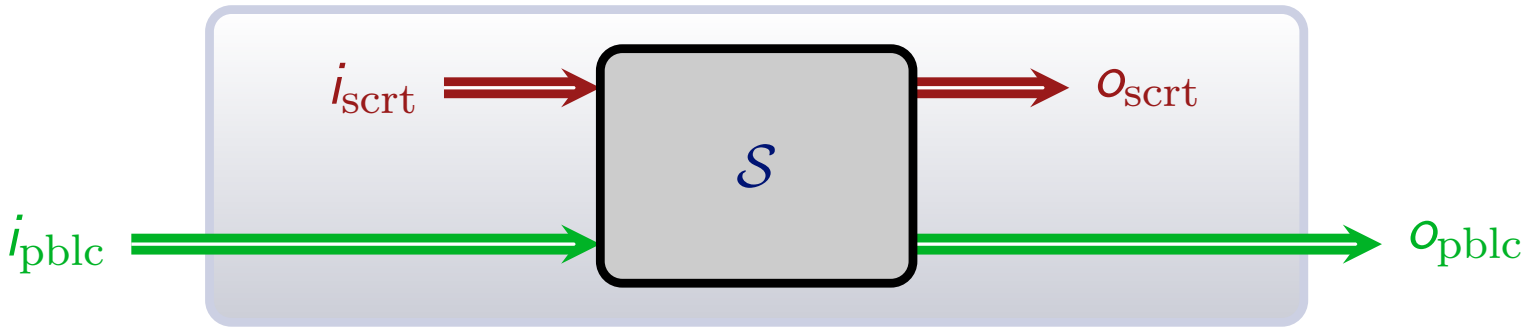

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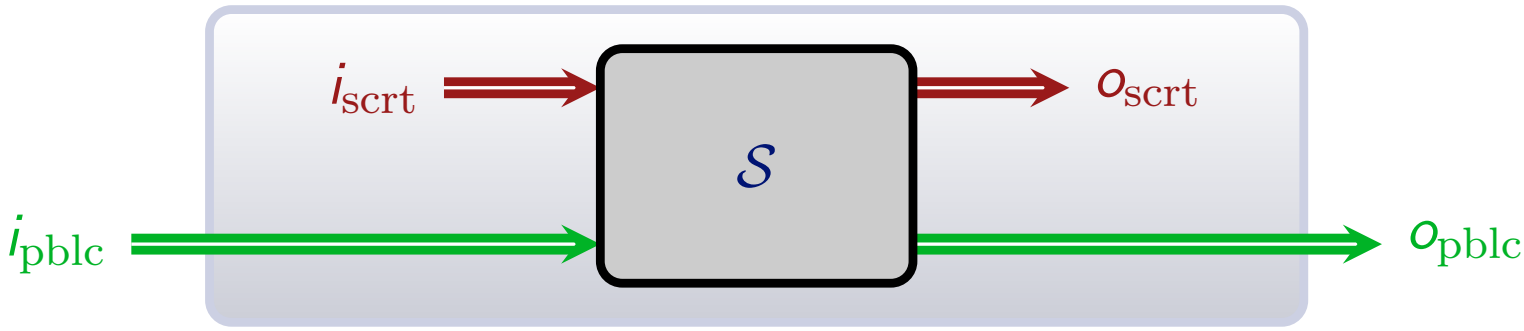

Trace-based view on $\mathcal{S}$: observe execution traces, i.e., infinite sequences over 2^{AP} for some set AP of atomic propositions.

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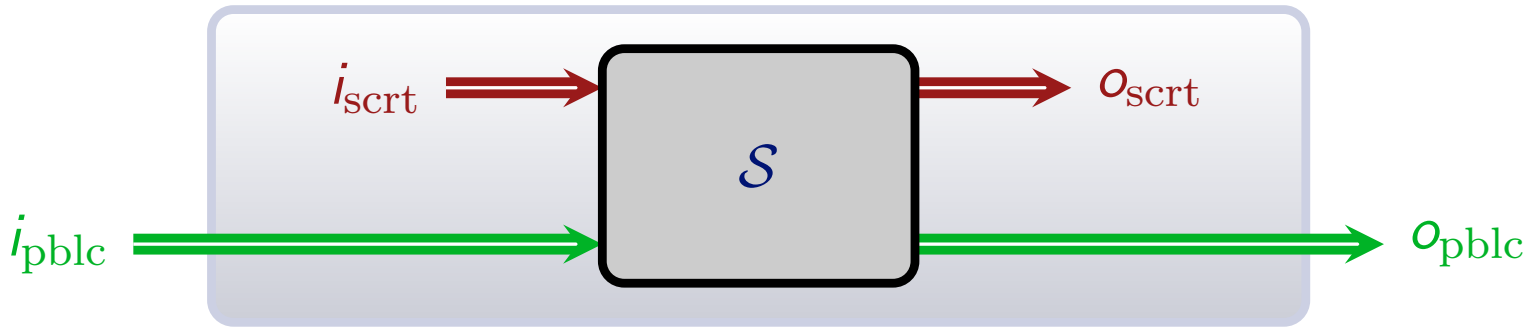

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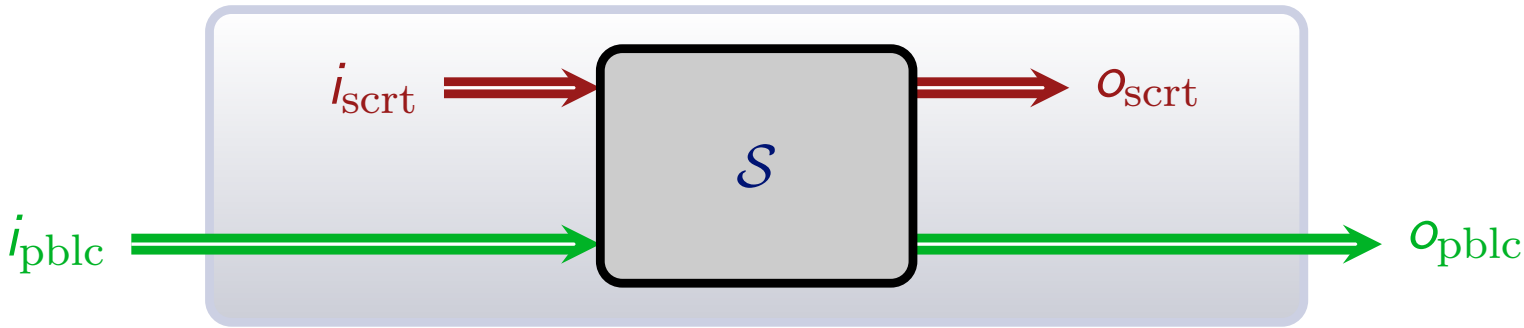

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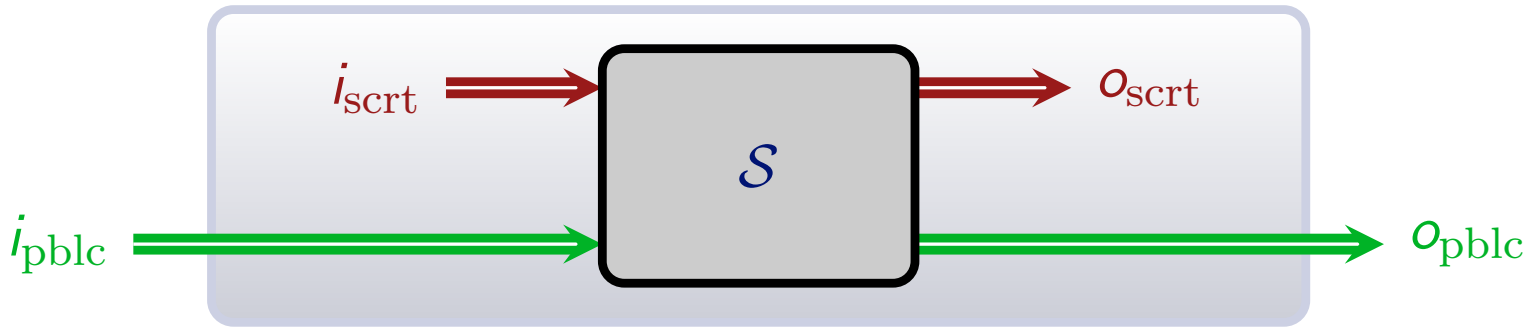

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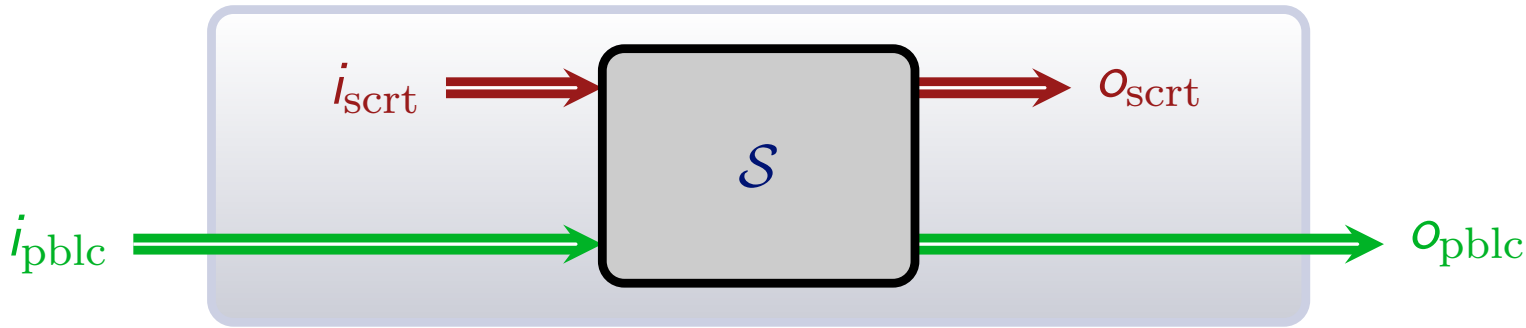

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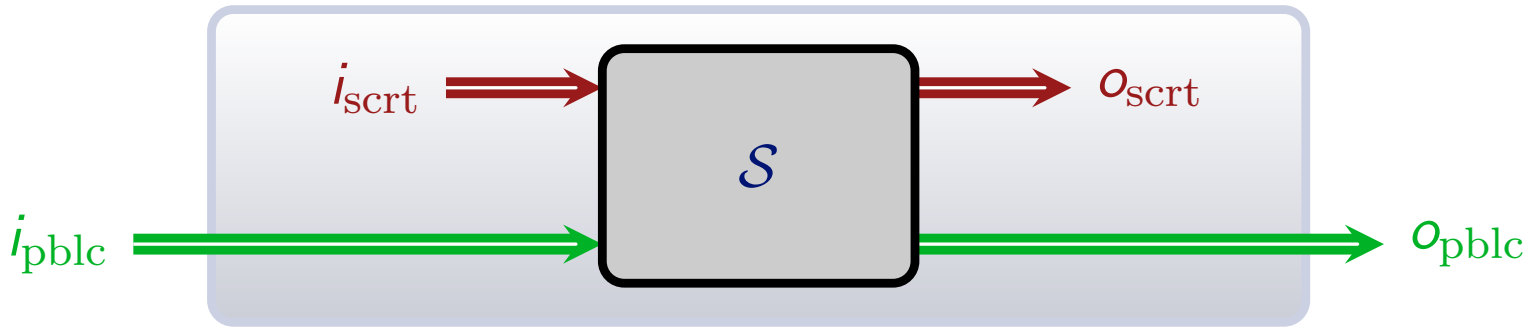

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Remark

- The first property can be checked by inspecting each trace in isolation (a trace property), but
- the other three properties can only be checked by reasoning about multiple traces simultaneously (hyperproperties).

LTL in One Slide

Syntax

$$\varphi ::= a \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{X}\,\varphi \mid \varphi\,\mathsf{U}\,\varphi \qquad \text{where } a \in \mathrm{AP}$$

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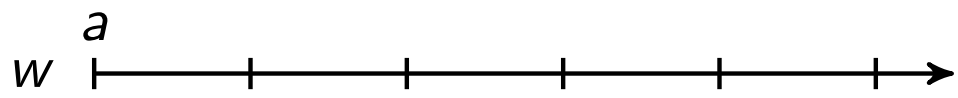

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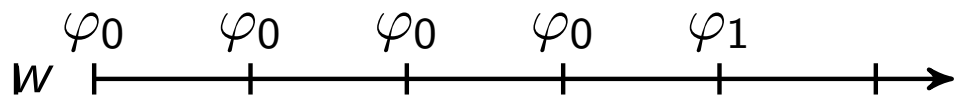

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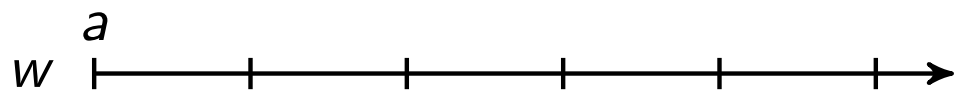

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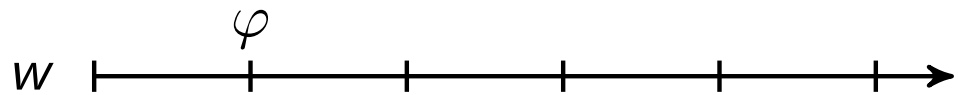

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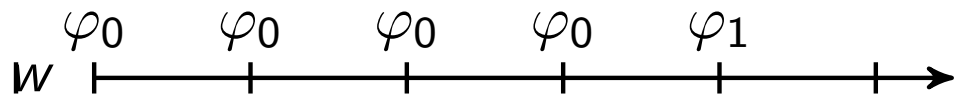

Syntactic Sugar

- $\mathsf{F}\,\psi = \mathsf{tt}\,\mathsf{U}\,\psi$
- $\mathsf{G}\,\psi = \neg\,\mathsf{F}\,\neg\psi$

HyperLTL

HyperLTL = LTL + trace quantification

$$\varphi ::= \exists \pi.\ \varphi \mid \forall \pi.\ \varphi \mid \psi$$
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- Prenex normal form, but
- closed under boolean combinations.
- Time passes synchronously on the quantified traces.

Examples

- Noninterference:

$$\forall\pi\forall\pi'.\ \mathsf{G}((i_{\mathsf{pblc}})_\pi \leftrightarrow (i_{\mathsf{pblc}})_{\pi'}) \rightarrow \mathsf{G}((o_{\mathsf{pblc}})_\pi \leftrightarrow (o_{\mathsf{pblc}})_{\pi'})$$

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- $\mathcal{S}$ terminates within a uniform time bound. Not expressible in HyperLTL.

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The HyperLTL model-checking problem:

Given a finite transition system $\mathcal{S}$ and φ, does $\mathrm{Traces}(\mathcal{S}) \models \varphi$?

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Bottleneck of the "classical" algorithm: complementation of Büchi automata.

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  - For $\neg\theta$ complement automaton for θ.

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Remark

- The game is sound and complete,
- but it is in general not algorithmically solvable.

Sequential Games..

.. are algorithmically more appealing..

π_0

π_1

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π_0 t_0^0

π_1

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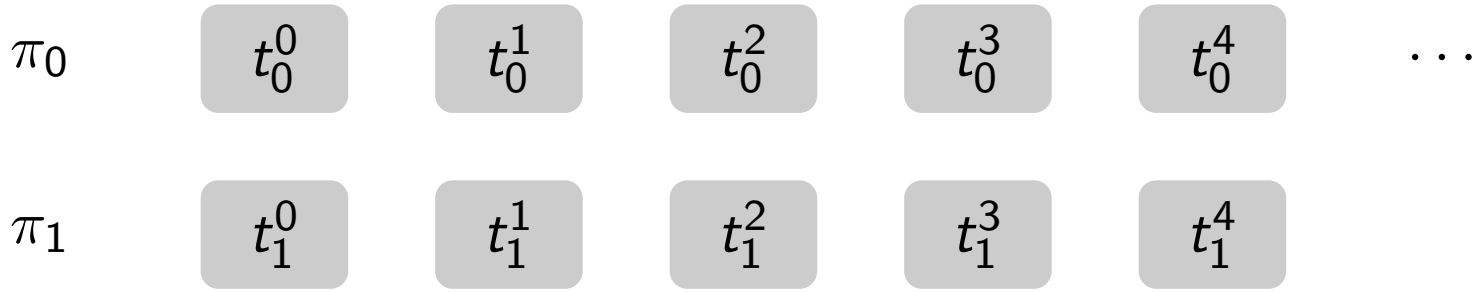

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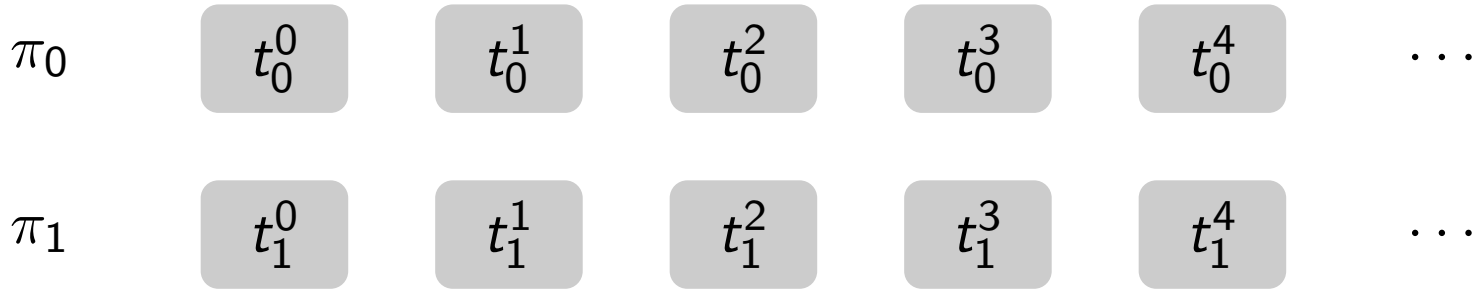

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Example

Consider the formula

$$\varphi = \forall\pi\exists\pi'.\ (\mathsf{F}\, a_\pi) \leftrightarrow a_{\pi'}.$$

$(2^{\{a\}})^\omega$ satisfies φ, but Prover loses the sequential game.

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Theorem [Beutner and Finkbeiner '22]
For every transition system $\mathcal{S}$ and every $\forall^*\exists^*$ HyperLTL formula φ there is a finite set of prophecies such that $\mathcal{S} \models \varphi$ if and only if Prover wins the induced game with prophecies (which is a finite parity game).

What About General Quantifier Prefixes?

Consider a formula of the form $\forall\pi_0\exists\pi_1\forall\pi_2\exists\pi_3.\ \psi$ with quantifier-free ψ.

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We need imperfect information to ensure that π_1 does not depend on π_2 and π_3.

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- Also, we complement Büchi automata.