Exploiting Assumptions for Effective Monitoring of Real-Time Properties under Partial Observability*

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Abstract. Runtime verification of temporal properties over timed sequences of observations is crucial in various applications within cyber-physical systems ranging from autonomous vehicles over smart grids to medical devices. In this paper, we are addressing the challenge of effectively predicting the failure or success of properties in a continuous real-time setting. Our approach allows predictions to exploit assumptions on the system being monitored and supports predictions of non-observable system behaviour (e.g. internal faults). More concretely, in our approach properties are expressed in Metric Interval Temporal Logic (MITL), assumptions on the monitored system are specified in terms of Timed Automata, and observations are to be provided in terms of sequences of timed constraints. We present an assumption-based runtime verification algorithm and its implementation on top of the real-time verification tool UPPAAL. We show experimentally that assumptions can be effective in anticipating the satisfaction/violation of timed properties and in handling monitoring properties that predicate over unobservable events.

Keywords: Assumption-based runtime verification · Real-Time · MITL · Timed Automata.

1 Introduction

The problem of monitoring timed properties has gained significant attention due to its crucial role in ensuring the correctness and reliability of real-time systems. The runtime verification of temporal properties over timed sequences of observations is crucial in various applications ranging from cyber-physical systems including autonomous vehicles and beyond. While different solutions for runtime verification of timed temporal properties have been presented [6, 5,

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4, 22, 20], some challenges remain to be addressed, in particular extending these solutions with prognosis and diagnosis capabilities. More specifically, we are here interested in effectively predicting in advance the failure of properties and in handling partially observable systems.

In the discrete-time setting, these challenges have been addressed with Assumption-Based Runtime Verification (ABRV) [12–15]. ABRV uses assumptions about the behavior of the system to predict the future behavior of the system and to relate observable and non-observable variables. These assumptions can be derived, for example, from models produced during the system design, or from the data collected from the system in operation. Exploiting assumptions, the monitor can anticipate the detection of property failures. Moreover, the specification is no more limited to the interface of black box systems as in traditional runtime verification, but can be extended to constrain also the internal non-observable parts (such as, for example, internal faults).

In ABRV, the output of the monitor has four possible values:

- $\top$ (Satisfied): given the sequence of observations, the system satisfies the specified temporal properties under the given assumption.
- $\bot$ (Violated): this value indicates that the observed behavior of the system violates the specified temporal property, under the given assumption.
- $\times$ (Out-of-model): the observed behavior violates the assumptions, i.e., there is no run of the assumption compatible with the observations.
- *?* (Unknown): given the current observations and assumption, it is not possible to determine definitively whether the property is satisfied or violated.

Here, we enhance the monitoring of timed systems with assumptions. We define and solve the problem of ABRV for timed properties for the following setting:

- The *properties* to be monitored are specified in Metric Interval Temporal Logic (MITL), which allows for the expression of temporal properties over timed words, making it suitable for real-time systems.
- The *assumptions* about the system are specified in terms of Timed Automata which relate observable events with non-observable events, locations, and clocks.
- The *observations* are specified in terms of sequences of timed constraints which predicate over the assumption automaton defining the set of its runs that are compatible with the observations.

Like in the discrete-time case, the assumption allows the monitor to give a $\top$ or $\bot$ verdict even if the property contains future operators and non-observable events. For example, suppose we monitor the MITL property $\varphi = F_{[0,10]}a \wedge G_{[0,20]}\neg b$ (expressing that there is an a in the first ten units of time, but no b in the first 20 units of time) and we assume that the system satisfies the property $\psi = G_{[0,1]}\neg b \wedge G(a \rightarrow G_{[0,10]}\neg b)$ (expressing that there is no b in the first unit of time and no a is followed by a b within ten units of time). Then, the monitor can output a $\top$ verdict even before time 20, for instance at time 10 when b is false in the interval $[0, 10]$ and a is true at time 10. Further, it can even give the verdict $\top$ if b is not observable, e.g., when a is true at time 0 and 10.

One of our main contributions is a rich definition of observations that take into account both data and time uncertainty. As in [15], the observations are represented by formulas that can capture the uncertainty on data. For example, $\neg a$ means that a is not seen but b can be true or false. The approach is further extended to have uncertainty on time, taking into account potential errors in the timestamps with which the monitor receives data from the system. This is represented in the observations with time intervals that are associated to observation formulas. Thus, for example, we can say that a is seen in an interval $[6, 7]$ but we do not know exactly when. Finally, we concatenate these pairs of formulas and time intervals to form complex observation patterns. For example, the sequence $o = (a, [0, 0], !)(\neg a, [0, 7], *)(a, [6, 7], !)(\neg a, [6, 16], *)(a, [15, 16], !)$ says that we see three a's, one at time 0, another in the interval $[6, 7]$, and a final one in the interval $[15, 16]$ and that we do not know anything about b (intuitively, an observation with an ! ($*$) indicates exactly one occurrence (zero or more occurrences)). If the system satisfies the assumption ψ from above, we can conclude at time 16 that the property φ is true despite the uncertainty about time and b.

We propose a *zone-based online* algorithm that at any time provides a monitoring verdict saying if the property is satisfied or violated given the assumption and a sequence of observations. We implemented the algorithm on top of UPPAAL and show the feasibility of the approach. Especially, we demonstrate how the assumptions can be effective in anticipating the satisfaction/violation of timed properties and in handling properties that predicate over unobservable events. We also report on the influence of unobservable events on the response-time, the time it takes to compute a verdict when given a new observation.

2 Preliminaries

The set of natural numbers (excluding zero) is $\mathbb{N}$, we define $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, the set of non-negative rational numbers is $\mathbb{Q}_{\geq 0}$, and the set of non-negative real numbers is $\mathbb{R}_{\geq 0}$. The powerset of a set S is denoted by 2^S.

Timed Words. A timed word over a finite alphabet Σ is a pair $\rho = (\sigma, \tau)$ where σ is a nonempty word over Σ and τ is a sequence of non-decreasing non-negative real numbers of the same length as σ. Timed words may be finite or infinite. In the latter case, we require $\limsup \tau = \infty$, i.e., time diverges. The set of finite timed words is denoted by $T\Sigma^*$ and the set of infinite timed words by $T\Sigma^\omega$. We also represent a timed word as a sequence of pairs $(\sigma_1, \tau_1)(\sigma_2, \tau_2)\ldots$. If $\rho = (\sigma_1, \tau_1)(\sigma_2, \tau_2)\cdots(\sigma_n, \tau_n)$ is a finite timed word, we denote by $\tau(\rho)$ the total time duration of ρ, i.e., τ_n. We lift this to languages $L \subseteq T\Sigma^*$ by defining $\tau(L) = \sup_{\rho \in L} \tau(\rho)$, which can be infinite.

If $\rho_1 = (\sigma_1^1, \tau_1^1)\ldots(\sigma_n^1, \tau_n^1)$ is a finite timed word, $\rho_2 = (\sigma_1^2, \tau_1^2)(\sigma_2^2, \tau_2^2)\ldots$ a finite or infinite timed word, and $t \in \mathbb{R}_{\geq 0}$ then the concatenation $\rho_1 \cdot_t \rho_2$ is defined iff $t \geq \tau(\rho_1)$. Then, we define $\rho_1 \cdot_t \rho_2 = (\sigma_1, \tau_1)(\sigma_2, \tau_2)\cdots$ such that

$$\sigma_i = \begin{cases} \sigma_i^1 & \text{if } i \leq n \\ \sigma_{i-n}^2 & \text{else} \end{cases} \quad \text{and} \quad \tau_i = \begin{cases} \tau_i^1 & \text{if } i \leq n \\ \tau_{i-n}^2 + t & \text{else.} \end{cases}$$

We lift this definition to sets $L_1 \subseteq T\Sigma^*$ and $L_2 \subseteq T\Sigma^* \cup T\Sigma^\omega$ via

$$L_1 \cdot_t L_2 = \{\rho_1 \cdot_t \rho_2 \mid \rho_1 \in L_1 \text{ and } \rho_2 \in L_2\},$$

provided we have $t \geq \tau(L_1)$.

Timed Automata. A timed Büchi automaton (TBA) $\mathcal{B} = (Q, Q_0, \Sigma, C, \Delta, \mathcal{F})$ consists of a finite alphabet Σ , a finite set Q of locations, a set $Q_0 \subseteq Q$ of initial locations, a finite set C of clocks, a finite set $\Delta \subseteq Q \times Q \times \Sigma \times 2^C \times G(C)$ of transitions with $G(C)$ being the set of clock constraints over C, and a set $\mathcal{F} \subseteq Q$ of accepting locations. A transition (q, q', a, λ, g) is an edge from q to q' on input symbol a, where λ is the set of clocks to reset and g is a clock constraint over C. A clock constraint is a conjunction of atomic constraints of the form $c \sim n$, where c is a clock, $n \in \mathbb{N}_0$, and $\sim \in \{<, \leq, =, \geq, >\}$.

A state of $\mathcal{B}$ is a pair (q, v) where q is a location in Q and $v \colon C \to \mathbb{R}_{\geq 0}$ is a valuation mapping clocks to their values. For any $d \in \mathbb{R}_{\geq 0}$, $v+d$ is the valuation $x \mapsto v(x) + d$. A run of $\mathcal{B}$ from a state (q_0, v_0) over a timed word $(\sigma_1, \tau_1)(\sigma_2, \tau_2) \cdots$ is a sequence of steps $(q_0, v_0) \stackrel{(\sigma_1,\tau_1)}{\longrightarrow} (q_1, v_1) \stackrel{(\sigma_2,\tau_2)}{\longrightarrow} (q_2, v_2) \stackrel{(\sigma_3,\tau_3)}{\longrightarrow} \cdots$ where for all $i \geq 1$ there is a transition $(q_{i-1}, q_i, \sigma_i, \lambda_i, g_i)$ such that $v_i(c) = 0$ for all c in λ_i and $v_i(c) = v_{i-1}(c) + (\tau_i - \tau_{i-1})$ otherwise, and g_i is satisfied by the valuation $v_{i-1} + (\tau_i - \tau_{i-1})$. Here, we use $\tau_0 = 0$. Given a run r, we denote the set of locations visited infinitely many times by r as $\mathrm{Inf}(r)$. A run r of $\mathcal{B}$ is accepting if $\mathrm{Inf}(r) \cap \mathcal{F} \neq \emptyset$. The language of $\mathcal{B}$ from a starting state (q, v), denoted $L(\mathcal{B}, (q, v))$, is the set of all timed words with an accepting run in $\mathcal{A}$ starting from (q, v). We define the language of $\mathcal{B}$, written $L(\mathcal{B})$, to be $\bigcup_q L(\mathcal{B}, (q, v_0))$, where q ranges over all locations in Q_0 and where $v_0(c) = 0$ for all $c \in C$.

Proposition 1 ([2]). *For all TBA $\mathcal{B}$, $\mathcal{B}'$ there is a TBA $\mathcal{B} \otimes \mathcal{B}'$ with $L(\mathcal{B} \otimes \mathcal{B}') = L(\mathcal{B}) \cap L(\mathcal{B}')$. The set of states of $\mathcal{B} \otimes \mathcal{B}'$ is $Q \times Q' \times \{0, 1\}$, where Q and Q' are the sets of states of $\mathcal{B}$ and $\mathcal{B}'$, respectively.*

Logic. We use Metric Temporal Interval Logic (MITL) to formally express properties to be monitored; these are subsequently translated into equivalent TBA which we use in our monitoring algorithm. The syntax of MITL formulas over a finite alphabet Σ is defined as

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid X_I \varphi \mid \varphi\ U_I \varphi$$

where $p \in \Sigma$ and I ranges over non-singular intervals over $\mathbb{R}_{\geq 0}$ with endpoints in $\mathbb{N}_0 \cup \{\infty\}$. Note that we often write $\sim n$ for $I = \{d \in \mathbb{R} \mid d \sim n\}$ where $\sim \in \{<, \leq, \geq, >\}$, and $n \in \mathbb{N}$. We also define the standard syntactic sugar $\mathtt{true} = p \vee \neg p$, $\mathtt{false} = \neg\mathtt{true}$, $\varphi \wedge \psi = \neg(\neg\varphi \vee \neg\psi)$, $\varphi \to \psi = \neg\varphi \vee \psi$, $F_I \varphi = \mathtt{true}\ U_I \varphi$, and $G_I \varphi = \neg F_I \neg\varphi$.

The semantics of MITL is defined over infinite timed words. Given such a timed word $\rho = (\sigma_1, \tau_1)(\sigma_2, \tau_2) \cdots \in T\Sigma^\omega$, a position $i \geq 1$, and an MITL formula φ, we inductively define the satisfaction relation $\rho, i \models \varphi$ as follows:

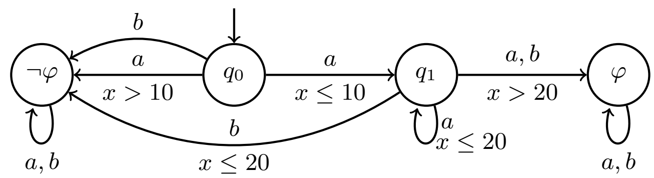

Fig. 1: A TBA for the language of the formula $\varphi = F_{[0,10]} a \wedge G_{[0,20]} \neg b$ and its negation: If location φ ($\neg\varphi$) is accepting then it accepts $L(\varphi)$ ($L(\neg\varphi)$).

- $\rho, i \models p$ iff $p = \sigma_i$.
- $\rho, i \models \neg\varphi$ iff $\rho, i \not\models \varphi$.
- $\rho, i \models \varphi \vee \psi$ if $\rho, i \models \varphi$ or $\rho, i \models \psi$.
- $\rho, i \models X_I \varphi$ iff $\rho, (i+1) \models \varphi$ and $\tau_{i+1} - \tau_i \in I$.
- $\rho, i \models \varphi\, U_I \psi$ iff there exists $k \geq i$ s.t. $\rho, k \models \psi$, $\tau_k - \tau_i \in I$, and $\rho, j \models \varphi$ for all $i \leq j < k$.

We write $\rho \models \varphi$ whenever $\rho, 1 \models \varphi$. The language $L(\varphi)$ of an MITL formula φ is the set of all infinite timed words that satisfy φ.

Theorem 1 ([3, 9]). *For each MITL formula φ there exists a TBA $\mathcal{B}$ such that $L(\varphi) = L(\mathcal{B})$.*

Example 1. Fig. 1 illustrates the above theorem providing a TBA for the formula $F_{[0,10]} a \wedge G_{[0,20]} \neg b$ and its negation.

3 Monitoring under Assumptions

Monitoring timed properties [6, 20] requires to determine whether every extension of a finite observation (a finite timed word) satisfies a given property (yielding the verdict $\top$), whether every extension violates the property (yielding the verdict $\bot$), or neither is true (yielding the verdict *?*). Monitoring under assumptions involves two changes over the classical monitoring framework.

Firstly, the assumption itself: In its most general form, it is a set $A \subseteq T\Sigma^\omega$ of infinite timed words. Intuitively, A contains the executions we assume to be possibly generatable by the system we are monitoring. Hence, every execution that is not in A does not need to be taken into account when determining a verdict, i.e., the assumption refines verdicts. However, this also means that our assumption can be invalidated if we observe an execution prefix that is not consistent with our assumption. This requires a new verdict, denoted by $\times$. In this case, the assumption needs to be refined as it does not match our observation.

Secondly, we allow inexact observations: In the classical setting, we observe a finite timed word $(\sigma_1, \tau_1) \cdots (\sigma_n, \tau_n)$ and reason about its possible extensions. Hence, we implicitly presume that no other events occurred between time 0 and

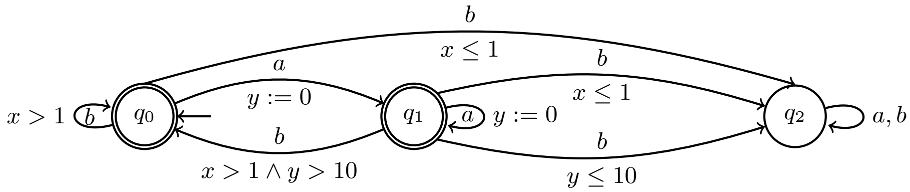

Fig. 2: A TBA for the language of the formula $G_{[0,1]}\neg b \wedge G(a \rightarrow G_{[0,10]}\neg b)$ with accepting locations q_0 and q_1.

τ_n and that the timepoints are exact. In the following, we allow for some imperfect information about the observation. In the most general form, an observation is then a set $O \subseteq T\Sigma^*$ of finite timed words. Intuitively, O contains those words that are consistent with our (imperfect) observation.

Example 2. Consider the property "$F_{[0,10]}a \wedge G_{[0,20]}\neg b$" of Example 1. Monitoring this property on a timed word, a conclusive verdict is given in the following cases:

- The property is false at any time in the interval $[0, 20]$ a "b" is observed;
- The property is false after time 10 if "a" was not previously observed;
- The property is true after time 20 if "b" was not previously observed and "a" was observed in the interval $[0, 10]$.

Consider now the assumption "$G_{[0,1]}\neg b \wedge G(a \rightarrow G_{[0,10]}\neg b)$" which corresponds to the TBA in Fig. 2. Then, if "a" was observed in the interval $[0, 10]$, as soon as we see another "a" within the interval $[10, 20]$ and no b was observed before, we can conclude that the property is true. On the other hand, if a "b" is observed at time 0, then the observation violates the assumption.

Example 3. Let us consider again the property "$F_{[0,10]}a \wedge G_{[0,20]}\neg b$" but now we observe "$a$" with uncertainty on the timestamps and "b" is unobservable. For example, we observe "a" at time 0, another time in the interval $[6, 7]$ and a final time in the interval $[15, 16]$, and now is time 30. The words that are consistent with these observations have the form $\rho_0(a, 0)\rho_1(a, t_1)\rho_2(a, t_2)\rho_3$ where

- $t_1 \in [6, 7]$ and $t_2 \in [15, 16]$,
- ρ_0 is a (possibly empty) finite timed word $(b, 0) \cdots (b, 0)$,
- ρ_1 is a (possibly empty) finite timed word $(b, t_{1,1}) \cdots (b, t_{1,n_1})$ with $t_{1,j} \in [0, t_1]$ for all $1 \leq j \leq n_1$,
- ρ_2 is a (possibly empty) finite timed word $(b, t_{2,1}) \cdots (b, t_{2,n_2})$ with $t_{2,j} \in [t_1, t_2]$ for all $1 \leq j \leq n_2$, and
- ρ_3 is a (possibly empty) finite timed word $(b, t_{3,1}) \cdots (b, t_{3,n_3})$ with $t_{3,j} \in [t_2, 30]$ for all $1 \leq j \leq n_3$.

Without assumptions we cannot have any conclusive verdict, because we do not know if a "b" occurred before timepoint 20 or not. But with the assumption from the previous example, we can conclude at time 16 that the property is true:

- ρ_0 must be empty, as there cannot be a b within the first unit of time.
- ρ_1 must be empty, as there cannot be a b for ten units of time after the a at timepoint 0 and $t_1 \leq 7 \leq 10$.
- ρ_2 must be empty, as there cannot be a b for ten units of time after the a at timepoint t_1 and $t_2 \leq 16 \leq t_1 + 10$.
- ρ_3 cannot contain a b with timestamp $t_{3,j} \leq 20$, as this would imply that a b has occurred less than ten units of time after the a at t_2.

Thus, under the assumption, we can make a definitive verdict, which we could not without the assumption.

In the following, we formalize this intuition. To develop the theory as general as possible, we allow real timepoints in the observations. Later, when we are concerned with algorithms, we will restrict ourselves to rational inputs. In the same spirit, we begin with a very abstract definition of monitoring under assumptions. Later, we will explain how to represent the property, the assumption, and the observation finitely.

Definition 1. *Let $\mathbb{B}_4 = \{\top, \bot, ?, \times\}$. Given a property $\varphi \subseteq T\Sigma^\omega$ of infinite timed words, an assumption $A \subseteq T\Sigma^\omega$, a nonempty observation $O \subseteq T\Sigma^*$, and a current time instant $t \geq \tau(O)$, the function $\mathcal{V}\colon (2^{T\Sigma^\omega} \times 2^{T\Sigma^\omega}) \to (2^{T\Sigma^*} \times \mathbb{R}_{\geq 0}) \to \mathbb{B}_4$ evaluates to a verdict with the following definition:*

$$\mathcal{V}(\varphi, A)(O, t) = \begin{cases} \times & \text{if } O \cdot_t T\Sigma^\omega \cap A = \emptyset, \\ \top & \text{if } O \cdot_t T\Sigma^\omega \cap A \neq \emptyset \text{ and } O \cdot_t T\Sigma^\omega \cap A \subseteq \varphi, \\ \bot & \text{if } O \cdot_t T\Sigma^\omega \cap A \neq \emptyset \text{ and } O \cdot_t T\Sigma^\omega \cap A \subseteq T\Sigma^\omega \setminus \varphi, \\ ? & \text{otherwise.} \end{cases}$$

$\mathcal{V}(\varphi, A)(O, t)$ is undefined when $t < \tau(O)$.

In the following, we present an algorithm computing $\mathcal{V}$ in the setting where

- the property φ and its complement is accepted by a TBA (this covers in particular the case of φ being given in MITL due to Theorem 1),
- the assumption A is given by a TBA, and
- the observation O is given by a sequence of pairs of time-intervals and propositional formulas over the locations, the clock constraints, and the alphabet of the assumption automaton.

We begin by introducing the assumption and observations. The former is given by a TBA, which we typically denote by $\mathcal{A}$ to distinguish it from other TBA. Thus, let $\mathcal{A} = (Q, Q_0, \Sigma, C, \Delta, \mathcal{F})$ be a TBA, i.e., Q is the set of locations, Σ is the alphabet, and C is the set of clocks. Recall that $G(C)$ denotes the clock constraints over C, i.e., conjunctions of atomic constraints of the form $c \sim n$, where $c \in C$ is a clock, $n \in \mathbb{N}_0$, and $\sim \in \{<, \leq, =, \geq, >\}$. Let ϕ be a propositional formula over the set $\Sigma \cup Q \cup G(C)$ of propositions (which is infinite!), let $\sigma \in \Sigma$, and let (q, v) be a state of $\mathcal{A}$. We define $\sigma, (q, v) \models \phi$ as follows:

- For $\sigma' \in \Sigma$, $\sigma, (q, v) \models \sigma'$ iff $\sigma' = \sigma$.
- For $q' \in Q$, $\sigma, (q, v) \models q'$ iff $q' = q$.
- For $g \in G(C)$, $\sigma, (q, v) \models g$ iff g is satisfied by v.
- The semantics of Boolean connectives is defined as usual.

An $\mathcal{A}$-observation is a finite sequence $o = (\phi_1, I_1, m_1) \cdots (\phi_n, I_n, m_n)$ where the ϕ_j are propositional formulas over $\Sigma \cup Q \cup G(C)$, the I_j are bounded intervals of $\mathbb{R}_{\geq 0}$ (which may overlap), and the multiplicities m_j are in $\{!, *\}$. It defines the language $\mathcal{C}_{\mathcal{A}}(o) \subseteq T\Sigma^*$ of (consistent) finite timed words $(\sigma_1, \tau_1) \cdots (\sigma_{n'}, \tau_{n'})$ such that there is a prefix

$$r = (q_0, v_0) \xrightarrow{(\sigma_1,\tau_1)} (q_1, v_1) \xrightarrow{(\sigma_2,\tau_2)} \cdots \xrightarrow{(\sigma_{n'-1},\tau_{n'-1})} (q_{n'-1}, v_{n'-1}) \xrightarrow{(\sigma_{n'},\tau_{n'})} (q_{n'}, v_{n'})$$

of a run of $\mathcal{A}$ with $q_0 \in Q_0$, $v_0(c) = 0$ for all $c \in C$, and there is a function $h \colon \{1, 2, \ldots, n'\} \to \{1, 2, \ldots, n\}$ such that

1. $h(1) \leq h(2) \leq \cdots \leq h(n')$,
2. for every $j \in \{1, 2, \ldots, n\}$ with $m_j =!$, there is a unique $j' \in \{1, 2, \ldots, n'\}$ such that $h(j') = j$ (i.e., observations with an ! must appear exactly once, observations with an $*$ may appear zero or more times),
3. $\tau_j \in I_{h(j)}$ for all $j \in \{1, 2, \ldots, n'\}$, and
4. $\sigma_j, (q_j, v_j) \models \phi_{h(j)}$ for all $j \in \{1, 2, \ldots, n'\}$.

Thus, a finite sequence of such formulas and intervals yields a language of finite timed words, those that are consistent with the formulas and intervals.

Example 4. Let us continue Example 3 and let $\mathcal{A}$ be the assumption automaton shown in Fig. 2. Consider the $\mathcal{A}$-observation

$$o = (a, [0, 0], !)(\neg a, [0, 7], *)(a, [6, 7], !)(\neg a, [6, 16], *)(a, [15, 16], !)(\neg a, [0, 30], *).$$

Then, as argued in Example 3, $\mathcal{C}_{\mathcal{A}}(o)$ is the language

$$\{(a, 0)(a, t_1)(a, t_2)(b, t_{3,1}) \cdots (b, t_{3,n_3}) \mid \\ t_1 \in [6, 7], t_2 \in [15, 16], \text{ and } t_2 + 10 < t_{3,1} \leq \cdots \leq t_{3,n_3} \leq 30\}.$$

For example, given the run prefix (we ignore the clock x as it is never reset and thus is always equal to the timestamp on the transition leading to a state)

$$r = (q_0, y = 0) \xrightarrow{(a,0)} (q_1, y = 0) \xrightarrow{(a,6)} (q_1, y = 0) \xrightarrow{(a,15)} (q_1, y = 0)$$

we can define h as follows: $h(1) = 1, h(2) = 3, h(3) = 5$. For the run prefix

$$r = (q_0, y = 0) \xrightarrow{(a,0)} (q_1, y = 0) \xrightarrow{(a,6)} (q_1, y = 0) \xrightarrow{(b,15)} (q_2, y = 9) \xrightarrow{(a,16)} (q_2, y = 10)$$

we can define the function h as follows: $h(1) = 1, h(2) = 3, h(3) = 4, h(4) = 5$. Finally, the run prefix

$$r = (q_0, y = 0) \xrightarrow{(a,0)} (q_1, y = 0) \xrightarrow{(a,6)} (q_1, y = 0) \xrightarrow{(a,15)} (q_2, y = 9) \xrightarrow{(a,16)} (q_2, y = 10)$$

is the prefix of a run of $\mathcal{A}$ but it is not compatible with the observation o. In fact, any h satisfying the conditions 1), 3), and 4) should assign $h(3) = 5$ and $h(4) = 5$ violating condition 2).

Remark 1. We have $\tau(\mathcal{C}_{\mathcal{A}}((\phi_1, I_1, m_1) \cdots (\phi_n, I_n, m_n))) \leq \sup I_n$ by definition.

4 A Zone-Based Monitoring Algorithm

In this section, we present an algorithm computing the monitoring function $\mathcal{V}$. To this end, we first need to introduce some notation for TBA and zones to represent subsets of states of TBA, which may be uncountable. Recall that we have defined the theory of monitoring under assumptions with respect to arbitrary, i.e., real, timepoints. However, as we are now dealing with algorithms, we have to restrict ourselves to rational inputs (which are finitely representable). Thus, we say that an $\mathcal{A}$-observation $(\phi_1, I_1, m_1) \cdots (\phi_n, I_n, m_n)$ is rational, if each I_j is an interval over $\mathbb{R}_{\geq 0}$ with rational endpoints.

For the monitoring algorithm, we use – as is standard in analysing timed automata models – symbolic states being pairs (q, Z) of locations and zones. A zone is a finite conjunction of constraints of the form $x \sim t$ and $x - x' \sim t$ for clocks x, x', constants $t \in \mathbb{Q}_{\geq 0}$, and $\sim \in \{<, \leq, =, \geq, >\}$. Given two zones Z and Z' over a set C of clocks, and a set $\lambda \subseteq C$ of clocks, we define the following operations on zones (which can be efficiently implemented using the DBM data-structure [8]):

- $Z[\lambda] = \{v \mid \exists v' \models Z \text{ s.t. } v(x) = 0 \text{ if } x \in \lambda, \text{ otherwise } v(x) = v'(x)\}$
- $Z^{\nearrow} = \{v \mid \exists v' \models Z \text{ s.t. } v = v' + d \text{ for some } d \in \mathbb{R}_{\geq 0}\}$
- $Z \wedge Z' = \{v \mid v \models Z \textit{ and } v \models Z'\}$.

To describe our algorithm, we first define the set of states of a TBA from where it is possible to reach an accepting location infinitely many times in the future, i.e., those states from which an accepting run is possible. This is useful, because if processing a finite timed word leads to such a state, then the timed word can be extended to an infinite one in the language of the automaton, a notion that underlies Definition 1. Given a TBA $\mathcal{B} = (Q, Q_0, \Sigma, C, \Delta, \mathcal{F})$, the set of states with nonempty language is:

$$S^{ne}_{\mathcal{B}} = \{(q, v) \mid q \in Q, v \in C \to \mathbb{R}_{\geq 0} \text{ s.t. } L(\mathcal{B}, (q, v)) \neq \emptyset\}.$$

Proposition 2 ([20]). $S^{ne}_{\mathcal{B}}$ *can be computed using a zone-based algorithm.*

We continue by capturing the set of states of a TBA that can be reached by processing a finite timed word. In the following definition, we write $(q_0, v_0) \xrightarrow{\rho}_{\mathcal{B}} (q_n, v_n)$ for a finite timed word $\rho = (\sigma, \tau) \in T\Sigma^*$ to denote the existence of a finite sequence of states

$$(q_0, v_0) \xrightarrow{(\sigma_1, \tau_1)} (q_1, v_1) \xrightarrow{(\sigma_2, \tau_2)} \cdots \xrightarrow{(\sigma_n, \tau_n)} (q_n, v_n)$$

where for all $1 \leq i \leq n$ there is a transition $(q_{i-1}, q_i, \sigma_i, \lambda_i, g_i)$ such that $v_i(c) = 0$ for all c in λ_i and $v_{i-1}(c)+(\tau_i-\tau_{i-1})$ otherwise, and g is satisfied by the valuation $v_{i-1} + (\tau_i - \tau_{i-1})$, where we use $\tau_0 = 0$. Given a TBA $\mathcal{B}$, a finite timed word $\rho \in T\Sigma^*$, and a time-point $t \in \mathbb{R}_{\geq 0}$ with $t \geq \tau(\rho)$, the set of possible states a run over ρ starting from initial states of $\mathcal{B}$ can end in after time t has passed is

$$\mathcal{T}_{\mathcal{B}}(\rho, t) = \bigcup\nolimits_{q_0 \in Q_0} \{(q, v + (t - \tau(\rho))) \mid (q_0, v_0) \xrightarrow{\rho}_{\mathcal{B}} (q, v)\},$$

where v_0 is the clock valuation mapping every clock to 0. We call $\mathcal{T}_{\mathcal{B}}(\rho, t)$ the reach-set of $\mathcal{B}$ over (ρ, t). The above definition is adapted from [20] to take into account the time that has passed since the last observation, i.e., the input t.

Next, we lift this definition to sets $L \subseteq T\Sigma^*$ of finite words via

$$\mathcal{T}_{\mathcal{B}}(L, t) = \bigcup\nolimits_{\rho \in L} \mathcal{T}_{\mathcal{B}}(\rho, t),$$

assuming $t \geq \tau(L)$. Otherwise, $\mathcal{T}_{\mathcal{B}}(L, t) = \emptyset$ by convention.

We now show how to compute reach-sets using zones. First, we use the zone operations introduced above to compute the successor states of an input letter with a given target location. Fix a TBA $(Q, Q_0, \Sigma, C, \Delta, \mathcal{F})$. For a symbolic state (q, Z), a letter $\sigma \in \Sigma$ and target location $q' \in Q$, we define

$$\text{Post}((q, Z), \sigma, q') = \{(q', Z') \mid (q, q', \sigma, \lambda, g) \in \Delta, Z' = (Z^{\nearrow} \wedge g)[\lambda]\},$$

being the set of states one can reach by taking a σ-transition at some point in the future from (q, Z) with q' as target-location. Using Post we can compute the successor states of a time-uncertain letter/target location (σ, q', I), where $\sigma \in \Sigma$, $q' \in Q$ and $I \subseteq \mathbb{R}_{\geq 0}$ is a time interval with rational endpoints. For this, we extend zones with an additional clock *time* just recording time since system start. The successors of a symbolic state (q, Z) are

$$\text{Succ}((q, Z), (\sigma, q', I)) = \{(q', Z') \mid (q', Z'') \in \text{Post}((q, Z), \sigma, q'), Z' = Z'' \wedge time \in I\}$$

and the successors of a set of symbolic states S are

$$\text{Succ}(S, (\sigma, q', I)) = \bigcup\nolimits_{(q,Z) \in S} \text{Succ}((q, Z), (\sigma, q', I)).$$

Now, our main technical lemma below exploits the above to effectively compute reach-sets. More precisely, given a rational $\mathcal{A}$-observation o (i.e., the TBA $\mathcal{A}$ represents the assumption), we can compute the reach-set of the set $\mathcal{C}_{\mathcal{A}}(o)$ in the product $\mathcal{B} \otimes \mathcal{A}$, for any given TBA $\mathcal{B}$, i.e., we compute the words consistent with the observation o in the TBA $\mathcal{A}$ (the assumption), while the reach-set of that language is computed in $\mathcal{B} \otimes \mathcal{A}$ (this will later be the product of the property (or its negation) and the assumption).

Lemma 1. *Fix TBA $\mathcal{A}, \mathcal{B}$. There is a zone-based online algorithm computing*

$$\mathcal{T}_{\mathcal{B} \otimes \mathcal{A}}(\mathcal{C}_{\mathcal{A}}((\phi_1, I_1, m_1) \cdots (\phi_n, I_n, m_n)), t)$$

for every rational observation $(\phi_1, I_1, m_1) \cdots (\phi_n, I_n, m_n)$, and every $t \in \mathbb{Q}_{\geq 0}$ with $t \geq \sup I_n$.

Proof. Let $o^i = (\phi_1, I_1, m_1)\cdots(\phi_i, I_i, m_i)$, and let us denote by $S^i_{\mathcal{B}\otimes\mathcal{A}}$ the set $\mathcal{T}_{\mathcal{B}\otimes\mathcal{A}}(\mathcal{C}_{\mathcal{A}}(o^i))$ of successors of o^i in $\mathcal{B}\otimes\mathcal{A}$. We will show inductively in i, that $S^i_{\mathcal{B}\otimes\mathcal{A}}$ can be obtained effectively using zone operations. For the base case $i = 0$, we note that $\mathcal{C}_{\mathcal{A}}(o^0) = \{\varepsilon\}$, thus $\mathcal{T}_{\mathcal{B}\otimes\mathcal{A}}(\mathcal{C}_{\mathcal{A}}(o^0))$ is the set of initial states of $\mathcal{B}\otimes\mathcal{A}$, which is clearly effectively representable using zones.

For the inductive case, let us assume that $S^{i-1}_{\mathcal{B}\otimes\mathcal{A}}$ is effectively computable using zone operations. Now consider consider (ϕ_i, I_i, m_i). Given that Σ, $Q_{\mathcal{A}}$ and $Q_{\mathcal{B}}$ are finite, ϕ_i is equivalent to a finite disjunction of simple formulas of the form $\psi_{i,j} = \sigma_{i,j} \wedge q^a_{i,j} \wedge g_{i,j}$, where $\sigma_{i,j} \in \Sigma$, $q^a_{i,j} \in Q_{\mathcal{A}}$, and $g_{i,j} \in G(C_{\mathcal{A}})$. Now in the case $m_i = !$, the set of successors of $\psi_{i,j}$ is simply

$$S^i_{\mathcal{B}\otimes\mathcal{A}} = \bigcup_j \bigcup_{q^b\in Q_{\mathcal{B}}} \bigcup_{k\in\{0,1\}} \mathrm{Succ}(S^{i-1}_{\mathcal{B}\otimes\mathcal{A}}, (\sigma_{i,j}, (q^b, q^a_{i,j}, k), I_i)) \wedge g_{i,j}.$$

In the case $m_i = *$, $S^i_{\mathcal{B}\otimes\mathcal{A}}$ is the least fixed-point $\mathbf{X}$, satisfying the equality

$$\mathbf{X} = S^{i-1}_{\mathcal{B}\otimes\mathcal{A}} \cup \bigcup_j \bigcup_{q^b\in Q_{\mathcal{B}}} \bigcup_{k\in\{0,1\}} \left[\mathrm{Succ}(\mathbf{X}, (\sigma_{i,j}, (q^b, q^a_{i,j}, k), I_i)) \wedge g_{i,j}.\right]$$

Given the upper bounds of the interval I_i, the least fixed-point will be found in a finite number of iterations of the right-hand-side of the above equation (starting from the empty set).

The above inductive proof provides in an obvious manner the basis for an effective online construction of the sets $\mathcal{T}_{\mathcal{B}\otimes\mathcal{A}}(\mathcal{C}_{\mathcal{A}}(o^i))$. □

Now, we are able to present our algorithm to compute $\mathcal{V}$ for a property φ (given by two TBA $\mathcal{B}_\varphi$ and $\mathcal{B}_{\neg\varphi}$ such that $L(\mathcal{B}_\varphi) = L(\varphi)$ and $L(\mathcal{B}_{\neg\varphi}) = L(\neg\varphi)$) and an assumption A (given by a TBA $\mathcal{A}$): Given $o = (\phi_1, I_1, m_1)\cdots(\phi_n, I_n, m_n)$ (a rational observation) and $t > \sup I_n$, do the following:

1. Compute $\mathcal{T}_{\mathcal{A}}(\mathcal{C}_{\mathcal{A}}(o), t)$. If it is nonempty (which is the case iff $\mathcal{C}_{\mathcal{A}}(o)$ is nonempty), but has an empty intersection with $S^{ne}_{\mathcal{A}}$, then return $\times$. This checks whether there is some some finite word that is consistent with the observation and can be extended to satisfy the assumption. If this is not the case, then the assumption was wrong.
2. Compute $\mathcal{T}_{\mathcal{B}_{\neg\varphi}\otimes\mathcal{A}}(\mathcal{C}_{\mathcal{A}}(o), t)$. If it has an empty intersection with $S^{ne}_{\mathcal{B}_{\neg\varphi}\otimes\mathcal{A}}$, then return $\top$: If there is a finite word consistent with the observation that can be extended to satisfy the assumption, but no such extension satisfies the complement of the property, then every such extension must satisfy the property. Hence, we can return $\top$.
3. Compute $\mathcal{T}_{\mathcal{B}_{\varphi}\otimes\mathcal{A}}(\mathcal{C}_{\mathcal{A}}(o), t)$. If it has an empty intersection with $S^{ne}_{\mathcal{B}_{\varphi}\otimes\mathcal{A}}$, then return $\bot$: If there is a finite word consistent with the observation that can be extended to satisfy the assumption, but no such extension satisfies the property, then every such extension must satisfy the complement of the property. Hence, we can return $\bot$.
4. Return *?*. Otherwise, there is both a finite word that is consistent with the observation that can be extended to satisfy the property and a finite word that is consistent with the observation that can be extended to satisfy the complement of the property. Consequently, we return *?*.

Theorem 2. *The algorithm described above computes $\mathcal{V}(\varphi, A)$.*

As argued above, our algorithm can be implemented using zones: both the reach-sets and the sets of nonempty states can be computed using zones, zones are closed under intersection, and can be tested effectively for emptiness [7].

Furthermore, our algorithm is online in the following sense: The set of nonempty states only needs to be computed once for each of the three automata and the symbolic states capturing

$$\mathcal{T}_{\mathcal{A}}(\mathcal{C}_{\mathcal{A}}((\phi_1, I_1, m_1)\cdots(\phi_n, I_n, m_n)(\phi_{n+1}, I_{n+1}, m_{n+1})), t')$$

can be computed from the symbolic states capturing

$$\mathcal{T}_{\mathcal{A}}(\mathcal{C}_{\mathcal{A}}((\phi_1, I_1, m_1)\cdots(\phi_n, I_n, m_n)), t),$$

as evident from the proof of Lemma 1. The same is true for the reach-sets in the other two automata $\mathcal{B}_{\neg\varphi} \otimes \mathcal{A}$ and $\mathcal{B}_{\varphi} \otimes \mathcal{A}$.

Remark 2. Our algorithm requires TBA both for the property *and* its complement, but TBA are in general not closed under complementation [2]. For the important case of MITL properties, such automata always exist, as MITL is closed under negation and can be translated into equivalent TBA (Theorem 1).

5 Evaluation

We implemented our assumption-based online monitoring algorithm described in Section 4 by extending the UPPAAL tool component MoniTAal[3], thereby demonstrating how the use of assumptions and unobservable events can enhance monitoring capabilities. In the following, we report on two proof-of-concept cases.

Task sequence. We first experiment with a system under monitoring that produces a finite sequence of events $a_1, \ldots, a_k$. Each a_i, with $1 \leq i < k$, is followed by a_{i+1} with a time within the interval $[l_i, u_i]$. The assumption is formalized by the TBA shown in Fig. 3. The domain is parameterized on k, and the l_i and u_i. Further, depending on the experiment, not all the a_i will be observable. We consider the bounded response property $G(a_1 \rightarrow F_{[0,B]} a_k)$. Suppose that we observe a timed word $(a_1, t_1)\cdots(a_k, t_k)$. If for some j in the range $1 < j \leq k$, we have $t_j + \sum_{j \leq i < k} u_i \leq B + t_1$, then the verdict at time t_j is $\top$. On the other hand, if $t_j + \sum_{j \leq i < k} l_i > B + t_1$, then the verdict at time t_j is $\bot$. As a corner case, if $\sum_{1 \leq i < k} u_i \leq B$ or $\sum_{1 \leq i < k} l_i > B$, the verdict is respectively $\top$ and $\bot$ at time 0, since all words of the assumption respectively satisfy and violate the property. We run several experiments to show the effect of the assumption and study the scalability under a sequence of unobservable events.

First we show how unobservable events can affect the response-time, the time between receiving an event and outputting a verdict. We pick $k = 100$ and

[3] https://github.com/DEIS-Tools/MoniTAal

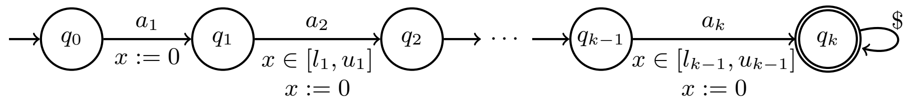

Fig. 3: A TBA representing the assumption for the bounded response example.

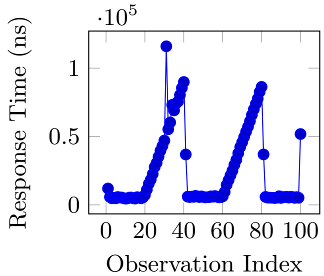

Fig. 4: Response time of the monitoring implementation, when monitoring the task sequence example with $\{a_i \mid i \in \{21, \ldots, 40\} \cup \{61, \ldots, 80\}\}$ being unobservable.

Table 1: Verdict distribution for monitoring the task sequence example a thousand times with $k = 10$, $B = 675$, $l_i = 50$ and and $u_i = 100$ for all i, with and without assumption. Each row specifies the number of times each verdict is given after an observation.

	Verdicts					
	No Assumption			With Assumption		
Observation	$\top$	$\bot$	?	$\top$	$\bot$	?
a_5	0	0	1000	0	0	1000
a_6	0	0	1000	0	1	999
a_7	0	0	1000	17	15	967
a_8	0	0	1000	81	90	796
a_9	0	33	967	165	153	478
a_{10}	0	457	510	246	232	0

$l_i = 50$ and $u_i = 100$ for all i. The events $\{a_i \mid i \in \{21, \ldots, 40\} \cup \{61, \ldots, 80\}\}$ are unobservable within the interval $[0, 10000]$. In Fig. 4 we see that for each consecutive unobservable event, the response time grows linearly. This is due to the reach-set growing. Nevertheless, the reach-set shrinks when an observable event is received. The minimum response time is 4583 nanoseconds (ns), the maximum is 115926 ns and the average is 24753 ns. For reference, if we monitor 5000 consecutive unobservable events, the maximum response time is 32 milliseconds.

To show the effect of the assumption, we monitor the bounded response property $G(a_1 \rightarrow F_{[0,675]} a_{10})$ a thousand times, with and without the assumption where $l_i = 50$ and $u_i = 100$ for all i. The observed words are random, but within the assumption. The results in Table 1 show that verdicts are computed earlier with the assumption than without. Without the assumption the earliest verdicts were in 33 cases $\bot$ after observing a_9, while with the assumption we saw a $\top$ or $\bot$ verdict in 522 cases before observing a_{10}.

Thus, when monitoring a live system in an online setting (compared to evaluating a log history), a verdict can be reached earlier, because of the restrictions the assumption inhibits. Furthermore, we demonstrate in this case how unobservable events can affect the size of the reach-set, as the number of words that are consistent with an observation can increase with the number of consecutive unobservable events. This in turn affects the response-time.

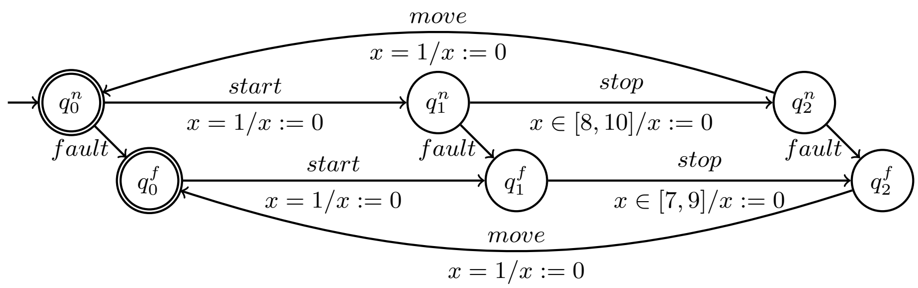

Fig. 5: A TBA representing the assumption for the conveyor belt example.

Conveyor belt example. This example represents a conveyor belt that moves an item through different stations, where the item is processed according to some task. The task in the nominal case takes between 8 and 10 times units. However, if the process is faulty, it finishes earlier and takes between 7 and 9 time units: it may sometimes complete correctly on time, but it may in other cases stop too early. The fault can happen at any time and is permanent. Our assumption automaton is shown in Fig. 5. Our monitoring property is simply $G\neg fault$.

Consider that we observe the events *start*, *stop*, and *move* with precise information on the time. If the *stop* signal happens less than 8 time units after *start*, we detect a violation of the property. If instead *stop* happens between 8 and 9 time units, we cannot say if there was a fault or not.

Suppose now that we have uncertainty on the time of the observations like in the following observation sequence:

$$(start, [1,1], !)(fault, [1,11], *)(stop, [8,10], !)(fault, [8,11], *)(move, [9,11], !)$$
$$(fault, [9,12], *)(start, [10,12], !)(fault, [11,22], *)(stop, [16,18], !)$$

The first *stop* happens at a time between 8 and 10, thus between 7 and 9 time units after the first *start*. This is compatible with both a nominal (with *stop* occurring between times 9 and 10) and a faulty execution (with *stop* occurring between times 8 and 9): after the first stop, we do not know if there was a fault.

The second *start* happens in the time interval $[10,12]$ and has the same uncertainty: it is consistent with the nominal behavior if *start* actually occurred in $[11,12]$ and with a faulty behavior if *start* occurred in $[10,11]$. The second stop happens in the time interval $[16,18]$. Thus, the difference with the previous start is between 4 and 8 time units. This seems compatible with a nominal delay ($[8,10]$). However, from the reasoning done above, if there were no fault the second start would have occurred in the interval $[11,12]$ and the second *stop* would have occurred in the interval $[19,22]$, which is not compatible with the observation. Thus we can conclude there was a fault.

We monitored the property $G\neg fault$ with the assumption from Fig. 5 by simulating the conveyor belt with an unbounded repeating pattern $\rho_1 \cdot_{\tau(\rho_1)} \rho_2 \cdot_{\tau(\rho_1)+\tau(\rho_2)} \cdots$ with each ρ_i having the form

$$(fault, [0,1], *), (start, 1, !), (fault, [1, t_i{+}2], *), (stop, [t_i, t_i{+}2], !), (move, t_i{+}2, !)$$

Table 2: Distribution of verdicts when monitoring 1000 random words of the conveyor belt assumption. Each column shows the number of times a conclusive verdict is given after observing the pattern a number of times. The longest run had 24 repetitions of the pattern before a verdict is given.

Repetitions	1	2	3	4	5	6	7	8	9	10	11
#Verdicts	251	185	125	121	90	47	48	31	24	20	13

Repetitions	12	13	14	15	16	17	18	19	20	24
#Verdicts	13	9	4	5	4	2	2	2	3	1

for some uniformly chosen $t_i \in \{7, 8, 9, 10\}$. The assumption is never violated, thus the only conclusive verdict reported is $\bot$ i.e. the property does not hold. The pattern essentially randomly selects whether *stop* is observed after 7, 8, 9 or 10 time units after *start*. Since 7 is only possible after a fault, there is a 1 in 4 chance, per repetition, of violating the property. The results in Table 2 show that in 251 out of 1000 cases a definitive verdict is given after observing the pattern once, and that the longest is 24 repetitions.

With this example, we see how an assumption makes it possible to monitor properties over unobservable events. Without an assumption, reasoning about unobservable behaviour would not be possible for such a property.

6 Related Work

Our automata-based monitoring of finite words against specifications over infinite words follows the seminal work of Bauer et al. [6], who presented monitoring algorithms for LTL and timed LTL. Their algorithm for timed LTL is based on clock regions [2], while we follow the approach of Grosen et al. [20] and use clock zones [8], whose performance is an order of magnitude faster. Also, they translated timed LTL into event-clock automata, which are less expressive than the timed Büchi automata (TBA) used both by Grosen et al. [20] and here. This approach has also been applied to monitoring under delayed observations [17].

As our algorithms work with TBA, we also support MITL specifications, as these can be compiled into TBA. The monitoring problem for MITL has been investigated before. Baldor et al. showed how to construct a monitor for dense-time MITL formulas by constructing a tree of timed transducers [4]. Ho et al. split unbounded and bounded parts of MITL formulas for monitoring, using traditional LTL monitoring for the unbounded parts and permitting a simpler construction for the (finite-word) bounded parts [22].

There is also a large body of work on monitoring with finite-word semantics. Roşu et al. focussed on discrete-time finite-word MTL [28], while Basin et al. proposed algorithms for monitoring real-time finite-word properties [5] and compared different time models. Donzé et al. [16] focussed on monitoring a quantitative semantics for STL, a variant of MTL with predicates over real-valued signals. André et al. consider monitoring finite logs of parameterized timed and hybrid systems [31]. Finally, Ulus et al. described monitoring timed regular expressions over finite words using unions of two-dimensional zones [29, 30].

The contribution of this paper is focused on extending the monitoring of timed properties with assumptions, framing the problem as defined in [12–15] for the discrete-time setting. Assumptions were first used in [23] for extending the monitoring of LTL with predictive capabilities. In [32], the assumption for predictive RV is computed applying static analysis to the monitored program. Pinisetty et al. further extend the predictive RV idea to support RV of timed properties [25], where the *a priori knowledge* is also expressed as a timed property. As in [15], we adopt a four-valued semantics for timed properties and we support partial observability. Besides the complexity of moving from discrete to dense time semantics, the ABRV framework is extended with a rich notion of observations that take into account uncertainty on the time.

The research of partial observability in Discrete-Event Systems is usually connected with diagnosability [26] and predictability [18, 19]. These notions have been extended to timed systems (see, e.g., [10, 11]). Moreover, they are related to monitorability, an important topic in RV and other related fields [1, 27, 24], which has been studied taking into account assumptions in [21].

7 Conclusion

In this paper, we extended runtime verification of timed properties with assumptions. These are used for anticipating or predicting a property failure or success, as well as for considering partial observability of the monitored system. A key contribution is to enable runtime verification to consider an observation sequence that has uncertainty on both the states/events and on the timing information. We provided an effective zone-based algorithm to compute the states that can be reached with such an observation sequence considering a property specified in MITL and an assumption as a TBA. Thus, such a computation can be used for online monitoring of timed properties under assumptions. The algorithm was implemented on top of UPPAAL and experimented with a few examples to show the feasibility of the approach.

For future work, further investigation is needed to check the scalability of the approach and to apply and optimize it to real-world case studies.

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