Synchronous Team Semantics for Temporal Logics

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We present team semantics for two of the most important linear and branching time specification languages, Linear Temporal Logic (LTL) and Computation Tree Logic (CTL).

With team semantics, LTL is able to express hyperproperties, which have in the last decade been identified as a key concept in the verification of information flow properties. We study basic properties of the logic and classify the computational complexity of its satisfiability, path, and model checking problem. Further, we examine how extensions of the basic logic react to adding additional atomic operators. Finally, we compare its expressivity to the one of HyperLTL, another recently introduced logic for hyperproperties. Our results show that LTL with team semantics is a viable alternative to HyperLTL, which complements the expressivity of HyperLTL and has partially better algorithmic properties.

For CTL with team semantics, we investigate the computational complexity of the satisfiability and model checking problem. The satisfiability problem is shown to be EXPTIME-complete while we show that model checking is PSPACE-complete.

CCS Concepts: • **Theory of computation → Modal and temporal logics**; **Problems, reductions and completeness**.

Additional Key Words and Phrases: Temporal Logics, Team Semantics, Model Checking, Satisfiability

1 Introduction

Guaranteeing security and privacy of user information is a key requirement in software development. However, it is also one of the hardest goals to accomplish. One reason for this difficulty is that such requirements typically amount to reasoning about the flow of information and relating different execution traces of the system in question. Many of the requirements of interest are not trace properties, that is, properties whose satisfaction can be verified by considering each computation trace in isolation. Formally, a trace property φ is a set of traces and a system satisfies φ if each of its traces is in φ. An example of a trace property is the property "the system terminates eventually", which is satisfied if every trace eventually reaches a terminating state. In contrast, the property "the system terminates within a bounded amount of time" is no longer a trace property. Consider a system that has a trace t_n for every n, so that t_n only reaches a terminating state after n steps. This system does not satisfy the bounded termination property, but each individual trace t_n could also stem from a system that does satisfy it. Thus, satisfaction of the property cannot be verified by considering each trace in isolation. Properties with this characteristic were termed *hyperproperties* by Clarkson and Schneider [17]. Formally, a hyperproperty φ is a set of sets of traces and a system satisfies φ if its set of traces is contained in φ. The conceptual difference to trace properties allows hyperproperties to specify a much richer landscape of properties including information flow properties capturing security and privacy specifications. Furthermore, one can also express specifications for symmetric access to critical resources in distributed protocols and Hamming distances between code words in coding theory [57]. However, the increase in expressiveness requires novel approaches to specification and verification.

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HyperLTL. Trace properties are typically specified in temporal logics, most prominently in Linear Temporal Logic (LTL) [55]. Verification of LTL specifications is routinely employed in industrial settings and marks one of the most successful applications of formal methods to real-life problems. Recently, this work has been extended to hyperproperties: HyperLTL, LTL equipped with trace quantifiers, has been introduced to specify hyperproperties [16]. A model of a HyperLTL-formula is a set of traces and the quantifiers range over these traces. This logic is able to express the majority of the information flow properties found in the literature (we refer to Section 3 of [16] for a comprehensive list). The satisfiability problem for HyperLTL is highly undecidable [26] while the model checking problem is decidable, albeit of non-elementary complexity [16, 23]. In view of this, the full logic is too strong for practical applications. Fortunately most information flow properties found in the literature can be expressed with at most one quantifier alternation and consequently belong to decidable (and tractable) fragments. Further works on HyperLTL have studied runtime verification [11, 21], connections to first-order logic [24], provided tool support [20, 23], and presented applications to "software doping" [18] and the verification of web-based workflows [22]. Recent works have also considered asynchronous extensions of HyperLTL [5, 8, 13, 30] and verification tools for full HyperLTL [10] (most of the previous tools were designed for fragments without quantifier alternation). In contrast, there are natural properties, e.g., bounded termination, which are not expressible in HyperLTL (which is an easy consequence of a much stronger non-expressibility result [12]).

Team Semantics. Intriguingly, there exists another modern family of logics which operates on sets of objects instead of objects alone. In 1997, Hodges introduced compositional semantics for Hintikka's Independence-friendly logic [35] where the semantical object is a set of first-order assignments. Hence formulae, in this setting, define sets of sets of first-order assignments. We could call these definable sets *first-order hyperproperties*, while Hodges himself called them *trumps*. A decade later Väänänen [63] introduced *Dependence logic* that adopted Hodges' semantics and reimagined Independence-friendly logic. Dependence logic extends first-order logic by atoms expressing that "the value of a variable x functionally determines the value of a variable y". Obviously, such statements only make sense when being evaluated over a set of assignments. Therefore, they are, using the parlance introduced above, hyperproperties. In the language of dependence logic, such sets are called *teams* and the semantics is termed *team semantics*.

After the introduction of dependence logic, a whole family of logics with different atomic statements have been introduced in this framework: *independence logic* [28] and *inclusion logic* [27] being the most prominent. Interest in these logics is rapidly growing and connections to a plethora of disciplines have been drawn, e.g., to database theory [33], real valued computation [32], quantum foundations [1, 2], and to the study of argumentation [49] and causation [4].

Our Contribution. The conceptual similarities between hyperproperties and team properties raise the question *what is the natural team semantics for temporal logics*. In this paper, we develop team semantics for LTL and CTL, obtaining the logics TeamLTL and TeamCTL. We analyse their expressive power and the complexity of their satisfiability and model checking problems and subsequently compare TeamLTL to HyperLTL. While team semantics has previously been defined for propositional and modal logic [64], we are the first to consider team semantics for temporal logics.

Our complexity results are summarised in Figure 1. We prove that the satisfiability problem (TSAT) for TeamLTL is PSPACE-complete by showing that the problem is equivalent to LTL satisfiability under classical semantics. We consider two variants of the model checking problem. As there are uncountably many traces, we have to represent teams, i.e., sets of traces, in a finitary manner. The path checking problem (TPC) asks to check whether a finite team of ultimately periodic traces satisfies a given formula. As our main technical result, we establish this problem to be PSPACE-complete. In the (general) model checking problem (TMC), a team is represented by a finite transition system. Formally, given a transition system and a formula, the model checking problem asks to determine whether the set of traces of the system satisfies the formula. We give a polynomial space algorithm for the model checking problem for the disjunction-free fragment, while we leave open the complexity of the general problem. Disjunction plays a special role in team semantics, as it splits a team into two. As a result, this operator is commonly called *splitjunction*. In our setting, the splitjunction requires us to deal with possibly infinitely many splits of uncountable teams, if a splitjunction is under the scope of a G-operator, which raises interesting language-theoretic questions. Additionally, we study the effects for complexity that follow when our logics are extended by dependence atoms, so-called *generalised atoms* [45], and the contradictory negation. Finally, we show that TeamLTL is able to specify properties which are not expressible in HyperLTL and *vice versa*.

Recall that satisfiability for HyperLTL is highly undecidable [26] and model checking is of non-elementary complexity [51, 57]. Our results show that similar problems for TeamLTL have a much simpler complexity while some hyperproperties are still expressible (e.g., input determinism, see page 13, or bounded termination). This demonstrates that TeamLTL is a viable alternative for the specification and verification of hyperproperties that complements HyperLTL.

In the second part of the paper, we develop team semantics for CTL. We establish that the satisfiability problem for the resulting logic TeamCTL is EXPTIME-complete while the model checking problem is PSPACE-complete. While CTL satisfiability is already EXPTIME-complete for classical semantics [25, 56], CTL model checking is P-complete [15, 59] for classical semantics, i.e., team semantics increases the complexity of the problem. For the team semantics setting, we extend our model checking result to cover also finite sets of FO-definable generalised atoms. Finally, we compare the expressiveness of TeamCTL and CTL with classical semantics.

Prior Work. Preliminary versions of this work have been published in the proceedings of the 43rd International Symposium on Mathematical Foundations of Computer Science, MFCS 2018 [42], and in the proceedings of the 22nd International Symposium on Temporal Representation and Reasoning, TIME 2015 [40]. The following list summarises how this article extends the conference versions:

- The presentation of the article has been thoroughly revised. Moreover, we now include an overview of the subsequent works on temporal team semantics published after our conference articles.
- The proof of Lemma 4.1 has been considerably extended and now contains also the correctness part of the reduction.
- The proof of Lemma 4.2 was omitted in the conference version and is now included.
- The full proof of Theorem 4.4 is now included.
- The formulation of Theorem 5.2 is slightly strengthened and a proof in now included.
- Theorem 8.3 is new.
- The construction and figures in the proof of Lemma 9.2 have been improved.
- The proof of Theorem 9.5 has been corrected.
- Theorem 9.7 is new.

Related work on TeamLTL. Since the publication of the original conference articles [40, 42] several follow-up works have been published. Krebs et al. [42] also introduced an asynchronous variant of TeamLTL. This line of work has been continued by Kontinen et al. [37–39]. In particular, Kontinen and Sandström [37] considered the extension of TeamLTL with the contradictory negation and

Logic	TSAT		TPC		TMC	
LTL	PSPACE	(Thm. 5.5)	PSPACE	(Thm. 4.3)	PSPACE-hard	(Thm. 4.3)
LTL(dep)	PSPACE	(Prop. 5.5)	PSPACE	(Thm. 5.2)	NEXPTIME-hard	(Thm. 5.7)
LTL($\otimes$, $\mathcal{D}$)	Σ^0_1-hard	[67]	PSPACE	(Thm. 5.2)	Σ^0_1-hard	[67]
LTL($\mathcal{D}$, $\sim$)	third-order arithmetic	[48]	PSPACE	(Thm. 5.2)	third-order arithmetic	[48]
LTL $-\vee$	?		?		$\in$ PSPACE	(Thm. 4.4)
CTL	EXPTIME	(Thm. 9.9)	–		PSPACE	(Thm. 9.2)
CTL($\mathcal{D}$, $\sim$)	?		–		PSPACE	(Thm. 9.7)

Fig. 1. Overview of complexity results for TeamLTL and TeamCTL. 'dep' refers to dependence atoms, '$\sim$' refers to the contradictory negation, $\mathcal{D}$ refers to any finite set of first-order definable generalised atoms, and 'LTL $-\vee$' refers to disjunction free LTL. All results are completeness results unless otherwise stated. We write '?' for open cases and '–' if the problem is not meaningful for the logic.

studied its complexity and translations to first-order and second-order logic. Then, Kontinen et al. [38] fixed a mistake in the original definition of the asynchronous TeamLTL, introduced a novel set-based semantics for TeamLTL and studied its complexity and expressivity. Hence, for the definition of asynchronous TeamLTL, please refer to the work of Kontinen et al. [38]. In their recent work on this topic, Kontinen et al. [39] revealed a tight connection between set-based asynchronous TeamLTL and the one-variable fragment of HyperLTL.

Lück [48] showed that the satisfiability and model checking problems of synchronous TeamLTL with the contradictory negation are complete for third-order arithmetic. Virtema et al. [67] studied the expressivity and complexity of various extensions of synchronous TeamLTL. In particular, they identified undecidable cases and cases when team logics can be translated to HyperQPTL and HyperQPTL$^+$. By doing so, they mapped the undecidability landscape of synchronous TeamLTL.

Gutsfeld et al. [29] introduced a flexible team-based formalism to logically specify asynchronous hyperproperties based on so-called *time evaluation functions*, which subsumes synchronous TeamLTL and can be used to model diverse forms of asynchronicity.

Bellier et al. [9] introduced a team-based formalism for quantified propositional temporal logic that takes inspiration from first-order team logics which hence is orthogonal to TeamLTL.

Finally, while there exists a deep understanding of the complexity of the satisfiability and model checking problems for classical LTL [6, 7] and CTL [41, 52, 53], these investigations do neither directly nor generally carry over to team semantics.

2 Preliminaries

The non-negative integers are denoted by $\mathbb{N}$ and the power set of a set S is denoted by 2^S. Throughout the paper, we consider a countably infinite set AP of atomic propositions.

Computational Complexity. We will make use of standard notions in complexity theory. In particular, we will use the complexity classes P, PSPACE, EXPTIME, and NEXPTIME.

An alternating Turing machine (ATM) is a non-deterministic Turing machine whose state space is partitioned into two types of states: existential and universal. Acceptance for ATMs is defined in an inductive way on any computation tree with respect to a given input as follows. A halting configuration is accepting if and only if it contains an accepting state. An 'inner' configuration (which is not a halting configuration) is accepting depending on its type of state: if it is existential then it is accepting if at least one of its children is accepting, and if it is universal then it is accepting if all of its children are accepting. An ATM accepts its input if and only if its initial configuration is accepting. Finally, the complexity class ATIME-ALT(x, y) is the set of problems solvable by ATMs in a runtime of x with y-many alternations.

Most reductions used in the paper are $\leq_m^p$-reductions, that is, polynomial time, many-to-one reductions.

Traces. A *trace* over AP is an infinite sequence from $(2^{AP})^\omega$; a finite trace is a finite sequence from $(2^{AP})^*$. The length of a finite trace t is denoted by $|t|$. The empty trace is denoted by ε and the concatenation of a finite trace t_0 and a finite or infinite trace t_1 by t_0t_1. Unless stated otherwise, a trace is always assumed to be infinite.

A *team* is a (potentially infinite) set of traces. Given a trace $t = t(0)t(1)t(2)\cdots$ and $i \geq 0$, we define $t[i, \infty) \coloneqq t(i)t(i+1)t(i+2)\cdots$, which we lift to teams $T \subseteq (2^{AP})^\omega$ by defining $T[i, \infty) \coloneqq \{\, t[i, \infty) \mid t \in T \,\}$. A trace t is *ultimately periodic*, if it is of the form $t = t_0 \cdot t_1^\omega = t_0t_1t_1t_1\cdots$ for two finite traces t_0 and t_1 with $|t_1| > 0$. As a result, an ultimately periodic trace t is finitely represented by the pair (t_0, t_1); we define $[\![(t_0, t_1)]\!] = t_0t_1^\omega$. Given a set $\mathcal{T}$ of such pairs, we define $[\![\mathcal{T}]\!] = \{\, [\![(t_0, t_1)]\!] \mid (t_0, t_1) \in \mathcal{T} \,\}$, which is a team of ultimately periodic traces. We call $\mathcal{T}$ a team encoding of $[\![\mathcal{T}]\!]$.

Linear Temporal Logic. The formulae of Linear Temporal Logic (LTL) [55] are defined via the grammar

$$\varphi ::= \top \mid \bot \mid p \mid \neg p \mid \varphi \land \varphi \mid \varphi \lor \varphi \mid \mathsf{X}\varphi \mid \varphi\mathsf{U}\varphi \mid \varphi\mathsf{R}\varphi,$$

where p ranges over the atomic propositions in AP. We define the following usual shorthands: $\top \coloneqq p \lor \neg p$, $\bot \coloneqq p \land \neg p$, $\mathsf{F}\varphi \coloneqq \top\mathsf{U}\varphi$, $\mathsf{G}\varphi \coloneqq \bot\mathsf{U}\varphi$. The length of a formula is defined to be the number of Boolean and temporal connectives occurring in it. Often, the length of an LTL-formula is defined to be the number of syntactically different subformulae, which might be exponentially smaller. Here, we need to distinguish syntactically equal subformulae which becomes clearer after defining the semantics. As we only consider formulae in negation normal form, we use the full set of temporal operators.

Next, we recall the classical semantics of LTL before we introduce team semantics. For traces $t \in (2^{AP})^\omega$ we define the following:

$t \models p$ if $p \in t(0)$,
$t \models \neg p$ if $p \notin t(0)$,
$t \models \psi \land \varphi$ if $t \models \psi$ and $t \models \varphi$,
$t \models \psi \lor \varphi$ if $t \models \psi$ or $t \models \varphi$,
$t \models \mathsf{X}\varphi$ if $t[1, \infty) \models \varphi$,
$t \models \psi\mathsf{U}\varphi$ if $\exists k \geq 0$ such that $t[k, \infty) \models \varphi$ and $\forall k' < k$ we have that $t[k', \infty) \models \psi$, and
$t \models \psi\mathsf{R}\varphi$ if $\forall k \geq 0$ we have that $t[k, \infty) \models \varphi$ or $\exists k' < k$ such that $t[k', \infty) \models \psi$.

3 Team Semantics for LTL

Next, we introduce (synchronous) team semantics for LTL, obtaining the logic TeamLTL. For teams $T \subseteq (2^{AP})^\omega$ we define the following:

$T \models p$ if $\forall t \in T$ we have that $p \in t(0)$,
$T \models \neg p$ if $\forall t \in T$ we have that $p \notin t(0)$,
$T \models \psi \land \varphi$ if $T \models \psi$ and $T \models \varphi$,
$T \models \psi \lor \varphi$ if $\exists T_1 \cup T_2 = T$ such that $T_1 \models \psi$ and $T_2 \models \varphi$,
$T \models \mathsf{X}\varphi$ if $T[1, \infty) \models \varphi$,
$T \models \psi\mathsf{U}\varphi$ if $\exists k \geq 0$ such that $T[k, \infty) \models \varphi$ and $\forall k' < k$ we have that $T[k', \infty) \models \psi$, and
$T \models \psi\mathsf{R}\varphi$ if $\forall k \geq 0$ we have that $T[k, \infty) \models \varphi$ or $\exists k' < k$ such that $T[k', \infty) \models \psi$.

property	definition	
empty team property	$\emptyset \models \varphi$	✓
downward closure	$T \models \varphi$ implies $\forall T' \subseteq T\colon T' \models \varphi$	✓
union closure	$T \models \varphi$ and $T' \models \varphi$ implies $T \cup T' \models \varphi$	×
flatness	$T \models \varphi$ if and only if $\forall t \in T\colon \{t\} \models \varphi$	×
singleton equivalence	$\{t\} \models \varphi$ if and only if $t \models \varphi$	✓

Fig. 2. Structural properties overview for TeamLTL.

We call expressions of the form $\psi \vee \varphi$ *splitjunctions* to emphasise that in team semantics disjunction splits a team into two parts. Similarly, the $\vee$-operator is referred to as a *splitjunction*. Notice that the object (a trace or a team) left of $\models$ determines which of the above semantics is used (classical or team semantics). Some subsequent works consider team semantics over multisets of traces [29]. For synchronous TeamLTL (as introduced here) the generalisation to multisets would allow the implementation of new quantitative dependency statements as atomic formulae. From the above semantics only disjunction would need to be reinterpreted via disjoint (multiset) unions. Since having multiset teams would not have any meaningful impact on TeamLTL, we adopt the slightly simpler set-based semantics.

Example 3.1. If p is an atomic proposition encoding that a computation has ended, then $\mathsf{F}p$ defines the hyperproperty *bounded termination*. In particular, a possibly infinite team T satisfies $\mathsf{F}p$, if there is a natural number $n \in \mathbb{N}$ such that $p \in t(n)$, for every $t \in T$.

We consider several standard properties of team semantics (cf., e.g. [19]) and verify which of these hold for our semantics for LTL. These properties are later used to analyse the complexity of the satisfiability and model checking problems. See Figure 2 for the definitions of the properties and a summary for which of the properties hold for our semantics. The positive results follow via simple inductive arguments. For the fact that team semantics is not union closed, consider teams $T = \{\{p\}\emptyset^\omega\}$ and $T' = \{\emptyset\{p\}\emptyset^\omega\}$. Then, we have $T \models \mathsf{F}p$ and $T' \models \mathsf{F}p$ but $T \cup T' \not\models \mathsf{F}p$.

A Kripke structure $\mathcal{K} = (W, R, \eta, w_I)$ consists of a finite set W of worlds, a left-total transition relation $R \subseteq W \times W$, a labeling function $\eta\colon W \longrightarrow 2^{\mathrm{AP}}$, and an initial world $w_I \in W$. A path π through $\mathcal{K}$ is an infinite sequence $\pi = \pi(0)\pi(1)\pi(2)\cdots \in W^\omega$ such that $\pi(0) = w_I$ and $(\pi(i), \pi(i+1)) \in R$ for every $i \geq 0$. The trace of π is defined as $t(\pi) = \eta(\pi(0))\eta(\pi(1))\eta(\pi(2))\cdots \in (2^{\mathrm{AP}})^\omega$. A Kripke structure $\mathcal{K}$ induces the team $T(\mathcal{K}) = \{\, t(\pi) \mid \pi \text{ is a path through } \mathcal{K} \,\}$.

Next, we define the most important verification problems for TeamLTL, namely satisfiability and two variants of the model checking problem. For classical LTL, one studies the path checking problem and the model checking problem. The difference between these two problems lies in the type of structures one considers. Recall that a model of an LTL-formula is a single trace. In the path checking problem, a trace t and a formula φ are given, and one has to decide whether $t \models \varphi$. This problem has applications to runtime verification and monitoring of reactive systems [43, 50]. In the model checking problem, a Kripke structure $\mathcal{K}$ and a formula φ are given, and one has to decide whether every execution trace t of $\mathcal{K}$ satisfies φ.

The satisfiability problem of TeamLTL is defined as follows.

Problem:	TSAT(LTL) — LTL satisfiability w.r.t. teams.
Input:	An LTL-formula φ.
Question:	Is there a non-empty team T such that $T \models \varphi$?

The non-emptiness condition is necessary, as otherwise every formula is satisfiable due to the empty team property (see Figure 2). Also note that, due to downward closure (see Figure 2), an

LTL-formula φ is satisfiable with team semantics if and only if there is a singleton team that satisfies the formula. From this and singleton equivalence (see Figure 2), we obtain the following result from the identical result for LTL under classical semantics [50, 61].

Proposition 3.2. TSAT(LTL) *is* PSPACE*-complete w.r.t.* $\leq^{\mathrm{p}}_{\mathrm{m}}$*-reductions.*

We consider the generalisation of the path checking problem for LTL (denoted by PC(LTL)), which asks for a given ultimately periodic trace t and a given formula φ, whether $t \models \varphi$ holds. In the team semantics setting, the corresponding question is whether a given finite team comprised of ultimately periodic traces satisfies a given formula. Such a team is given by a team encoding $\mathcal{T}$. To simplify our notation, we will write $\mathcal{T} \models \varphi$ instead of $[\![\mathcal{T}]\!] \models \varphi$.

Problem:	TPC(LTL) — LTL Team Path Checking.
Input:	An LTL-formula φ and a finite team encoding $\mathcal{T}$.
Question:	Does $\mathcal{T} \models \varphi$?

Now, consider the generalised model checking problem where one checks whether the team of traces of a Kripke structure satisfies a given formula. This is the natural generalisation of the model checking problem for classical semantics, denoted by MC(LTL), which asks, for a given Kripke structure $\mathcal{K}$ and a given LTL-formula φ, whether $t \models \varphi$ for every trace t of $\mathcal{K}$.

Problem:	TMC(LTL) — LTL Team Model Checking.
Input:	An LTL-formula φ and a Kripke structure $\mathcal{K}$.
Question:	Does $T(\mathcal{K}) \models \varphi$?

4 Complexity Results for TeamLTL

Next, we examine the computational complexity of path and model checking with respect to team semantics.

4.1 Path Checking

The following problem for quantified Boolean formulae (qBf) is well-known [46, 62] to be PSPACE-complete:

Problem:	QBF-VAL — Validity problem for quantified Boolean formulae.
Input:	A quantified Boolean formula φ.
Question:	Is φ valid?

Lemma 4.1. TPC(LTL) *is* PSPACE*-hard w.r.t.* $\leq^{\mathrm{p}}_{\mathrm{m}}$*-reductions.*

Proof. To prove the claim of the lemma, we will show that QBF-VAL $\leq^{\mathrm{p}}_{\mathrm{m}}$ TPC(LTL). Given a quantified Boolean formula φ, we stipulate, w.l.o.g., that φ is of the form $\exists x_1 \forall x_2 \cdots Q x_n \chi$, where $\chi = \bigwedge_{j=1}^{m} \bigvee_{k=1}^{3} \ell_{jk}$, $Q \in \{\exists, \forall\}$, and $x_1, \ldots, x_n$ are exactly the free variables of χ and pairwise distinct.

In the following we define a reduction which is composed of two functions f and g. Given a qBf φ, the function f will define an LTL-formula and g will define a team such that φ is valid if and only if $g(\varphi) \models f(\varphi)$. Essentially, the team $g(\varphi)$ will contain three kinds of traces, see Figure 3, that will be used to encode variables $i \in \{1, \ldots, n\}$, clauses $j \in \{1, \ldots, m\}$, and literal positions $k \in \{1, 2, 3\}$:

(1) traces which are used to mimic universal quantification ($U(i)$ and $E(i)$),
(2) traces that are used to simulate existential quantification ($E(i)$), and
(3) traces used to encode the matrix of φ ($L(j, k)$).

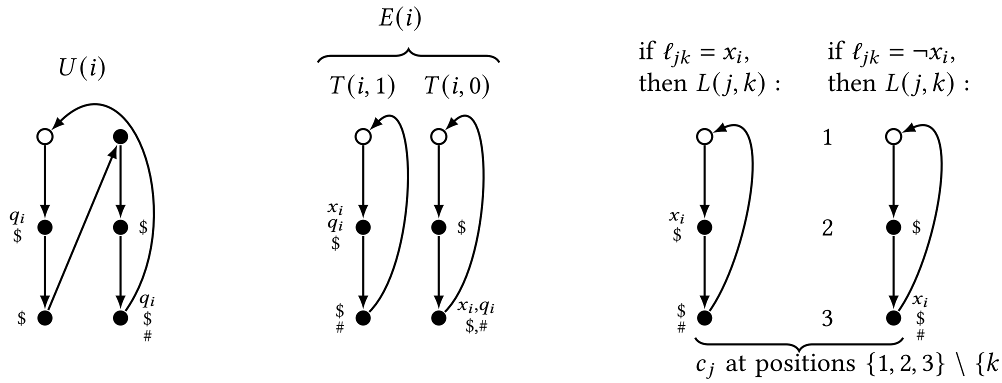

Fig. 3. Traces for the reduction presented in the proof of Lemma 4.1.

Moreover the trace $T(i,1)$ ($T(i,0)$, resp.) is used inside the proof to encode an assignment that maps the variable x_i to true (false, resp.). Note that, $U(i), T(i,1), T(i,0), L(j,k)$ are technically singleton sets of traces. For convenience, we identify them with the traces they contain.

Next, we inductively define the reduction function f that maps qBfs to LTL-formulae:

$$f(\chi) \coloneqq \bigvee_{i=1}^{n} \mathsf{F}x_i \vee \bigvee_{i=1}^{m} \mathsf{F}c_i,$$

where χ is the 3CNF-formula $\bigwedge_{j=1}^{m} \bigvee_{k=1}^{3} \ell_{jk}$ with free variables $x_1, \ldots, x_n$,

$$f(\exists x_i \psi) \coloneqq (\mathsf{F}q_i) \vee f(\psi), \qquad f(\forall x_i \psi) \coloneqq \big(\$ \vee (\neg q_i \mathsf{U} q_i) \vee \mathsf{F}[\# \wedge \mathsf{X} f(\psi)]\big)\mathsf{U}\#.$$

Intuitively, the idea is that the left part of the formula $f(\chi)$ is used to satisfy one literal per clause and the right part is to take care of the remaining literals in a trivial way.

The reduction function g that maps qBfs to teams is defined as follows with respect to the traces in Figure 3:

$$g(\chi) \coloneqq \bigcup_{j=1}^{m} L(j,1) \cup L(j,2) \cup L(j,3),$$

$$g(\exists x_i \psi) \coloneqq E(i) \cup g(\psi), \qquad g(\forall x_i \psi) \coloneqq U(i) \cup E(i) \cup g(\psi).$$

In Figure 3, the first position of each trace is marked with a white circle. For instance, the trace of $U(i)$ is then encoded via

$$(\varepsilon, \emptyset\{q_i, \$\}\{\$\}\emptyset\{\$\}\{q_i, \$, \#\}).$$

The reduction function showing QBF-VAL $\leq_{\mathrm{m}}^{\mathrm{p}}$ TPC(LTL) is then $\varphi \mapsto \langle g(\varphi), f(\varphi)\rangle$. Clearly $f(\varphi)$ and $g(\varphi)$ can be computed in linear time with respect to $|\varphi|$.

Intuitively, for the existential quantifier case, the formula $(\mathsf{F}q_i) \vee f(\psi)$ allows to continue in $f(\psi)$ with exactly one of $T(i,1)$ or $T(i,0)$. If $b \in \{0,1\}$ is a truth value then selecting $T(i,b)$ in the team is the same as setting x_i to b. For the case of $f(\forall x_i \psi)$, the formula $(\neg q_i \mathsf{U} q_i) \vee \mathsf{F}[\# \wedge \mathsf{X} f(\psi)]$ with respect to the team $(U(i) \cup E(i))[0, \infty)$ is similar to the existential case choosing x_i to be 1 whereas for $(U(i) \cup E(i))[3, \infty)$ one selects x_i to be 0. The use of the until operator in combination with \$ and # then forces both cases to happen.

Let $\varphi' = Q' x_{n'+1} \cdots Q x_n \chi$, where $Q', Q \in \{\exists, \forall\}$ and let I be an assignment of the variables in $\{x_1, \dots, x_{n'}\}$ for $n' \leq n$. Then, let

$$g(I, \varphi') \coloneqq g(\varphi') \cup \bigcup_{x_i \in \mathrm{Dom}(I)} T(i, I(x_i)).$$

We claim $I \models \varphi'$ if and only if $g(I, \varphi') \models f(\varphi')$.

Note that when $\varphi' = \varphi$ it follows that $I = \emptyset$ and that $g(I, \varphi') = g(\varphi)$. Accordingly, the lemma follows from the correctness of the claim above. The claim is proven by induction on the number of quantifier alternations in φ'.

Induction Basis. $\varphi' = \chi$, this implies that φ' is quantifier-free and $\mathrm{Dom}(I) = \{x_1, \dots, x_n\}$.

"$\Leftarrow$": Let $g(I, \varphi') = T_1 \cup T_2$ s.t. $T_1 \models \bigvee_{i=1}^{n} \mathsf{F} x_i$ and $T_2 \models \bigvee_{i=1}^{n} \mathsf{F} c_i$. We assume w.l.o.g. T_1 and T_2 to be disjoint, which is possible due to downward closure. We then have that

$$T_2 \subseteq \{\, L(j,k) \mid 1 \leq j \leq m, 1 \leq k \leq 3 \,\}$$

and

$$T_1 = (\{\, L(j,k) \mid 1 \leq j \leq m, 1 \leq k \leq 3 \,\} \setminus T_2) \cup \{\, T(i, I(x_i)) \mid 1 \leq i \leq n \,\},$$

where $1 \leq i, i' \leq m$, and c_i does not appear positively in the trace $T(i', I(x_{i'}))$. Due to construction of the traces, $L(j,k) \in T_2$ can only satisfy the subformula $\mathsf{F} c_{j'}$ for $j' = j$. Moreover, note that there exists no $s \in \mathbb{N}$ such that $L(j,k)(s) \ni c_j$ for all $1 \leq k \leq 3$; hence $\{L(j,1), L(j,2), L(j,3)\}$ falsifies $\mathsf{F} c_j$. These two combined imply that $T_2 \not\supseteq \{L(j,1), L(j,2), L(j,3)\}$, for each $1 \leq j \leq m$. However, for each $1 \leq j \leq m$, any two of $L(j,k)$, $1 \leq k \leq 3$, can belong to T_2 and hence exactly one belongs to T_1. This is true because, it is impossible to satisfy all three $L(j,k)$ per clause j simultaneously. One needs to decide for one literal per clause that is "the" satisfied one and is contained in T_1. The remaining literals are in T_2 (also if one or both are satisfied by the assignment).

Now, let $T_1 = T_1^1 \cup \cdots \cup T_1^n$ such that $T_1^i \models \mathsf{F} x_i$. Note that $\mathsf{F} x_i$ can be satisfied by $T(i', I(x_{i'}))$ only for $i' = i$. Since $T_1 \supseteq \{\, T(i, I(x_i)) \mid 1 \leq i \leq n \,\}$, it follows that $T(i, I(x_i)) \in T_1^i$, for each $1 \leq i \leq n$. Note also that, if $L(j,k) \in T_1$ it has to be in T_1^i where x_i is the variable of $\ell_{j,k}$. By construction of the traces, if $T(i,1) \in T_1^i$ we have $T_1^i(1) \models x_i$ and if $T(i,0) \in T_1^i$ then $T_1^i(2) \models x_i$. Thus, by construction of the traces $L(j,k)$, if $L(j,k) \in T_1$ then $I \models \ell_{j,k}$. Since, for each $1 \leq j \leq m$, there is a $1 \leq k \leq 3$ such that $L(j,k) \in T_1$ it follows that $I \models \varphi'$.

"$\Rightarrow$": Now assume that $I \models \varphi'$. As a result, pick for each $1 \leq j \leq m$ a *single* $1 \leq k \leq 3$ such that $I \models \ell_{jk}$. Denote this sequence of choices by $k_1, \dots, k_m$. Choose $g(I, \varphi') = T_1 \cup T_2$ as follows:

$$T_1 \coloneqq \{\, L(j, k_j) \mid 1 \leq j \leq m\} \cup \{T(i, I(x_i) \mid 1 \leq i \leq n \,\}$$
$$T_2 \coloneqq \{\, L(j,1), L(j,2), L(j,3) \mid 1 \leq j \leq m \,\} \setminus T_1$$

Then $T_2 \models \bigvee_{j=1}^{m} \mathsf{F} c_j$, for exactly two traces per clause are in T_2, and we can divide $T_2 = T_2^1 \cup \cdots \cup T_2^m$ where

$$T_2^j \coloneqq \{\, L(j,k), L(j,k') \mid k, k' \in \{1,2,3\} \setminus \{k_j\} \,\},$$

and, by construction of the traces, $T_2^j \models \mathsf{F} c_j$, for all $1 \leq j \leq m$. Further, note that $T_1 = T_1^1 \cup \cdots \cup T_1^n$, where

$$T_1^i \coloneqq \{\, L(j, k_j) \mid 1 \leq j \leq m, I(x_i) \models \ell_{jk} \,\} \cup \{T(i, I(x_i))\}.$$

There are two possibilities:

- $I(x_i) = 1$: then $x_i \in (L(j,k_j)(1) \cap T(i, I(x_i))(1))$.
- $I(x_i) = 0$: then $x_i \in (L(j,k_j)(2) \cap T(i, I(x_i))(2))$.

In both cases, $T_1^i \models \mathsf{F} x_i$, and thus $T_1 \models \bigvee_{i=1}^{n} \mathsf{F} x_i$. Hence it follows that $g(I, \varphi') \models f(\varphi')$ and the induction basis is proven.

Induction Step. "Case $\varphi' = \exists x_i\psi$." We show that $I \models \exists x_i\psi$ if and only if $g(I, \exists x_i\psi) \models f(\exists x_i\psi)$.

First note that $g(I, \exists x_i\psi) \models f(\exists x_i\psi)$ iff $E(i) \cup g(\psi) \models (\mathsf{F}q_i) \vee f(\psi)$, by the definitions of f and g. Clearly, $E(i) \not\models \mathsf{F}q_i$, but both $T(i,1) \models \mathsf{F}q_i$ and $T(i,0) \models \mathsf{F}q_i$. Observe that $E(i) = \{T(i,1), T(i,0)\}$ and q_i does not appear positively anywhere in $g(\psi)$. Accordingly, and by downward closure, $E(i) \cup g(\psi) \models (\mathsf{F}q_i) \vee f(\psi)$ if and only if

$$\exists b \in \{0,1\}: \; T(i, 1-b) \models \mathsf{F}q_i \text{ and } (E(i) \cup g(\psi)) \setminus T(i, 1-b) \models f(\psi). \tag{1}$$

Since $(E(i) \cup g(\psi)) \setminus T(i, 1-b) = T(i,b) \cup g(\psi) = g(I[x_i \mapsto b], \psi)$, Equation (1) holds if and only if $g(I[x_i \mapsto b], \psi) \models f(\psi)$, for some bit $b \in \{0,1\}$. By the induction hypothesis, the latter holds if and only if there exists a bit $b \in \{0,1\}$ s.t. $I[x_i \mapsto b] \models \psi$. Finally by the semantics of $\exists$ this holds if and only if $I \models \exists x_i\psi$.

"Case $\varphi' = \forall x_i\psi$." We need to show that $I \models \forall x_i\psi$ if and only if $g(I, \forall x_i\psi) \models f(\forall x_i\psi)$.

First note that, by the definitions of f and g, we have

$$g(I, \forall x_i\psi) \models f(\forall x_i\psi)$$

if and only if

$$U(i) \cup E(i) \cup g(\psi) \models \big(\$ \vee (\neg q_i \mathsf{U} q_i) \vee \mathsf{F}[\# \wedge \mathsf{X} f(\psi)]\big)\mathsf{U}\#. \tag{2}$$

In the following, we will show that (2) is true if and only if $T(i,b) \cup g(\psi) \models f(\psi)$ for all $b \in \{0,1\}$. From this the correctness follows analogously as in the case for the existential quantifier.

Notice first that each trace in $U(i) \cup E(i) \cup g(\psi)$ is periodic with period length either 3 or 6, and exactly the last element of each period is marked by the symbol #. Consequently, it is easy to see that (2) is true if and only if

$$(U(i) \cup E(i) \cup g(\psi))[j, \infty) \models \$ \vee (\neg q_i \mathsf{U} q_i) \vee \mathsf{F}[\# \wedge \mathsf{X} f(\psi)], \tag{3}$$

for each $j \in \{0,1,2,3,4\}$. Note that

$$(U(i) \cup E(i) \cup g(\psi))[j, \infty) \models \$,$$

for each $j \in \{1,2,4\}$, whereas no non-empty subteam of $(U(i) \cup E(i) \cup g(\psi))[j, \infty)$, $j \in \{0,3\}$ satisfies \$. Accordingly, (3) is true if and only if

$$(U(i) \cup E(i) \cup g(\psi))[j, \infty) \models (\neg q_i \mathsf{U} q_i) \vee \mathsf{F}[\# \wedge \mathsf{X} f(\psi)], \tag{4}$$

for both $j \in \{0,3\}$. Note that, by construction, q_i does not occur positively in $g(\psi)$. As a result, $X \cap g(\psi)[j, \infty) = \emptyset$, $j \in \{0,3\}$, for all teams X s.t. $X \models \neg q_i \mathsf{U} q_i$. Also, none of the symbols $x_{i'}$, $c_{i'}$, $q_{i''}$, for i', $i'' \in \mathbb{N}$ with $i'' \neq i$, occurs positively in $U(i)$. On that account, $X \cap U(i)[j, \infty) = \emptyset$, $j \in \{0,3\}$, for all X s.t. $X \models \mathsf{F}[\# \wedge \mathsf{X} f(\psi)]$, for eventually each trace in X will end up in a team that satisfies one of the formulae of the form $\mathsf{F}x_{i'}$, $\mathsf{F}c_{i'}$, or $\mathsf{F}q_{i''}$ (see the inductive definition of f). Moreover, it is easy to check that $(T(i,1) \cup U(i))[0, \infty) \models \neg q_i \mathsf{U} q_i$, $(T(i,0) \cup U(i))[0, \infty) \not\models \neg q_i \mathsf{U} q_i$, $(T(i,0) \cup U(i))[3, \infty) \models \neg q_i \mathsf{U} q_i$, and $(T(i,1) \cup U(i))[3, \infty) \not\models \neg q_i \mathsf{U} q_i$. From these, together with downward closure, it follows that (4) is true if and only if for $b_0 = 1$ and $b_3 = 0$

$$(U(i) \cup T(i, b_j))[j, \infty) \models \neg q_i \mathsf{U} q_i, \text{ for all } j \in \{0,3\} \tag{5}$$

and

$$(T(i, 1-b_j) \cup g(\psi))[j, \infty) \models \mathsf{F}[\# \wedge \mathsf{X} f(\psi)], \tag{6}$$

for both $j \in \{0,3\}$. In fact, as (5) always is the case, (4) is equivalent with (6). By construction, (6) is true if and only if $(T(i,b) \cup g(\psi))[6, \infty) \models f(\psi)$, for both $b \in \{0,1\}$. Now, since

$$(T(i,b) \cup g(\psi))[6, \infty) = T(i,b) \cup g(\psi)$$

the claim applies. □

Now we turn our attention to proving a matching upper bound. To this end, we need to introduce some notation to manipulate team encodings. Given a pair (t_0, t_1) of traces $t_0 = t_0(0)\cdots t_0(n)$ and $t_1 = t_1(0)\cdots t_1(n')$, we define $(t_0, t_1)[1, \infty)$ to be $(t_0(1)\cdots t_0(n), t_1)$ if $t_0 \neq \varepsilon$, and to be $(\varepsilon, t_1(1)\cdots t_1(n')t_1(0))$ if $t_0 = \varepsilon$. Furthermore, we inductively define $(t_0, t_1)[i, \infty)$ to be (t_0, t_1) if $i = 0$, and to be $((t_0, t_1)[1, \infty))[i-1, \infty)$ if $i > 0$. Then,

$$\llbracket (t_0, t_1)[i, \infty) \rrbracket = (\llbracket (t_0, t_1) \rrbracket)[i, \infty),$$

that is, we have implemented the prefix-removal operation on the finite representation. Furthermore, we lift this operation to team encodings $\mathcal{T}$ by defining $\mathcal{T}[i, \infty) = \{ (t_0, t_1)[i, \infty) \mid (t_0, t_1) \in \mathcal{T} \}$. As a result, we have $\llbracket \mathcal{T}[i, \infty) \rrbracket = (\llbracket \mathcal{T} \rrbracket)[i, \infty)$.

Given a finite team encoding $\mathcal{T}$, let

$$\mathrm{prfx}(\mathcal{T}) = \max\{ |t_0| \mid (t_0, t_1) \in \mathcal{T} \}$$

and let $\mathrm{lcm}(\mathcal{T})$ be the *least common multiple* of $\{ |t_1| \mid (t_0, t_1) \in \mathcal{T} \}$. Then, $\mathcal{T}[i, \infty) = \mathcal{T}[i + \mathrm{lcm}(\mathcal{T}), \infty)$ for every $i \geq \mathrm{prfx}(\mathcal{T})$.

Furthermore, observe that if $\mathcal{T}$ is a finite team and let $i \geq \mathrm{prfx}(\mathcal{T})$, then $\mathcal{T}[i, \infty)$ and $\mathcal{T}[i + \mathrm{lcm}(\mathcal{T}), \infty)$ satisfy exactly the same LTL-formulae under team semantics.

Remark 1. *In particular, we obtain the following consequences for temporal operators (for finite $\mathcal{T}$):*
$\mathcal{T} \models \psi \mathsf{U} \varphi$ *iff* $\exists k \leq \mathrm{prfx}(\mathcal{T}) + \mathrm{lcm}(\mathcal{T})$ *such that* $\mathcal{T}[k, \infty) \models \varphi$ *and* $\forall k' < k : \mathcal{T}[k', \infty) \models \psi$
$\mathcal{T} \models \psi \mathsf{R} \varphi$ *iff* $\forall k \leq \mathrm{prfx}(\mathcal{T}) + \mathrm{lcm}(\mathcal{T})$ *we have that* $\mathcal{T}[k, \infty) \models \varphi$ *or* $\exists k' < k : \mathcal{T}[k', \infty) \models \psi$

Accordingly, we can restrict the range of the temporal operators when model checking a finite team encoding. This implies that a straightforward recursive algorithm implementing team semantics solves TPC(LTL).

Lemma 4.2. TPC(LTL) *is in* PSPACE.

Proof. Consider Algorithm 1 where $\vee$ and $\bigvee$ denote classical (meta level) disjunctions, not splitjunctions, which are used to combine results from recursive calls.

Algorithm 1: Algorithm for TPC(LTL).

1 **Procedure** chk(*Team encoding* $\mathcal{T}$, *formula* φ);
2 **if** $\varphi = p$ **then return** $\bigwedge_{(t_0,t_1)\in\mathcal{T}}\ p \in t_0t_1(0)$;
3 **if** $\varphi = \neg p$ **then return** $\bigwedge_{(t_0,t_1)\in\mathcal{T}}\ p \notin t_0t_1(0)$;
4 **if** $\varphi = \psi \wedge \psi'$ **then return** $\mathsf{chk}(\mathcal{T}, \psi) \wedge \mathsf{chk}(\mathcal{T}, \psi')$;
5 **if** $\varphi = \psi \vee \psi'$ **then return** $\bigvee_{\mathcal{T}' \subseteq \mathcal{T}} \mathsf{chk}(\mathcal{T}', \psi) \wedge \mathsf{chk}(\mathcal{T} \setminus \mathcal{T}', \psi')$;
6 **if** $\varphi = \mathsf{X}\psi$ **then return** $\mathsf{chk}(\mathcal{T}[1, \infty), \psi)$;
7 **if** $\varphi = \psi \mathsf{U} \psi'$ **then return** ($\bigvee_{k \leq \mathrm{prfx}(\mathcal{T})+\mathrm{lcm}(\mathcal{T})} \mathsf{chk}(\mathcal{T}[k, \infty), \psi') \wedge \bigwedge_{k'<k} \mathsf{chk}(\mathcal{T}[k', \infty), \psi)$);
8 **if** $\varphi = \psi \mathsf{R} \psi'$ **then return** ($\bigwedge_{k \leq \mathrm{prfx}(\mathcal{T})+\mathrm{lcm}(\mathcal{T})} \mathsf{chk}(\mathcal{T}[k, \infty), \psi') \vee \bigvee_{k'<k} \mathsf{chk}(\mathcal{T}[k', \infty), \psi)$);

The algorithm is an implementation of the team semantics for LTL with slight restrictions to obtain the desired complexity. In line 5, we only consider strict splits, i.e., the team is split into two disjoint parts. This is sufficient due to downward closure. Furthermore, the scope of the temporal operators in lines 7 and 8 is restricted to the interval $[0, \mathrm{prfx}(\mathcal{T}) + \mathrm{lcm}(\mathcal{T})]$. This is sufficient due to Remark 1.

It remains to analyse the algorithm's space complexity. Its recursion depth is bounded by the size of the formula. Further, in each recursive call, a team encoding has to be stored. Additionally, in lines 5 and 7 to 10, a disjunction or conjunction of exponential arity has to be evaluated. In each case, this only requires linear space in the input to make the recursive calls and to aggregate the return value. Thus, Algorithm 1 is implementable in polynomial space. □

Combining Lemmas 4.1 and 4.2 settles the complexity of TPC(LTL).

Theorem 4.3. TPC(LTL) *is* PSPACE*-complete w.r.t.* $\leq_m^p$*-reductions.*

4.2 Model Checking

Here, we consider the model checking problem TMC for TeamLTL. We show that model checking for splitjunction-free formulae is in PSPACE (as is LTL model checking under standard semantics). Then, we discuss the challenges one has to overcome to generalize this result to formulae with splitjunctions, which we leave as an open problem.

Theorem 4.4. TMC(LTL) *restricted to splitjunction-free formulae is in* PSPACE*.*

Proof. Fix a Kripke structure $\mathcal{K} = (W, R, \eta, w_I)$ and a splitjunction-free formula φ. We define $S_0 = \{w_I\}$ and $S_{i+1} = \{ w' \in W \mid (w, w') \in R \text{ for some } w \in S_i \}$ for all $i \geq 0$. By the pigeonhole principle, this sequence is ultimately periodic with a characteristic (s, p) satisfying $s + p \leq 2^{|W|}$.[1] Next, we define a trace t over $\mathrm{AP} \cup \{ \overline{p} \mid p \in \mathrm{AP} \}$ via

$$t(i) = \{ p \in \mathrm{AP} \mid p \in \eta(w) \text{ for all } w \in S_i \} \cup \{ \overline{p} \mid p \notin \eta(w) \text{ for all } w \in S_i \}$$

that reflects the team semantics of (negated) atomic formulae, which have to hold in every element of the team.

An induction over the construction of φ shows that $T(\mathcal{K}) \models \varphi$ if and only if $t \models \overline{\varphi}$ (i.e., under classical LTL semantics), where $\overline{\varphi}$ is obtained from φ by replacing each negated atomic proposition $\neg p$ by $\overline{p}$. To conclude the proof, we show that $t \models \overline{\varphi}$ can be checked in non-deterministic polynomial space, exploiting the fact that t is ultimately periodic and of the same characteristic as $S_0S_1S_2\cdots$. However, as $s + p$ might be exponential, we cannot just construct a finite representation of t of characteristic (s, p) and then check satisfaction in polynomial space.

Instead, we present an on-the-fly approach which is inspired by similar algorithms in the literature. It is based on two properties:

(1) Every S_i can be represented in polynomial space, and from S_i one can compute S_{i+1} in polynomial time.
(2) For every LTL-formula $\overline{\varphi}$, there is an equivalent non-deterministic Büchi automaton $\mathcal{A}_{\overline{\varphi}}$ of exponential size (see, e.g., [3] for a formal definition of Büchi automata and for the construction of $\mathcal{A}_{\overline{\varphi}}$). States of $\mathcal{A}_{\overline{\varphi}}$ can be represented in polynomial space and given two states, one can check in polynomial time, whether one is a successor of the other.

These properties allow us to construct both t and a run of $\mathcal{A}_{\overline{\varphi}}$ on t on the fly.

In detail, the algorithm works as follows. It guesses a set $S^* \subseteq W$ and a state q^* of $\mathcal{A}_{\overline{\varphi}}$ and checks whether there are $i < j$ satisfying the following properties:

- $S^* = S_i = S_j$,
- q^* is reachable from the initial state of $\mathcal{A}_{\overline{\varphi}}$ by some run on the prefix $t(0)\cdots t(i)$, and
- q^* is reachable from q^* by some run on the infix $t(i+1)\cdots t(j)$. This run has to visit at least one accepting state.

By an application of the pigeonhole principle, we can assume w.l.o.g. that j is at most exponential in $|W|$ and in $|\varphi|$.

[1]The characteristic of an encoding (t_0, t_1) of an ultimately periodic trace $t_0t_1t_1t_1\cdots$ is the pair $(|t_0|, |t_1|)$. Slightly abusively, we say that $(|t_0|, |t_1|)$ is the characteristic of $t_0t_1t_1t_1\cdots$, although this is not unique.

Let us argue that these properties can be checked in non-deterministic polynomial space. Given some guessed S^*, we can check the existence of $i < j$ as required by computing the sequence $S_0S_1S_2\cdots$ on-the-fly, i.e., by just keeping the current set in memory, comparing it to S^*, then computing its successor, and then discarding the current set. While checking these reachability properties, the algorithm also guesses corresponding runs as required in the second and third property. As argued above, both tasks can be implemented in non-deterministic space. To ensure termination, we stop this search when the exponential upper bound on j is reached. This is possible using a counter with polynomially many bits and does not compromise completeness, as argued above.

It remains to argue that the algorithm is correct. First, assume $t \models \overline{\varphi}$, which implies that $\mathcal{A}_{\overline{\varphi}}$ has an accepting run on t. Recall that t is ultimately periodic with characteristic (s, p) such that $s + p \leq 2^{|W|}$ and that $\mathcal{A}_{\overline{\varphi}}$ is of exponential size. As a result, a pumping argument yields $i < j$ with the desired properties.

Secondly, assume the algorithm finds $i < j$ with the desired properties. Then, the run to q and the one from q to q can be turned into an accepting run of $\mathcal{A}_{\overline{\varphi}}$ on t. That being so, $t \models \overline{\varphi}$. □

Note that as long as we disallow splitjunctions our algorithm is able to deal with contradictory negations and other extensions; we will return to this topic shortly in the next section.

The complexity of the general model checking problem is left open. It is trivially PSPACE-hard, due to Theorem 4.3 and the fact that finite teams of ultimately periodic traces can be represented by Kripke structures. However, the problem is potentially much harder as one has to deal with infinitely many splits of possibly uncountable teams with non-periodic traces, if a split occurs under the scope of a G-operator.

5 Extensions of TeamLTL

Next, we take a brief look into extensions of our logic. Extensions present a flexible way to delineate the expressivity and complexity of team-based logics. The philosophy behind extensions is to consider what are the fundamental hyperproperties that we want a logic to be able to express and add those as new atomic expressions.

The most well studied atomic expressions considered in team semantics are dependence and inclusion atoms. Intuitively, the dependence atom $\mathrm{dep}(p_1, \ldots, p_n; q_1, \ldots, q_m)$ expresses that the truth values of the variables $q_1, \ldots, q_m$ are functionally determined by the truth values of $p_1, \ldots, p_n$. Formally, for Teams $T \subseteq (2^{\mathrm{AP}})^\omega$, the satisfaction of a dependence atom $T \models \mathrm{dep}(p_1, \ldots, p_n; q_1, \ldots, q_m)$ has the following meaning:

$$\forall t, t' \in T: \ (t(0) \overset{p_1}{\Leftrightarrow} t'(0) \wedge \cdots \wedge t(0) \overset{p_n}{\Leftrightarrow} t'(0)) \text{ implies } (t(0) \overset{q_1}{\Leftrightarrow} t'(0) \wedge \cdots \wedge t(0) \overset{q_m}{\Leftrightarrow} t'(0)),$$

where $t(i) \overset{p}{\Leftrightarrow} t(j)$ means the sets $t(i)$ and $t(j)$ agree on proposition p, i.e., both contain p or not. Observe that the formula $\mathrm{dep}(;p)$ merely means that p has to be constant on the team. Often, due to convenience we will write $\mathrm{dep}(p)$ instead of $\mathrm{dep}(;p)$. Note that the hyperproperty 'input determinism', i.e., feeding the same input to a system multiple times yields identical behaviour, now can be very easily expressed via the formula $\mathrm{dep}(i_1, \ldots, i_n; o_1, \ldots, o_m)$, where i_j are the (public) input variables and o_j are the (public) output variables.

Inclusion atoms $(p_1, \ldots, p_n) \subseteq (q_1, \ldots, q_n)$ on the other hand express the inclusion dependency that all the values occurring for $p_1, \ldots, p_n$ must also occur as truth values for $q_1, \ldots, q_m$. Hence, if $o_1, \ldots, o_n$ denote public observable bits and c is a bit revealing confidential information, then the atom $(o_1, \ldots o_n, c) \subseteq (o_1, \ldots o_n, \neg c)$ expresses a form of non-inference by stating that an observer cannot infer the value of the confidential bit from the public observables.

One can also take a more abstract view and consider atoms whose semantics can be written in first-order (FO) logic over some trace properties; this includes both dependence and inclusion atoms as special cases. The notion of generalised atoms in the setting of first-order team semantics was introduced by Kuusisto [45]. An n-ary generalised atom is an expression of the form $\#(\varphi_1, \dots, \varphi_n)$ that takes n LTL-formulae as parameters. We consider FO-formulae over the signature $(A_{x_i})_{1\leq i\leq n}$, where each A_{x_i} is a unary predicate, as defining formulae for n-ary generalised atoms. We interpret a team T as a relational structure $\mathfrak{A}(T)$ over the same signature with universe T. When we evaluate $\#(\varphi_1, \dots, \varphi_n)$, the interpretations of A_{x_i} in $\mathfrak{A}(T)$ are determined by the interpretation of the parameters of #. That is, $t \in T$ is in $A_{x_i}^{\mathfrak{A}}$ if and only if $t \models \varphi_i$ in the classical semantics of LTL.

Definition 5.1. An FO-formula ψ *defines* the n-ary generalised atom # if $T \models \#(\varphi_1, \dots, \varphi_n) \iff \mathfrak{A}(T) \models \psi$. In this case, # is also called an *FO-definable generalised atom.*

In the view of FO-definable atoms, the dependence atom $\mathrm{dep}(p; q)$ is FO-definable by

$$\forall t \forall t' \big((A_{x_1}(t) \leftrightarrow A_{x_1}(t')) \rightarrow (A_{x_2}(t) \leftrightarrow A_{x_2}(t')) \big)$$

We call an LTL-formula extended by a generalised atom # an LTL(#)-formula. Similarly, we lift this notion to *sets of generalised atoms* as well as to the corresponding decision problems, i.e., TPC(LTL(#)) is the team path checking problem for LTL-formulae which may use the generalised atom #. In the same way this notion is lifted to sets of generalised atoms $\mathcal{D}$.

Another way to extend TeamLTL is to introduce additional connectives. Here the usual connectives to consider are the Boolean disjunction $\varovee$ and the contradictory negation $\sim$. A team T satisfies $\varphi \varovee \psi$ if it satisfies φ or ψ (or both). Contradictory negation combined with team semantics allows for powerful constructions. For instance, the complexity of model checking for propositional logic jumps from NC^1 to PSPACE [54], whereas the complexity of validity and satisfiability jumps all the way to alternating exponential time with polynomially many alternations (ATIME-ALT(exp, pol)) [34]. Formally, we define that $T \models {\sim}\varphi$ if $T \not\models \varphi$. Note that the contradictory negation $\sim$ is not equivalent to the negation $\neg$ of atomic propositions defined earlier, i.e., ${\sim}p$ and $\neg p$ are not equivalent.

It turns out that the algorithm for TPC(LTL) (Algorithm 1 on page 11) is very robust to strengthenings of the logic via the aforementioned constructs. The result of Theorem 4.3 can be extended to facilitate also the Boolean disjunction, contradictory negation and first-order definable generalised atoms.

Theorem 5.2. *Let $\mathcal{D}$ be a finite set of first-order definable generalised atoms.* TPC(LTL($\varovee, \sim, \mathcal{D}$)) *is* PSPACE-*complete w.r.t.* $\leq_{\mathrm{m}}^{\mathrm{P}}$*-reductions.*

Proof. The lower bound applies from Theorem 4.3. For the upper bound, we extend the algorithm stated in the proof of Lemma 4.2 for the cases for the Boolean disjunction, contradictory negation, and FO-definable atoms. The case for the Boolean disjunction is obtained by adding the following line to the recursive algorithm of the proof of Lemma 4.2 (where the latter $\vee$ denotes the classical meta level disjunction):

$$\textbf{if } \varphi = \psi \varovee \psi' \textbf{ then return } \mathsf{chk}(\mathcal{T}, \psi) \vee \mathsf{chk}(\mathcal{T}, \psi')$$

The case for the contradictory negation is obtained by adding the following line to the recursive algorithm (where $\neg$ denotes the classical meta level negation):

$$\textbf{if } \varphi = {\sim}\varphi' \textbf{ then return } \neg\mathsf{chk}(\mathcal{T}, \varphi')$$

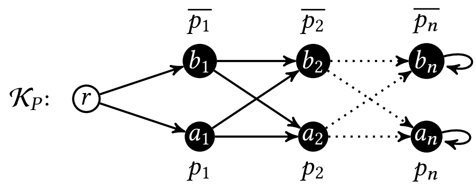

Fig. 4. Kripke structure for the proof of Theorem 5.4.

Note that, since the extension is not anymore a downward closed logic, we need to modify the case for the splitjunction to reflect this. Hence we use the case:

$$\textbf{if } \varphi = \psi \vee \psi' \textbf{ then return } \bigvee_{\mathcal{T}_1 \cup \mathcal{T}_2 = \mathcal{T}} \mathsf{chk}(\mathcal{T}_1, \psi) \wedge \mathsf{chk}(\mathcal{T}_2, \psi')$$

Finally, whenever a first-order definable atom # appears in the computation of the algorithm, we need to solve an FO model checking problem and classical LTL model checking problems. As FO model checking is solvable in logarithmic space for any fixed formula [36] and LTL model checking can be done in NC (combined complexity) [43] the theorem follows. □

The next proposition translates a result from Hannula et al. [34] to our setting. They show completeness for ATIME-ALT(exp, pol) for the satisfiability problem of propositional team logic with contradictory negation. This logic coincides with LTL-formulae without temporal operators with team semantics.

Proposition 5.3 ([34]). TSAT(LTL($\sim$)) *for formulae without temporal operators is complete for the class* ATIME-ALT(exp, pol) *w.r.t.* $\leq_m^p$*-reductions.*

The result from the previous proposition will be utilised in the proof of the next theorem. It shows that already a very simple fragment of LTL($\sim$) has an ATIME-ALT(exp, pol)-hard model checking problem. Lück has established that the general problem is complete for third-order arithmetic [48].

Theorem 5.4. TMC(LTL($\sim$)) *is* ATIME-ALT(exp, pol)*-hard w.r.t.* $\leq_m^p$*-reductions.*

Proof. We will state a reduction from the satisfiability problem of propositional team logic with the contradictory negation $\sim$ (short PL($\sim$)). The stated hardness then follows from Proposition 5.3.

For $P = \{p_1, \dots, p_n\}$, consider the traces starting from the root r of the Kripke structure $\mathcal{K}_P$ depicted in Figure 4 using proposition symbols $p_1, \dots, p_n, \overline{p_1}, \dots, \overline{p_n}$. Each trace in the model corresponds to a propositional assignment on P. For $\varphi \in$ PL($\sim$), let φ^* denote the LTL($\sim$)-formula obtained by simultaneously replacing each (non-negated) variable p_i by $\mathsf{F}p_i$ and each negated variable $\neg p_i$ by $\mathsf{F}\overline{p_i}$. Let P denote the set of variables that occur in φ. Recall that we defined $\top \coloneqq (p \vee \neg p)$ and $\bot \coloneqq p \wedge \neg p$, then $T(\mathcal{K}_P) \models \big(\top \vee ((\sim\bot) \wedge \varphi^*)\big)$ *if and only if* $T' \models \varphi^*$ for some non-empty $T' \subseteq T(\mathcal{K}_P)$. It is easy to check that $T' \models \varphi^*$ *if and only if* the propositional team corresponding to T' satisfies φ and thus the above holds if and only if φ is satisfiable. □

In the following, problems of the form TSAT(LTL(dep)), etc., refer to LTL-formulae with dependence atoms dep. The following proposition follows from the corresponding result for classical LTL using downward closure and the fact that on singleton teams dependence atoms are trivially fulfilled.

Proposition 5.5. TSAT(LTL) *and* TSAT(LTL(dep)) *are* PSPACE*-complete w.r.t.* $\leq_m^p$*-reductions.*

The following result from Virtema talks about the validity problem of propositional team logic.

Proposition 5.6 ([66]). *Validity of propositional logic with dependence atoms is* NEXPTIME-*complete w.r.t.* $\leq_m^p$*-reductions.*

We use this result to obtain a lower bound on the complexity of TMC(dep).

Theorem 5.7. TMC(dep) *is* NEXPTIME-*hard w.r.t.* $\leq_m^p$*-reductions.*

Proof. The proof of this result uses the same construction idea as in the proof of Theorem 5.4, but this time from a different problem, namely, validity of propositional logic with dependence atoms which settles the lower bound by Proposition 5.6. Due to downward closure the validity of propositional formulae with dependence atoms boils down to model checking the maximal team in the propositional (and not in the trace) setting, which essentially is achieved by $T(\mathcal{K})$, where $\mathcal{K}$ is the Kripke structure from the proof of Theorem 5.4. □

6 TeamLTL vs. HyperLTL

TeamLTL expresses hyperproperties [17], that is, sets of teams, or equivalently, sets of sets of traces. HyperLTL [16], which extends LTL by trace quantification, is another logic expressing hyperproperties. For example, input determinism can be expressed as follows: every pair of traces that coincides on their input variables, also coincides on their output variables (this can be expressed in TeamLTL by a dependence atom dep as sketched in Section 5). This results in a powerful formalism with vastly different properties than LTL [24]. After introducing syntax and semantics of HyperLTL, we compare the expressive power of TeamLTL and HyperLTL.

The formulae of HyperLTL are given by the grammar

$$\varphi ::= \exists \pi.\varphi \mid \forall \pi.\varphi \mid \psi, \qquad \psi ::= p_\pi \mid \neg\psi \mid \psi \vee \psi \mid \mathsf{X}\psi \mid \psi \mathsf{U} \psi,$$

where p ranges over atomic propositions in AP and where π ranges over a given countable set $\mathcal{V}$ of *trace variables*. The other Boolean connectives and the temporal operators release R, eventually F, and always G are derived as usual, due to closure under negation. A sentence is a closed formula, i.e., one without free trace variables.

The semantics of HyperLTL is defined with respect to trace assignments that are a partial mappings $\Pi\colon \mathcal{V} \to (2^{\mathrm{AP}})^\omega$. The assignment with empty domain is denoted by $\Pi_\emptyset$. Given a trace assignment Π, a trace variable π, and a trace t, denote by $\Pi[\pi \to t]$ the assignment that coincides with Π everywhere but at π, which is mapped to t. Further, $\Pi[i,\infty)$ denotes the assignment mapping every π in Π's domain to $\Pi(\pi)[i,\infty)$. For teams T and trace-assignments Π we define the following:[2]

$(T,\Pi) \models p_\pi$ if $p \in \Pi(\pi)(0)$,
$(T,\Pi) \models \neg\psi$ if $(T,\Pi) \not\models \psi$,
$(T,\Pi) \models \psi_1 \vee \psi_2$ if $(T,\Pi) \models \psi_1$ or $(T,\Pi) \models \psi_2$,
$(T,\Pi) \models \mathsf{X}\psi$ if $(T,\Pi[1,\infty)) \models \psi$,
$(T,\Pi) \models \psi_1 \mathsf{U} \psi_2$ if $\exists k \geq 0$ such that $(T,\Pi[k,\infty)) \models \psi_2$ and $\forall 0 \leq k' < k$ we have that $(T,\Pi[k',\infty)) \models \psi_1$,
$(T,\Pi) \models \exists \pi.\psi$ if $\exists t \in T$ such that $(T,\Pi[\pi \to t]) \models \psi$, and
$(T,\Pi) \models \forall \pi.\psi$ if $\forall t \in T$ we have that $(T,\Pi[\pi \to t]) \models \psi$.

We say that T satisfies a sentence φ, if $(T,\Pi_\emptyset) \models \varphi$, and write $T \models \varphi$.

The semantics of HyperLTL is synchronous; this can be seen from how the until and next operators are defined. Hence, one could expect that HyperLTL is closely related to TeamLTL as defined here, which is synchronous as well. In the following, we refute this intuition. In fact, HyperLTL is closely related to the asynchronous variant of TeamLTL [39].

[2]Note that we use the same $\models$ symbol for HyperLTL that we used for TeamLTL. It is clear from the context which semantics is used.

Formally, a HyperLTL sentence φ and an LTL-formula φ' with team semantics are equivalent, if for all teams T we have that $T \models \varphi$ if and only if $T \models \varphi'$.

Theorem 6.1.

(1) *No* LTL*-formula with team semantics is equivalent to* $\exists\pi.p_\pi$.
(2) *No* HyperLTL *sentence is equivalent to* $\mathsf{F}p$ *with team semantics.*

Proof. (1) Consider $T = \{\emptyset^\omega, \{p\}\emptyset^\omega\}$. We have $T \models \exists\pi.p_\pi$. Assume there is an equivalent LTL-formula with team semantics, call it φ. Then, $T \models \varphi$ and thus $\{\emptyset^\omega\} \models \varphi$ by downward closure. Hence, by equivalence, $\{\emptyset^\omega\} \models \exists\pi.p_\pi$, yielding a contradiction.

(2) Bozzelli et al. proved that the property encoded by $\mathsf{F}p$ with team semantics cannot be expressed in HyperLTL [12]. □

Note that these separations are obtained by very simple formulae. Furthermore, the same separation hold for LTL(dep), using the same arguments.

Corollary 6.2. HyperLTL *and* TeamLTL *are of incomparable expressiveness.*

7 Team Semantics for CTL

After having presented team semantics for LTL, arguably the most important specification language for linear time properties, we now turn our attention to branching time properties. Here, we present team semantics for CTL.

Recall that in the LTL setting teams were sets of traces, as LTL is evaluated on a single trace. CTL is evaluated on a vertex of a Kripke structure. Consequently, teams in the CTL setting are (multi-)sets of vertices. For the complexity results of this paper, it does not really matter whether team semantics for CTL is defined with respect to multi-sets or plain sets. However, we have chosen multi-sets since the subsequent logic will be conceptually simpler than a variant based on sets. See the work of Kontinen et al. [38] for a development of asynchronous TeamLTL under set-based semantics. The challenges there to define the right set-based semantics for asynchronous TeamLTL are analogous to what would rise from the use of path quantifiers in CTL.

We will reuse some notions from LTL, e.g., the set countably infinite set of propositions is AP. The set of all CTL-formulae is defined inductively via the following grammar:

$$\varphi ::= p \mid \neg p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \mathsf{PX}\varphi \mid \mathsf{P}[\varphi \mathsf{U} \varphi] \mid \mathsf{P}[\varphi \mathsf{R} \varphi],$$

where $\mathsf{P} \in \{\mathsf{A}, \mathsf{E}\}$ and $p \in \mathrm{AP}$. We define the following usual shorthands: $\top \coloneqq p \vee \neg p$, $\bot \coloneqq p \wedge \neg p$, $\mathsf{F}\varphi \coloneqq [\top \mathsf{U} \varphi]$, and $\mathsf{G}\varphi \coloneqq [\bot \mathsf{R} \varphi]$. Note that the formulae are in negation normal form (NNF) which is the convention in team semantics. In the classical setting this is not a severe restriction as transforming a given formula into its NNF requires linear time in the input length. In team semantics, the issue is more involved as here $\neg\varphi$ does not have an agreed compositional semantics and the contradictory negation often increases the complexity and expressiveness of a logic considerably as was seen in Section 5.

We will again use Kripke structures as previously defined for LTL. When the initial world is not important, we will omit it from the tuple definition. By $\Pi(w)$, for a world w in a Kripke structure $\mathcal{K}$, we denote the (possibly infinite) set of all paths π for which $\pi(0) = w$. For a set $V \subseteq W$, we define $\Pi(V) \coloneqq \bigcup_{w \in V} \Pi(w)$.

Definition 7.1. Let $\mathcal{K} = (W, R, \eta)$ be a Kripke structure and let $w \in W$ a world. The satisfaction relation $\models$ for CTL is defined as follows:

$$
\begin{array}{ll}
\mathcal{K}, w \models p & \text{iff } p \in \eta(w),\\
\mathcal{K}, w \models \neg p & \text{iff } p \notin \eta(w),\\
\mathcal{K}, w \models \varphi \wedge \psi & \text{iff } \mathcal{K}, w \models \varphi \text{ and } \mathcal{K}, w \models \psi,\\
\mathcal{K}, w \models \varphi \vee \psi & \text{iff } \mathcal{K}, w \models \varphi \text{ or } \mathcal{K}, w \models \psi,\\
\mathcal{K}, w \models \mathsf{EX}\varphi & \text{iff } \exists \pi \in \Pi(w) \text{ such that } \mathcal{K}, \pi(1) \models \psi,\\
\mathcal{K}, w \models \mathsf{E}[\varphi \mathsf{U} \psi] & \text{iff } \exists \pi \in \Pi(w) \exists k \in \mathbb{N} \text{ such that } \mathcal{K}, \pi(k) \models \psi \text{ and}\\
& \quad \forall i < k \text{ we have that } \mathcal{K}, \pi(i) \models \varphi, \text{ and}\\
\mathcal{K}, w \models \mathsf{E}[\varphi \mathsf{R} \psi] & \text{iff } \exists \pi \in \Pi(w) \forall k \in \mathbb{N} \text{ we have that } \mathcal{K}, \pi(k) \models \psi \text{ or}\\
& \quad \exists i < k \text{ such that } \mathcal{K}, \pi(i) \models \varphi,\\
\mathcal{K}, w \models \mathsf{AX}\varphi & \text{iff } \forall \pi \in \Pi(w) \text{ such that } \mathcal{K}, \pi(1) \models \psi,\\
\mathcal{K}, w \models \mathsf{A}[\varphi \mathsf{U} \psi] & \text{iff } \forall \pi \in \Pi(w) \exists k \in \mathbb{N} \text{ such that } \mathcal{K}, \pi(k) \models \psi \text{ and}\\
& \quad \forall i < k \text{ we have that } \mathcal{K}, \pi(i) \models \varphi, \text{ and}\\
\mathcal{K}, w \models \mathsf{A}[\varphi \mathsf{R} \psi] & \text{iff } \forall \pi \in \Pi(w) \forall k \in \mathbb{N} \text{ we have that } \mathcal{K}, \pi(k) \models \psi \text{ or}\\
& \quad \exists i < k \text{ such that } \mathcal{K}, \pi(i) \models \varphi.
\end{array}
$$

Next, we will introduce team semantics for CTL based on multisets, obtaining the logic TeamCTL. Notice that for TeamLTL, a team is a set of traces while here it is a multiset of worlds mimicking the behaviour of satisfaction in vanilla CTL. A *multiset* is a generalisation of the concept of a set that allows for having multiple instances of the same element. Technically, we define that a multiset is a functional set of pairs (i, x), where i is an element of some sufficiently large set $\mathbb{I}$ of indices, and x is an element whose multiplicities we encode in the multiset. The *support* of a multiset M is the image set $M[\mathbb{I}] \coloneqq \{x \mid (i, x) \in M, i \in \mathbb{I}\}$, while the *multiplicity* of an element x in a multiset M is the cardinality of the pre-image $M^{-1}[\{x\}] \coloneqq \{i \in \mathbb{I} \mid (i, x) \in M\}$. We often omit the index, and write simply x instead of (i, x) to simplify the notation. We also write $\{\!\{\dots\}\!\}$ to denote a multiset with its elements inside the curly brackets omitting the indices. For multisets A and B, we write $A \uplus B \coloneqq \{((i, 0), x) \mid (i, x) \in A\} \cup \{((i, 1), x) \mid (i, x) \in B\}$ to denote the disjoint union of A and B. Note that $A \cup B$ denotes the standard union of the sets A and B, which is not always a (functional) multiset. In the following, we introduce some notation for multisets.

Definition 7.2. Let A, B be two multisets over some index set $\mathbb{I}$. We write $A \overset{\#}{=} B$ be if there exists a permutation $\pi\colon \mathbb{I} \to \mathbb{I}$ such that $A = \{\, (\pi(i), x) \mid (i, x) \in B \,\}$. Likewise, we write $A \overset{\#}{\subseteq} B$ if there exists a permutation $\pi\colon \mathbb{I} \to \mathbb{I}$ such that $A \subseteq \{\, (\pi(i), x) \mid (i, x) \in B \,\}$.

Now, we turn to the definition of teams in the setting of CTL. Note that this notion is different from the notion of a team in the context of LTL, where a team is a possibly infinite set of traces.

Definition 7.3. Let $\mathcal{K} = (W, R, \eta)$ be a Kripke structure. Any multiset T where $T[\mathbb{I}] \subseteq W$ is called a *(multi)team of* $\mathcal{K}$.

By abuse of notation, if the indices are irrelevant, we will write $w \in T$ instead of $(i, w) \in T$.

Let $\mathcal{K} = (W, R, \eta)$ be a Kripke structure and T be a team of $\mathcal{K}$. If $f\colon T \to \Pi(W)$ is a function that maps elements of the team to paths in W such that $f((i, x)) \in \Pi(x)$ for all $(i, x) \in T$ then we call f *T-compatible*, and define $T[f]$ to be the following multiset of traces

$$T[f] \coloneqq \{\, (i, f((i, x))) \mid (i, x) \in T \,\}.$$

Let $n \in \mathbb{N}$. We define $T[f, n]$ to be the team of $\mathcal{K}$

$$T[f, n] \coloneqq \{\, (i, t(n)) \mid (i, t) \in T[f] \,\},$$

that is, the multiset of worlds reached by synchronously proceeding to the n-th element of each trace.

Next, we define (synchronous) team semantics for CTL.

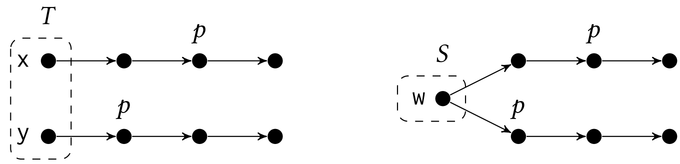

Fig. 5. (left) A team T does not satisfy EFp with team semantics while all singleton teams $\{t\} \subseteq T$ individually do satisfy the formula. (right) The multiplicity of elements in a team matters: If the multiplicity of the world w in S is 1 then S satisfies AFp, while for larger multiplicities S falsifies AFp.

Definition 7.4. Let $\mathcal{K} = (W, R, \eta)$ be a Kripke structure, T be a team of $\mathcal{K}$, and φ, ψ be CTL-formulae. The satisfaction relation $\models$ for CTL is defined as follows.

$\mathcal{K}, T \models p$	iff	$\forall w \in T$ we have that $p \in \eta(w)$.
$\mathcal{K}, T \models \neg p$	iff	$\forall w \in T$ we have that $p \notin \eta(w)$.
$\mathcal{K}, T \models \varphi \wedge \psi$	iff	$\mathcal{K}, T \models \varphi$ and $\mathcal{K}, T \models \psi$.
$\mathcal{K}, T \models \varphi \vee \psi$	iff	$\exists T_1 \uplus T_2 = T$ such that $\mathcal{K}, T_1 \models \varphi$ and $\mathcal{K}, T_2 \models \psi$.
$\mathcal{K}, T \models \mathsf{EX}\varphi$	iff	$\exists\, T$-compatible f such that $\mathcal{K}, T[f, 1] \models \varphi$.
$\mathcal{K}, T \models \mathsf{E}[\varphi \mathsf{U} \psi]$	iff	$\exists\, T$-compatible f $\exists k \in \mathbb{N}$ such that $\mathcal{K}, T[f, k] \models \psi$ and $\forall 1 \leq i < k : \mathcal{K}, T[f, i] \models \varphi$.
$\mathcal{K}, T \models \mathsf{E}[\varphi \mathsf{R} \psi]$	iff	$\exists\, T$-compatible f $\forall k \in \mathbb{N}$ we have that $\mathcal{K}, T[f, k] \models \psi$ or $\exists 1 \leq i < k : \mathcal{K}, T[f, i] \models \varphi$.
$\mathcal{K}, T \models \mathsf{AX}\varphi$	iff	$\forall\, T$-compatible f we have that $\mathcal{K}, T[f, 1] \models \varphi$.
$\mathcal{K}, T \models \mathsf{A}[\varphi \mathsf{U} \psi]$	iff	$\forall\, T$-compatible f $\exists k \in \mathbb{N}$ such that $\mathcal{K}, T[f, k] \models \psi$ and $\forall 1 \leq i < k$ we have that $\mathcal{K}, T[f, i] \models \varphi$.
$\mathcal{K}, T \models \mathsf{A}[\varphi \mathsf{R} \psi]$	iff	$\forall\, T$-compatible f $\forall k \in \mathbb{N}$ we have that $\mathcal{K}, T[f, k] \models \psi$ or $\exists 1 \leq i < k$ such that $\mathcal{K}, T[f, i] \models \varphi$.

Basic Properties. In the following, we investigate several fundamental properties of the satisfaction relation. Notice that these properties also hold for TeamLTL as defined in Section 2 on page 6. Observe that $\mathcal{K}, T \models \bot$ if and only if $T = \emptyset$. The proof of the following proposition is trivial.

Proposition 7.5 (Empty team property). *For every Kripke structure $\mathcal{K}$ we have that $\mathcal{K}, \emptyset \models \varphi$ holds for every* CTL*-formula φ.*

When restricted to singleton teams, team semantics coincides with the traditional semantics of CTL defined via pointed Kripke structure.

Proposition 7.6 (Singleton equivalence). *For every Kripke structure $\mathcal{K} = (W, R, \eta)$, every world $w \in W$, and every* CTL*-formula φ the following equivalence holds:*

$$\mathcal{K}, \{\!\{w\}\!\} \models \varphi \Leftrightarrow \mathcal{K}, w \models \varphi.$$

Proof. Let $\mathcal{K} = (W, R, \eta)$ be an arbitrary Kripke structure. We first prove the claim via induction on the structure of φ:

Assume that φ is a (negated) proposition symbol p. Now

$$\begin{aligned} &\mathcal{K}, w \models \varphi \\ &\quad\text{iff}\quad p \text{ is (not) in } \eta(w') \\ &\quad\text{iff}\quad \text{for all } w' \in \{\!\{w\}\!\} \text{ it holds that } p \text{ is (not) in } \eta(w') \\ &\quad\text{iff}\quad \mathcal{K}, \{\!\{w\}\!\} \models \varphi. \end{aligned}$$

The case $\wedge$ trivial. For the $\vee$ case, assume that $\varphi = \psi \vee \theta$. Now it holds that

$$\begin{aligned} &\mathcal{K}, w \models \psi \vee \theta \\ &\quad\text{iff}\quad \mathcal{K}, w \models \psi \text{ or } \mathcal{K}, w \models \theta \\ &\quad\text{iff}\quad \mathcal{K}, \{\!\{w\}\!\} \models \psi \text{ or } \mathcal{K}, \{\!\{w\}\!\} \models \theta \\ &\quad\text{iff}\quad (\mathcal{K}, \{\!\{w\}\!\} \models \psi \text{ and } \mathcal{K}, \emptyset \models \theta) \text{ or } (\mathcal{K}, \emptyset \models \psi \text{ and } \mathcal{K}, \{\!\{w\}\!\} \models \theta) \\ &\quad\text{iff}\quad \exists T_1 \cup T_2 = \{\!\{w\}\!\} \text{ s.t. } \mathcal{K}, T_1 \models \psi \text{ and } \mathcal{K}, T_2 \models \theta \\ &\quad\text{iff}\quad \mathcal{K}, \{\!\{w\}\!\} \models \psi \vee \theta. \end{aligned}$$

Here the first equivalence holds by the semantics of disjunction, the second equivalence follows by the induction hypothesis, the third via the empty team property, the fourth via the empty team property in combination with the semantics of "or", and the last by the team semantics of disjunction.

The cases for EX and AX, until and weak until are all similar and straightforward. We show here the case for EX. Assume $\varphi = \mathsf{EX}\psi$. Then, $\mathcal{K}, w \models \mathsf{EX}\psi$ iff there exists a trace $\pi \in \Pi(w)$ such that $\mathcal{K}, \pi(1) \models \psi$. Thus, by the induction hypothesis $\mathcal{K}, \pi(1) \models \psi$ iff $\mathcal{K}, \{\!\{\pi(1)\}\!\} \models \psi$. Hence, equivalently, there exists a T-compatible function f such that $\mathcal{K}, T[f, 1] \models \psi$ which is equivalent to $\mathcal{K}, T \models \mathsf{EX}\varphi$. □

TeamCTL is *downward closed* if the following holds for every Kripke structure $\mathcal{K}$, for every CTL-formula φ, and for every team T and T' of $\mathcal{K}$:

$$\text{If } \mathcal{K}, T \models \varphi \text{ and } T' \Subset T \text{ then } \mathcal{K}, T' \models \varphi.$$

The proof of the following lemma is analogous with the corresponding proofs for modal and first-order dependence logic (see [64, 65]).

Proposition 7.7 (Downward closure). TeamCTL *is downward closed.*

Proof. The proof is by induction on the structure of φ. Let $\mathcal{K} = (W, R, \eta)$ be an arbitrary Kripke structure and $T' \Subset T$ be some team of $\mathcal{K}$. Hence, there is a bijection π witnessing $T' \Subset T$, i.e., it satisfies $T' \subseteq \{ (\pi(i), x) \mid (i, x) \in T \}$.

The cases for literals are trivial: Assume $\mathcal{K}, T \models p$. Then by definition $p \in \eta(w)$ for every $w \in T$. Now since $T' \Subset T$, clearly $p \in \eta(w)$ for every $w \in T'$. Thus, we have that $\mathcal{K}, T' \models p$. The case for negated propositions symbols is identical.

The case for $\wedge$ is clear. For the case for $\varphi \vee \psi$ assume that $\mathcal{K}, T \models \varphi \vee \psi$. Now, by the definition of disjunction there exist $T_1 \uplus T_2 = T$ such that $\mathcal{K}, T_1 \models \varphi$ and $\mathcal{K}, T_2 \models \psi$. Define $T_1' = \{ (\pi(i), x) \in T' \mid (i, x) \in T_1 \}$ and T_2' analogously. Then, we have $T' = T_1' \cup T_2'$ and $T_1' \Subset T'$ as well as $T_2' \Subset T'$ (the inclusions are both witnessed by π from above). Hence, by induction hypothesis it then follows that $\mathcal{K}, T_1' \models \varphi$ and $\mathcal{K}, T_2' \models \psi$. Finally, it follows by the semantics of the disjunction that $\mathcal{K}, T' \models \varphi \vee \psi$.

Now consider $\mathsf{EX}\varphi$ and assume that $\mathcal{K}, T \models \mathsf{EX}\varphi$. We have to show that $\mathcal{K}, T' \models \mathsf{EX}\varphi$ for every $T' \Subset T$. Notice that by the semantics of EX there exists a T-compatible function f such that $\mathcal{K}, T[f, 1] \models \varphi$. Now, since $T' \Subset T$, the function $f \upharpoonright T'$ (i.e., the restriction of f to the domain T') is T'-compatible. Consequently, by induction hypothesis, $\mathcal{K}, T'[f \upharpoonright T', 1] \models \varphi$ and thus $\mathcal{K}, T' \models \mathsf{EX}\varphi$.

The proofs for the cases for all remaining operators are analogous. □

Similar to TeamLTL, CTL violates flatness and union closure. In this setting, a logic is *union closed* if it satisfies

$$\mathcal{K}, T \models \varphi \text{ and } \mathcal{K}, T' \models \varphi \text{ implies } \mathcal{K}, T \uplus T' \models \varphi,$$

and has the flatness property if

$$\mathcal{K}, T \models \varphi \text{ if and only if for all } w \in T \text{ it holds that } \mathcal{K}, \{\!\{w\}\!\} \models \varphi.$$

Proposition 7.8. CTL *is neither union closed nor satisfies flatness.*

Proof. The counter example for both is depicted in Figure 5 (left). □

In addition, as exemplified in the right-hand side of Figure 5 on Page 19, TeamCTL is sensitive for multiplicities.

Proposition 7.9. *Let T and T' be teams with the same support. In general, it need not hold that $\mathcal{K}, T \models \varphi$ if and only if $\mathcal{K}, T' \models \varphi$.*

Comparison to TeamCTL with Time Evaluation Functions. After the publication of the conference version of this work [40], Gutsfeld et al. [29] revisited the foundations of temporal team semantics and introduced several logics based on a concept they aptly named *time evaluation functions* (tef). Note first that, using a slightly different notation than in this paper, the satisfying element in (synchronous) TeamLTL is a pair (T, i), where T is a set of traces and $i \in \mathbb{N}$ is the current time step. In this paper, we write $T[i, \infty]$ instead of (T, i). Hence in synchronous TeamLTL each trace share access to a common clock. Simply put, time evaluation functions $\tau \colon T \times \mathbb{N} \to \mathbb{N}$ model asynchronous evaluation of time on traces which are equipped with local clocks. The idea is that $\tau(i, t)$ denotes the value of the local clock of trace t at global time i. Hence the satisfying element in this setting is a triplet (T, τ, n), where T is a set of traces, τ is a time evaluation function, and n is the value of the global clock. TeamLTL with tef's is then defined similar to our TeamLTL, where logical operators modify T and n. In particular, if τ is the synchronous tef (i.e., $\tau(i, t) = i$ for all i and t) then exactly synchronous TeamLTL, as defined in this paper, is obtained. Finally they consider a logic they also named TeamCTL which takes the syntax of CTL and reinterprets path quantifiers as quantifiers raging over tef's. For further details, we refer to the work of Gutsfeld et al. [29].

The semantics of Definition 7.4 for CTL can be modified to a semantics for the full branching time logic CTL* by working with multisets of traces with time steps $(T[f], n)$ instead of teams $T[f, n]$. This approach is essentially equivalent to synchronous time evaluation functions as in the work of Gutsfeld et al. [29], when interpreted over computation trees.

8 Expressive Power of TeamCTL

Next, we study about the expressiveness of TeamCTL and exemplify that some simple properties are not expressible in CTL with classical semantics. In order to relate the team semantics to the classical semantics, we first present a definition that lifts the classical semantics to multisets of worlds.

Definition 8.1. For every Kripke structure $\mathcal{K} = (W, R, \eta)$, every CTL-formula φ, and every team T of $\mathcal{K}$, we let

$$\mathcal{K}, T \models^c \varphi \text{ if and only if } \forall w \in T : \mathcal{K}, w \models \varphi.$$

We refer to this notion as *classical (multiset) semantics.*

Observe that classical multiset semantics is flat by definition (i.e., $\mathcal{K}, T \models^c \varphi$ if and only if for all $w \in T$ we have that $\mathcal{K}, \{w\} \models^c \varphi$), and hence union and downward closed. In the following, we introduce the notions of definability and k-definability.

Definition 8.2. For each CTL-formula φ, define

$$\mathfrak{F}^c_\varphi \coloneqq \{(\mathcal{K}, T) \mid \mathcal{K}, T \models^c \varphi \text{ under classical multiset semantics}\} \text{ and}$$
$$\mathfrak{F}^s_\varphi \coloneqq \{(\mathcal{K}, T) \mid \mathcal{K}, T \models \varphi \text{ under (synchronous) team semantics}\}.$$

We say that φ *defines the class* $\mathfrak{F}^c_\varphi$ *in classical multiset semantics* (of CTL, resp.). Analogously, we say that φ *defines the class* $\mathfrak{F}^s_\varphi$ *in team semantics* (of CTL, resp.).

A class $\mathfrak{F}$ of pairs of Kripke structures and teams is *definable in classical multiset semantics* (in team semantics), if there exists some CTL-formula ψ such that $\mathfrak{F} = \mathfrak{F}^c_\psi$ ($\mathfrak{F} = \mathfrak{F}^s_\psi$). Furthermore, for $k \in \mathbb{N}$, define

$$\mathfrak{F}^{c,k}_\varphi \coloneqq \{(\mathcal{K}, T) \mid \mathcal{K}, T \models^c \varphi \text{ and } |T| \leq k\} \text{ and } \mathfrak{F}^{s,k}_\varphi \coloneqq \{(\mathcal{K}, T) \mid \mathcal{K}, T \models \varphi \text{ and } |T| \leq k\}.$$

We say that φ *k-defines the class* $\mathfrak{F}^{c,k}_\varphi$ *($\mathfrak{F}^{s,k}_\varphi$, resp.) in classical multiset (team, resp.) semantics* (of CTL). The definition of *k-definability* is analogous to that of definability.

Next we will show that there exists a class $\mathfrak{F}$ which is definable in team semantics, but is not definable in classical multiset semantics.

THEOREM 8.3. *The class* $\mathfrak{F}^s_{\mathsf{EF}p}$ *is not definable in classical multiset semantics.*

PROOF. For the sake of a contradiction, assume that there is a CTL-formula φ such that $\mathfrak{F}^c_\varphi = \mathfrak{F}^s_{\mathsf{EF}p}$. Consider the Kripke structure from Figure 5 (left). Clearly $\mathcal{K}, \{\!\{\mathsf{x}\}\!\} \models \mathsf{EF}p$ and $\mathcal{K}, \{\!\{\mathsf{y}\}\!\} \models \mathsf{EF}p$. Thus by our assumption, it follows that $\mathcal{K}, \{\!\{\mathsf{x}\}\!\} \models^c \varphi$ and $\mathcal{K}, \{\!\{\mathsf{y}\}\!\} \models^c \varphi$. From Definition 8.1 it then follows that $\mathcal{K}, \{\!\{\mathsf{x}, \mathsf{y}\}\!\} \models^c \varphi$. But clearly $\mathcal{K}, \{\!\{\mathsf{x}, \mathsf{y}\}\!\} \not\models \mathsf{EF}p$. □

COROLLARY 8.4. *For $k > 1$, the class* $\mathfrak{F}^{s,k}_{\mathsf{EF}p}$ *is not k-definable in classical multiset semantics.*

We conjecture that the class $\mathfrak{F}^c_{\mathsf{EF}p}$ is not definable in team semantics. However, we will see that the class $\mathfrak{F}^{c,k}_{\mathsf{EF}p}$ is k-definable in team semantics.

THEOREM 8.5. *For every $k \in \mathbb{N}$ and every* CTL*-formula φ, the class* $\mathfrak{F}^{c,k}_\varphi$ *is k-definable in team semantics.*

PROOF. Fix $k \in \mathbb{N}$ and a CTL formula φ. We define

$$\varphi' \coloneqq \bigvee_{1 \leq i \leq k} \varphi.$$

We will show that $\mathfrak{F}^{c,k}_\varphi = \mathfrak{F}^{s,k}_{\varphi'}$. Let $\mathcal{K}$ be an arbitrary Kripke structure and T be a team of $\mathcal{K}$ of size at most k. Then the following is true

$$\begin{aligned} \mathcal{K}, T \models^c \varphi \quad &\Leftrightarrow \quad \forall w \in T : \mathcal{K}, w \models \varphi \\ &\Leftrightarrow \quad \forall w \in T : \mathcal{K}, \{\!\{w\}\!\} \models \varphi \\ &\Leftrightarrow \quad \mathcal{K}, T \models \varphi'. \end{aligned}$$

The first equivalence follows by Definition 8.1, the second is due to Proposition 7.6, and the last by the semantics of disjunction, empty team property, and downward closure. □

9 Complexity Results for TeamCTL

Next, we define the most important decision problems for TeamCTL and classify their computational complexity. Notice that, in comparison to TeamLTL, the problems are defined in a way that a team is explicitly given as part of the input, while for TeamLTL, the team is given implicitly by a Kripke structure.

Problem:	TMC(CTL) — CTL Team Model Checking.
Input:	A Kripke structure $\mathcal{K}$, a team T of $\mathcal{K}$, and a CTL-formula φ.
Question:	$\mathcal{K}, T \models \varphi$?

Problem:	TSAT(CTL) — CTL Team Satisfiability.
Input:	A CTL-formula φ.
Question:	Does there exist a Kripke structure $\mathcal{K}$ and a non-empty team T of $\mathcal{K}$ such that $\mathcal{K}, T \models \varphi$?

9.1 Model Checking

We first investigate the computational complexity of model checking. Our benchmark here is the complexity of model checking for classical CTL.

Proposition 9.1 ([15, 59]). *Model checking for* CTL*-formulae under classical semantics is* P*-complete.*

Now we turn to the model checking problem for TeamCTL. Here we show that the problem becomes intractable under reasonable complexity class separation assumptions, i.e., P ≠ PSPACE. The main idea is to exploit the synchronicity of the team semantics in a way to check in parallel all clauses of a given quantified Boolean formula for satisfiability by a set of relevant assignments.

Lemma 9.2. TMC(CTL) *is* PSPACE*-hard w.r.t.* $\leq^{\mathrm{p}}_{\mathrm{m}}$*-reductions.*

Proof. We will reduce from QBF-VAL. Let $\varphi \coloneqq \exists x_1 \forall x_2 \cdots Q x_n \bigwedge_{i=1}^{m} \bigvee_{j=1}^{3} \ell_{i,j}$ be a closed quantified Boolean formula (QBF) and $Q = \exists$ if n is odd, resp., $Q = \forall$ if n is even.

Now define the corresponding structure $\mathcal{K} \coloneqq (W, R, \eta)$ as follows (the structure is given in Figure 6). The structure $\mathcal{K}$ is constructed out of smaller structures in a modular way. Formally, if $\mathcal{K}_1 = (W_1, R_1, \eta_1)$ and $\mathcal{K}_2 = (W_2, R_2, \eta_2)$ are two Kripke structures, then let $\mathcal{K}_1 \cup \mathcal{K}_2$ be defined as the Kripke structure $(W_1 \cup W_2, R_1 \cup R_2, \eta_1 \cup \eta_2)$. Without loss of generality, we will always assume that $W_1 \cap W_2 = \emptyset$, $R_1 \cap R_2 = \emptyset$, and $\eta_1 \cap \eta_2 = \emptyset$. For each x_i define a Kripke structure $\mathcal{K}_{x_i} \coloneqq (W_{x_i}, R_{x_i}, \eta_{x_i})$, where

$$
\begin{aligned}
W_{x_i} &\coloneqq \{w_1^{x_i}, \dots, w_i^{x_i}\} \cup \{\, w_{j,1}^{x_i}, w_{j,2}^{x_i} \mid i \leq j \leq n+4 \,\},\\
R_{x_i} &\coloneqq \{\, (w_j^{x_i}, w_{j+1}^{x_i}) \mid 1 \leq j < i \,\} \cup \{\, (w_{j,a}^{x_i}, w_{j+1}^{x_i}, a) \mid a \in \{1,2\}, i \leq j < n+4 \,\}, \text{ and}\\
&\quad\ \cup \{\, ((w_{n+4,a}^{x_i}, w_{n+4,a}^{x_i})) \mid a \in \{1,2\} \,\}\\
\eta_{x_i} &\coloneqq \{\, (w, \{x_1, \dots, x_n\}) \mid w \in \{w_{n+3,1}^{x_i}, w_{n+4,2}^{x_i}\} \,\} \cup \{\, (w, \{x_1, \dots, x_n\} \setminus \{x_i\}) \mid w \in \{w_{n+4,1}^{x_i}, w_{n+3,2}^{x_i}\} \,\}.
\end{aligned}
$$

If ℓ is a literal, then we will write $\mathcal{V}(\ell)$ to denote the corresponding variable of ℓ. Furthermore, we define a Kripke structure $\mathcal{K}_\varphi \coloneqq (W_\varphi, R_\varphi, \eta_\varphi)$, where

$$
\begin{aligned}
W_\varphi &\coloneqq \{w_i^c \mid 1 \leq i \leq n+1\} \cup \{w^{c_j} \mid 1 \leq j \leq m\} \cup \{w_{j,i,k}^{c_j} \mid 1 \leq j \leq m, 1 \leq i \leq 3, 1 \leq k \leq 2\},\\
R_\varphi &\coloneqq \{\, (w_i^c, w_{i+1}^c) \mid 1 \leq i < n \,\} \cup \{\, (w_{n+1}^c, w^{c_j}) \mid 1 \leq j \leq m \,\}\\
&\quad\ \cup \{(w_i^c, w_{i+1}^c) \mid 1 \leq i < n\}\\
&\quad\ \cup \{(w_{n+1}^c, w^{c_j}) \mid 1 \leq j \leq m\}\\
&\quad\ \cup \{(w^{c_j}, w_{j,i,1}^{c_j}), (w_{j,i,1}^{c_j}, w_{j,i,2}^{c_j}) \mid 1 \leq i \leq 3, 1 \leq j \leq m\}\\
&\quad\ \cup \{(w_{j,i,2}^{c_j}, w_{j,i,2}^{c_j}) \mid 1 \leq i \leq 3, 1 \leq j \leq m\}, \text{ and}\\
\eta_\varphi &\coloneqq \Big\{(w_{j,i,1}^{c_j}, \{x_k \mid \ell_{j,i} = x_k\} \cup \{x_k \mid x_k \neq \mathcal{V}(\ell_{j,i})\}) \Bigm| 1 \leq j \leq m, 1 \leq i \leq 3\Big\}\\
&\quad\ \cup \Big\{(w_{j,i,2}^{c_j}, \{x_k \mid \ell_{j,i} = \neg x_k\} \cup \{x_k \mid x_k \neq \mathcal{V}(\ell_{j,i})\}) \Bigm| 1 \leq j \leq m, 1 \leq i \leq 3\Big\}.
\end{aligned}
$$

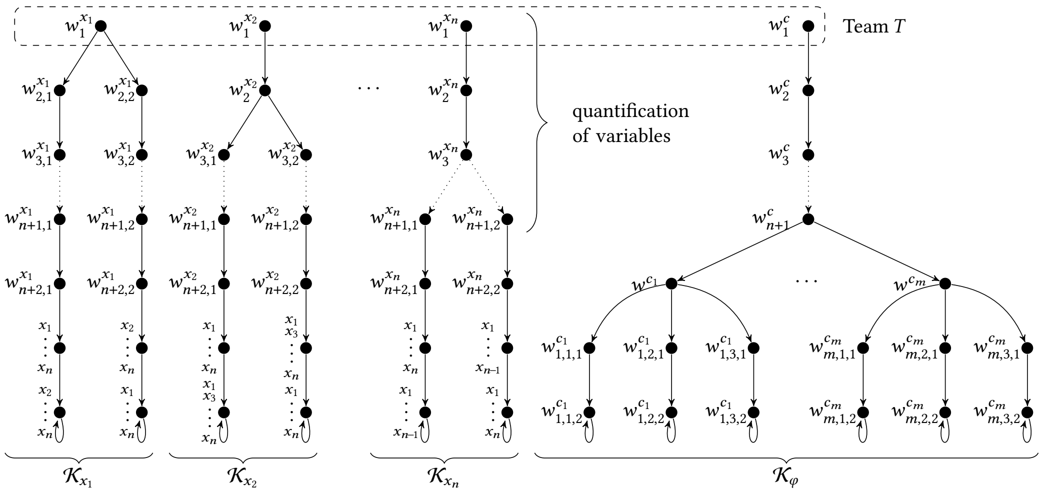

Fig. 6. General view on the created Kripke structure in the proof of Theorem 9.2. For $\mathcal{K}_{x_i}$ choosing the left path of the structure corresponds to setting x_i to 1, choosing the right path to setting x_i to 0. Synchronicity of the variable Kripke structures $\mathcal{K}_{x_i}$ together with the structure $\mathcal{K}_{\varphi}$ ensure choosing consistent values for the x_i's while satisfying all clauses.

Finally, let $\mathcal{K} = (W, R, \eta)$ be the Kripke structure defined as

$$\bigcup_{1\leq i\leq n} \mathcal{K}_{x_i} \cup \mathcal{K}_{\varphi}.$$

Furthermore, set

$$T \coloneqq \{\!\{w_1^{x_1}, \dots, w_1^{x_n}, w_1^c\}\!\} \text{ and } \psi \coloneqq \underbrace{\mathsf{EXAX}\cdots\mathsf{PX}}_{n}\mathsf{AXEX}\bigwedge_{i=1}^{n}\mathsf{EF}x_i,$$

where $\mathsf{P} = \mathsf{E}$ if n is odd and $\mathsf{P} = \mathsf{A}$ if n is even. We define the reduction as $f\colon \langle\varphi\rangle \mapsto \langle\mathcal{K}, T, \psi\rangle$.

Figure 7 shows an example of the reduction for the instance

$$\exists x_1\forall x_2\exists x_3(x_1 \lor \overline{x_2} \lor \overline{x_3}) \land (\overline{x_1} \lor x_2 \lor x_3) \land (\overline{x_1} \lor \overline{x_2} \lor \overline{x_3}).$$

Note that this formula is a valid QBF and hence belongs to QBF-VAL. The left three branching systems choose the values of the x_i's. A decision for the left (right, resp.) path should be understood as setting the variable x_i to the truth value 1 (0, resp.).

For the correctness of the reduction we need to show that $\varphi \in$ QBF-VAL iff $f(\varphi) \in$ TMC(CTL).

"$\Rightarrow$": Let $\varphi \in$ QBF-VAL, $\varphi = \exists x_1\forall x_2\cdots Qx_n\chi$, where $\chi = \bigwedge_{i=1}^{m}\bigvee_{j=1}^{3}\ell_{i,j}$. Thus, there exists a subtree S of the full binary assignment tree on the variable set, obtained by encoding the existential choices of the quantifier prefix $\exists x_1\forall x_2\cdots Qx_n$, such that $s \models \chi$ for every leaf s in the subtree S. The set S specifies the choices for each existential variable depending on the preceeding universal choices.

Now we will prove that $\mathcal{K}, T \models \psi$, where $T = \{\!\{w_1^{x_1}, \dots, w_1^{x_n}, w_1^c\}\!\}$. For w_1^c there is no choice in the next n steps defined by the prefix of φ. For $w_1^{x_1}, \dots, w_1^{x_n}$ we decide the successors as follows depending on the subtree S.

Note that during the evaluation of ψ w.r.t. T and $\mathcal{K}$ in the first n CTL operators of ψ, for each existential variable x_i, the choice for the corresponding successor $w_{2,1}^{x_i}$ or $w_{2,2}^{x_i}$ is determined by the subtree S. If x_i is mapped to 1 then choose in step i of this prefix from $w_i^{x_i}$ the successor world $w_{i+1,1}^{x_i}$. If x_i is mapped to 0 then choose $w_{i+1,2}^{x_i}$ instead.

Now fix an abritrary path in the subtree S which specifies a particular leaf s. Let us write $s(x)$ for the value of x in the leaf s. After n steps, the current team T' then is $\{\{w_{n+1}^c\}\} \cup \{\{w_{n+1,1}^{x_i} \mid s(x_i) = 1, 1 \le i \le n\}\} \cup \{\{w_{n+1,2}^{x_i} \mid s(x_i) = 0, 1 \le i \le n\}\}$ (note that now the team agrees with the assignment in s). In the next step, the team branches now on all clauses of χ and becomes $\{\{w^{c_j} \mid 1 \le j \le m\}\} \cup \{\{w_{n+2,1}^{x_i} \mid s(x_i) = 1, 1 \le i \le n\}\} \cup \{\{w_{n+2,2}^{x_i} \mid s(x_i) = 0, 1 \le i \le n\}\}$. Now continuing with an EX in ψ the team members w^{c_j} of the currently considered state of evaluation have to decide for a literal which satisfies the respective clause. As $s \models \chi$ this must be possible. W.l.o.g. assume that in clause C_j the literal $\ell_{j,i}$ satisfies C_j by $s(\ell_{j,i}) = 1$ for $1 \le j \le m$ and $i \in \{1, 2, 3\}$. Then we choose the world $w_{j,i,1}^{c_j}$ as a successor from w^{c_j} for $1 \le j \le m$.

For the ("variable" team members) $w_{n+2,k}^{x_i}$ with $k \in \{1, 2\}$ we have no choice and proceed to $w_{n+3,k}^{x_i}$. Now we have to satisfy the remainder of φ which is $\bigwedge_{i=1}^{n} \mathsf{EF}x_i$. Observe that for variable team members $w_{n+3,1}^{x_i}$ only has x_i labeled in the current world and not in the successor world $w_{n+4,1}^{x_i}$, i.e., $x_i \notin \eta(w_{n+4,1}^{x_i})$.

Symmetrically this is true for the $w_{n+3,2}^{x_i}$ worlds but $x_i \notin \eta(w_{n+3,2}^{x_i})$ and $x_i \in \eta(w_{n+4,2}^{x_i})$. Hence "staying" in the world (hence immediately satisfying the $\mathsf{EF}x_i$) means setting x_i to true by s whereas making a further step means setting x_i to false by s.

Further observe for the formula team members we have depending on the value of $s(\ell_{j,i})$ that $x \in \eta(w_{n+3,i,1}^{c_j})$ and $x \notin \eta(w_{n+3,i,2}^{c_j})$ if $s(\ell_{j,i}) = 1$, and $x \notin \eta(w_{n+3,i,1}^{c_j})$ and $x \in \eta(w_{n+3,i,2}^{c_j})$ if $s(\ell_{j,i}) = 0$. Thus according to the semantics the step depths w.r.t. an x_i have to be the same for every element of the team. Hence if we decided for the variable team member that $s(x_i) = 1$ then for the formula team members we cannot make a step to the successor world and therefore have to stay (similarly if $s(x_i) = 0$ then we have to do this step).

Note that this is not relevant for other worlds as there all variables are labelled as propositions and are trivially satisfied everywhere. Hence as $\ell_j \models C_j$ we have decided for the world $w_{n+3,2-i}^{x_i}$ and can do a step if $s(\ell_{j,i}) = 0$ and stay if $s(\ell_{j,i}) = 1$. Hence $\mathcal{K}, T \models \varphi$.

For the direction "$\Leftarrow$", observe that with similar arguments we can deduce from the "final" team in the end what has to be a satisfying assignment depending on the choices of $w_{n+3,k}^{x_i}$ and $k \in \{1, 2\}$. Hence by construction any of these assignments satisfies χ. Let again denote by S a satisfying subforest according to AXEX $\bigwedge_{i=1}^{n} x_i$. Then define a paths S' in the full binary assignment tree from S naturally by setting $s(x_i) = 1$ if there is a world $w_{n+1,1}^{x_i}$ in t and otherwise $s(x_i) = 0$. Then, obtain the constructed subtree S'' from these paths. Then it analogously follows that $s \models \chi$. S'' also agrees on the quantifier prefix of φ. Hence $\varphi \in$ QBF-VAL. □

Now, we will turn towards proving the PSPACE upper bound for TMC(CTL). Before, we will need a definition and two auxiliary lemmas. Given two teams T_1, T_2 of a Kripke structure $\mathcal{K}$, we say T_2 is a *successor team* of T_1, if there exists a T_1-compatible function f such that $T_2 \overset{\#}{=} T_1[f, 1]$.

Lemma 9.3. *Let $\mathcal{K}$ be a Kripke structure and T be a team of $\mathcal{K}$. Furthermore, let $T_1, T_2, \dots, T_m$ be a sequence of teams such that T_{i+1} is a successor team of T_i for all i. Then the largest m such that there exists no $1 \le i \ne j \le m$ with $T_i \overset{\#}{=} T_j$ is bounded from above by $|W|^{|T|}$.*

Proof. Let $T = \{\{t_1, \dots, t_n\}\}$ and $W = \{w_1, \dots, w_m\}$. Then, the number of different successor teams is bounded by the number of functions $f : T \to W$. As $|W|^{|T|}$ is the number of all functions from T to W, the claim follows. □

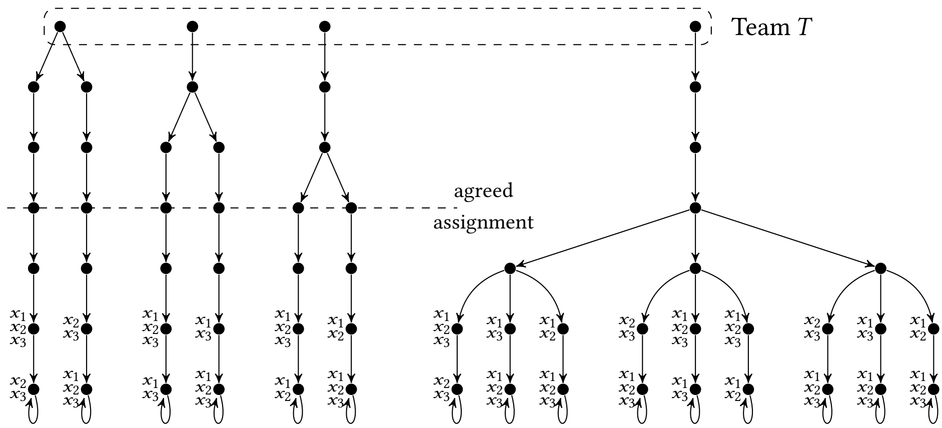

Fig. 7. Example structure built in proof of Lemma 9.2 for the qBf $\exists x_1 \forall x_2 \exists x_3 (x_1 \vee \neg x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3)$.

The following lemma shows that the successor relation over teams can be decided in polymonial time. Here, we measure the size of a team by the cardinality of the multiset and the size of the Kripke structure by the number of its worlds.

Lemma 9.4. *The question whether a given team T_2 is a successor team of a given team T_1 (w.r.t. a given Kripke structure $\mathcal{K}$) can be decided in polynomial time in $|T_1| + |T_2| + |\mathcal{K}|$.*

Proof. Let $T_1 = \{\!\{t_1, \dots, t_n\}\!\}$ and $T_2 = \{\!\{t'_1, \dots, t'_n\}\!\}$. Then, T_2 is a successor team of T_1 if and only if the following is true

(1) $|T_1| = |T_2|$,
(2) for all $w \in T_1$ there exists a $w' \in T_2$ such that wRw', and
(3) for all $w \in T_2$ there exists a $w' \in T_1$ such that $w'Rw$.

The first two items alone do not suffice because some z in T_2 could be reached from two different y's in T_1. The last item ensures that there is no x in T_2 that has no R-predecessor in T_1.

The first item can be checked in time $O(|T_1|)$. The second and third item can both be checked in polynomial time in $|T_1| \cdot |T_2| \cdot |\mathcal{K}|$. Together this is polynomial time in the input length. □

Lemma 9.5. TMC(CTL) *is in* PSPACE.

Proof. We construct an algorithm that runs in alternating polynomial time. As APTIME = PSPACE [14], this proves the desired upper bound.

Before we come to the model checking algorithm, we want to shortly discuss a subroutine that is used in it. This subroutine (depicted in Algorithm 2) is used in the cases for the binary temporal operators. It will recursively determine whether there exists a path of length c between two given teams such that a given formula φ is satisfied at each team of the path. Intuitively, the procedure works similarly as the path search method in the proof of Savitch's Theorem [58] (also, see the textbook [60, Section 8.1]). It will be called for path lengths c that are bound from above by $|W|^{|T|}$ (Lemma 9.3). As every recursion halfs the path length, the recursion depth is bound by $O(\log(|W|^{|T|})) = O(|T| \log |W|)$ and is hence polynomial in the input length.

Next, we define the main algorithm. Let φ be a CTL-formula, $\mathcal{K}$ be a Kripke structure, and T be a team of $\mathcal{K}$. Given these, the algorithmic call $\texttt{MC}(\mathcal{K}, T, \varphi)$ returns true if and only if $\mathcal{K}, T \models \varphi$. The algorithm is depicted in Algorithm 3.

Algorithm 2: Algorithm for PathSearch Procedure.

Procedure `PathSearch`(*Kripke structure* $\mathcal{K}$, *team* T_1, *team* T_2 *with* $|T_1| = |T_2|$, *formula* φ, *integer* c);
if not $\texttt{MC}(\mathcal{K}, T_1, \varphi)$ **or not** $\texttt{MC}(\mathcal{K}, T_2, \varphi)$ **then return** false;
if $c \leq 1$ **then return** ($T_1 \overset{\#}{=} T_2$ **or** T_2 is a successor team of T_1);
existentially branch on all teams T_{mid} of $\mathcal{K}$ with $|T_{\mathrm{mid}}| = |T_1| = |T_2|$;
return $\texttt{PathSearch}(\mathcal{K}, T_1, T_{\mathrm{mid}}, \varphi, \lfloor c/2 \rfloor) \vee \texttt{PathSearch}(\mathcal{K}, T_{\mathrm{mid}}, T_2, \varphi, \lceil c/2 \rceil)$

Intuitively, the algorithm is a recursive alternating tableaux algorithm that branches on the operators of the subformulae of φ. Notice that the recursion depth is bounded by the number of subformulae, hence linearly in the input length. Also note that the size of each guessed value is polynomial in the input size; for the value of c notice that it is encoded in binary. As for the correctness of the algorithm, each step directly implements the corresponding semantics. From Lemma 9.4 we know that the successor team check is in P for the guessed teams. The U (R, resp.) case makes use of Lemma 9.3 and thereby restricts the path length to an exponential value. Hence, the size of the binary encoding of the length is polynomial in the input length. This implies that this case can be checked in (alternating) polynomial time.

This overall guarantees polynomial runtime as well as the correctness of the alternating algorithm. This concludes the proof. □

Theorem 9.6. TMC(CTL) *is* PSPACE*-complete w.r.t.* $\leq_{\mathrm{m}}^{\mathrm{p}}$*-reductions.*

Proof. By Lemma 9.5 we know that TMC(CTL) is in PSPACE. By Theorem 9.2 we know that TMC(CTL) is PSPACE-hard. Thus, TMC(CTL) is PSPACE-complete. □

Similar to LTL, we can consider FO-definable atoms (see Definition 5.1) in the CTL-setting. Here we restrict the parameters to generalised atoms to be formulae in propositional logic. Analogous to the proof of Theorem 5.2, we can extend the model checking algorithm of Lemma 9.5 to deal with FO-definable generalised atoms and the contradictory negation.

Theorem 9.7. *Let* $\mathcal{D}$ *be a finite set of first-order definable generalised atoms.* TMC(CTL($\mathcal{D}$, $\sim$)) *is* PSPACE*-complete w.r.t.* $\leq_{\mathrm{m}}^{\mathrm{p}}$*-reductions.*

9.2 Satisfiability

Again our benchmark here is CTL satisfiability with classical semantics.

Proposition 9.8 ([25, 56]). *Satisfiability for* CTL*-formulae under classical semantics is* EXPTIME*-complete w.r.t.* $\leq_{\mathrm{m}}^{\mathrm{p}}$*-reductions.*

The same complexity result is easily transferred to TeamCTL.

Theorem 9.9. TSAT(CTL) *is* EXPTIME*-complete w.r.t.* $\leq_{\mathrm{m}}^{\mathrm{p}}$*-reductions.*

Proof. The problem merely asks whether there exists a Kripke structure $\mathcal{K}$ and a non-empty team T of $\mathcal{K}$ such that $\mathcal{K}, T \models \varphi$ for given formula CTL-formula φ. By downward closure (Proposition 7.7), it suffices to check whether φ is satisfied by a singleton team. By singleton equivalence (Proposition 7.6) we then immediately obtain the same complexity bounds as for classical CTL satisfiability. Hence, the claim follows from Proposition 9.8. □

10 Conclusions

We introduced and studied team semantics for the temporal logics LTL and CTL. We concluded that TeamLTL (with and without generalized atoms) is a valuable logic which allows to express

Algorithm 3: Algorithm for TMC(CTL).

Procedure MC(*Kripke structure* $\mathcal{K}$*, team* T*, formula* φ);
switch *formula* φ **do**
  case $\varphi = p$ *(resp.,* $\varphi = \neg p$*) and* $p \in \mathrm{AP}$ **do**
    if *for all* $w \in T$ *we have that* $p \in \eta(w)$ *(resp.,* $p \notin \eta(w)$*)* **then return** true **else return** false;
  case $\varphi = \psi_1 \wedge \psi_2$ **do** **return** $\mathsf{MC}(\mathcal{K}, T, \psi_1) \wedge \mathsf{MC}(\mathcal{K}, T, \psi_2)$;
  case $\varphi = \psi_1 \vee \psi_2$ **do**
    existentially branch on all possible splits $T_1 \uplus T_2 = T$ and **return** $\mathsf{MC}(\mathcal{K}, T_1, \psi_1) \wedge \mathsf{MC}(\mathcal{K}, T_2, \psi_2)$
  case $\varphi = \mathsf{EX}\psi$ **do** `// Lemma 9.4 ensures that this in in P`
    existentially branch on all possible successor teams T' of T and **return** $\mathsf{MC}(\mathcal{K}, T', \psi)$
  case $\varphi = \mathsf{AX}\psi$ **do** `// Lemma 9.4 ensures that this in in P`
    universally branch on all possible successor teams T' of T and **return** $\mathsf{MC}(\mathcal{K}, T', \psi)$
  case $\varphi = \mathsf{E}[\varphi \mathsf{U} \psi]$ **do**
    existentially branch on the possible path length $c \in [0, |W|^{|T|}]$ (Lemma 9.3);
    if $c = 0$ **then return** $\mathsf{MC}(\mathcal{K}, T, \psi)$;
    else
      existentially branch on possible $T_{\mathrm{end}-1}, T_{\mathrm{end}}$ such that T_{end} is a successor team of $T_{\mathrm{end}-1}$ and $|T| = |T_{\mathrm{end}}| = |T_{\mathrm{end}-1}|$;
      return $\mathsf{MC}(\mathcal{K}, T_{\mathrm{end}}, \psi) \wedge \mathsf{PathSearch}(\mathcal{K}, T, T_{\mathrm{end}-1}, \varphi, c-1)$
  case $\varphi = \mathsf{E}[\varphi \mathsf{R} \psi]$ **do**
    existentially branch on $\{\mathsf{no}\text{-}\varphi, \mathsf{some}\text{-}\varphi\}$;
    if no-φ **then**
      existentially branch on a possible loop length $c \in [1, |W|^{|T|} + 1]$ (Lemma 9.3);
      existentially branch on possible T_{end} with $|T| = |T_{\mathrm{end}}$ and **return** $\mathsf{PathSearch}(\mathcal{K}, T, T_{\mathrm{end}}, \psi, c)$
    else if some-φ **then**
      existentially branch on the possible path length $c \in [0, |W|^{|T|}]$ (Lemma 9.3);
      if $c = 0$ **then return** $\mathsf{MC}(\mathcal{K}, T, \psi) \wedge \mathsf{MC}(\mathcal{K}, T, \varphi)$;
      else
        existentially branch on possible $T_{\mathrm{end}-1}, T_{\mathrm{end}}$ with $|T| = |T_{\mathrm{end}}| = |T_{\mathrm{end}-1}|$ such that T_{end} is a successor team of $T_{\mathrm{end}-1}$;
        return $\mathsf{MC}(\mathcal{K}, T_{\mathrm{end}}, \psi) \wedge \mathsf{MC}(\mathcal{K}, T_{\mathrm{end}}, \varphi) \wedge \mathsf{PathSearch}(\mathcal{K}, T, T_{\mathrm{end}-1}, \psi, c-1)$
  case $\varphi = \mathsf{A}[\varphi \mathsf{U} \psi]$ **do** analogous to $\mathsf{E}[\varphi \mathsf{U} \psi]$ but using universally branching ;
  case $\varphi = \mathsf{A}[\varphi \mathsf{R} \psi]$ **do** analogous to $\mathsf{E}[\varphi \mathsf{R} \psi]$ but using universally branching ;

relevant hyperproperties and complements the expressiveness of HyperLTL while allowing for computationally simpler decision problems.

For TeamCTL, the complexity of the model checking problem increases from P-complete for usual CTL to PSPACE-complete for TeamCTL. This fact stems from the expressive notion of synchronicity between team members and is in line with the results of Kupferman et al. [44].

We conclude with some directions of future work and open problems.

10.1 Future Work for TeamLTL

We showed that some important properties that cannot be expressed in HyperLTL (such as uniform termination) can be expressed in TeamLTL. Moreover input determinism can be expressed in LTL (dep). Can we identify tractable LTL variants (i.e., syntactic fragments with particular dependency atoms) that can express a rich family of hyperproperties.

We showed that with respect to expressive power HyperLTL and TeamLTL are incomparable. However, the expressive power of HyperLTL and the different extensions of TeamLTL introduced here is left open. For example, the HyperLTL formula $\exists\pi.p_\pi$ is expressible in LTL($\sim$).

Another interesting questions is whether we can characterise the expressive power of relevant extensions of TeamLTL as has been done in first-order and modal contexts? Recent works have shown limits of expressivity of TeamLTL variants via translations to extensions and fragments of HyperLTL [39, 67] and FO [37]. It is also open whether the one-way translations in those papers can be strengthened to precise characterisations of expressivity as was done in the work of Kontinen et al. [39] for asynchronous set-based TeamLTL?

We studied the complexity of the path-checking, model checking, and satisfiability problem of TeamLTL and its extensions. However, many problems are still open: Can we identify matching upper and lower bounds for the missing cases and partial results of Figure 1 on page 4? In particular, what is the complexity of TMC when splitjunctions are allowed?

Finally, the complexity of the validity and implication problems are open for almost all cases.

10.2 Future Work for TeamCTL

We only scratched the surface of the complexity of the satisfiability problem of CTL with team semantics. There are two obvious directions for future work here. Complexity of CTL extended with atoms and connectives that preserve downward closure, and complexity of CTL extended with non-downward closed atoms and connectives. The complexity of the former is expected to stay relatively low and comparable to vanilla CTL, since there it still suffices to consider only singleton teams. The complexity of the latter is expected to be much higher and be perhaps even undecidable. Note that TeamLTL with the contradictory negation is highly undecidable [48].

The tautology or validity problem for this new logic is quite interesting and seems to have a higher complexity than the related satisfiability problem. This is due to alternation of set quantification: the validity problem quantifies over teams universally while the splitjunction implements an existential set quantification. We leave the considerations related to the validity problem as future work. Formally the corresponding problems are defined as follows:

Problem:	TVAL(CTL) — CTL Team Validity Problem.
Input:	A CTL-formula φ.
Question:	Does $\mathcal{K}, T \models \varphi$ hold for every Kripke structure $\mathcal{K}$ and every team T of $\mathcal{K}$?

In the context of team-based modal logics the computational complexity of the validity problem has been studied by Virtema [66], Hannula [31] and Lück [47]. Virtema and Hannula showed that the problem for modal dependence logic is NEXPTIME-complete whereas Lück established that the problem for modal logic extended with the contradictory negation is complete for the complexity class TOWER(poly), the class of problems that can be solved in time that is bounded by some n-fold exponential, where n itself is bounded polynomially in the input length.

It is well-known that there are several ways to measure the complexity of a model checking problem. In general, a model and a formula are given, and then one needs to decide whether the model satisfies the formula. *System complexity* considers the computational complexity for the case of a fixed formula whereas *specification complexity* fixes the underlying Kripke structure. We considered in this paper the *combined complexity* where both a formula and a model belong to the given input. Yet the other two approaches might give more specific insights into the intractability of the model checking case we investigated. In particular the study of so-to-speak *team complexity*, where the team or the team size is assumed to be fixed, might as well be of independent interest.

Finally this leads to the consideration of different kinds of restrictions on the problems. In particular for the quite strong PSPACE-completeness result for model checking in team semantics it is of interest how this intractability can be explained. Hence the investigation of fragments by means of allowed temporal operators and/or Boolean operators will lead to a better understanding of this presumably untameable high complexity.

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