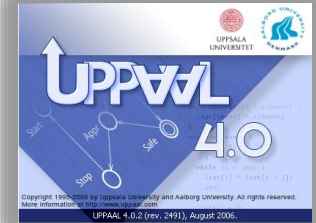


From **Timed Automata** to **Stochastic Hybrid Games**

Model Checking, Performance Analysis,
Optimization, Synthesis, and Machine Learning

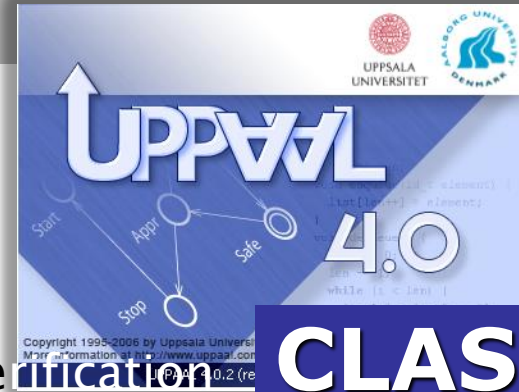
Kim G. Larsen

Aalborg University, DENMARK



Topics

- **Timed Automata**
 - Decidability (regions)
 - Symbolic Verification (zones)
- **Priced Timed Automata**
 - Decidability (priced regions)
 - Symbolic Verification (priced zones)
- **Stochastic Timed Automata**
 - Stochastic Semantics
 - Statistical Model Checking
 - Stochastic Hybrid Automata
- **Timed Games & Interfaces**
 - Strategies, Symbolic Synthesis
 - Refinement
- **Stochastic Priced Timed Games**
 - Strategies
 - Symbolic Synthesis (zones)
 - Stochastic Strategies
 - Reinforcement Learning



Verification

CLASSIC

1995

Optimization

CORA

2001

Synthesis

TIGA

2005

Component

ECDAR

2010

Testing

TRON

2004

Performance
Analysis

SMC

2011

Optimal Synthesis

STRATEGO

2014



Priced Timed Automata

Optimal Scheduling

Kim G. Larsen

CISS – Aalborg University
DENMARK



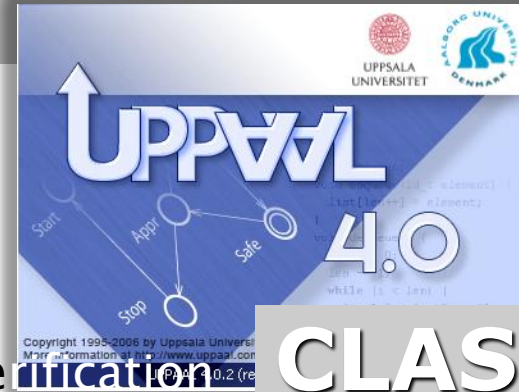
Overview

- Timed Automata
 - Scheduling

- **Priced** Timed Automata

- Optimal Reachability
- Optimal Infinite Scheduling
- Multi Objectives

- **Energy** Automata



Verification

CLASSIC

Optimization

CORA

Synthesis

TIGA

Component

ECDAR

Testing

TRON

Performance
Analysis

SMC

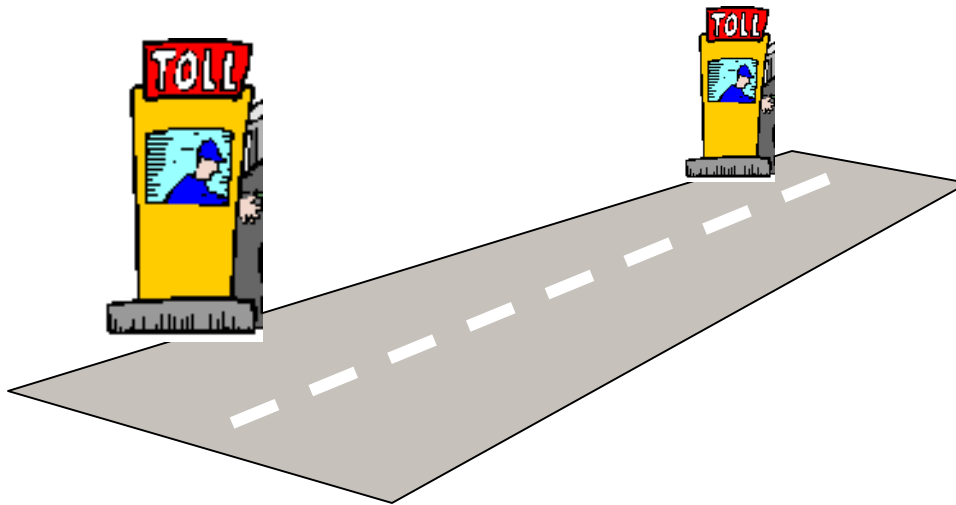
Optimal Synthesis

STRATEGO



Real Time Scheduling

- Only 1 "Pass"
- Cheat is possible
(drive close to car with "Pass")



UNSAFE



5

Crossing
Times



10



Pass



20



25

SAFE

**CAN THEY MAKE IT TO SAFE
WITHIN 70 MINUTES ???**



Let us play!

Solving scheduling problems using Uppaal

A number of cars are to pass a bridge. There is a toll for passing the bridge -- and a device (known as the 'BroBizz' or 'EasyPass') must be used in order to pass the bridge.

There is only one BroBizz available to the cars -- but luckily the toll booth system can be *cheated* if two cars drive close to each other. Only cars from the side at which BroBizz is located can pass the bridge. The toll booth at the side at which the BroBizz is located is colored green.

All cars must pass the bridge within a given time limit (shown at the center of the screen). Each car spends a given number of minutes passing the bridge. This *schedulability* problem can be solved using the Uppaal tool.

Using the buttons above you can:

- **Configure:**
Setup the number of cars, their speed, and the time limit
- **Interact:**
Try to solve the problem manually
- **Find some solution:**
Use Uppaal to solve the problem and display the solution
- **Find best solution:**
Use Uppaal to find the best solution to the problem

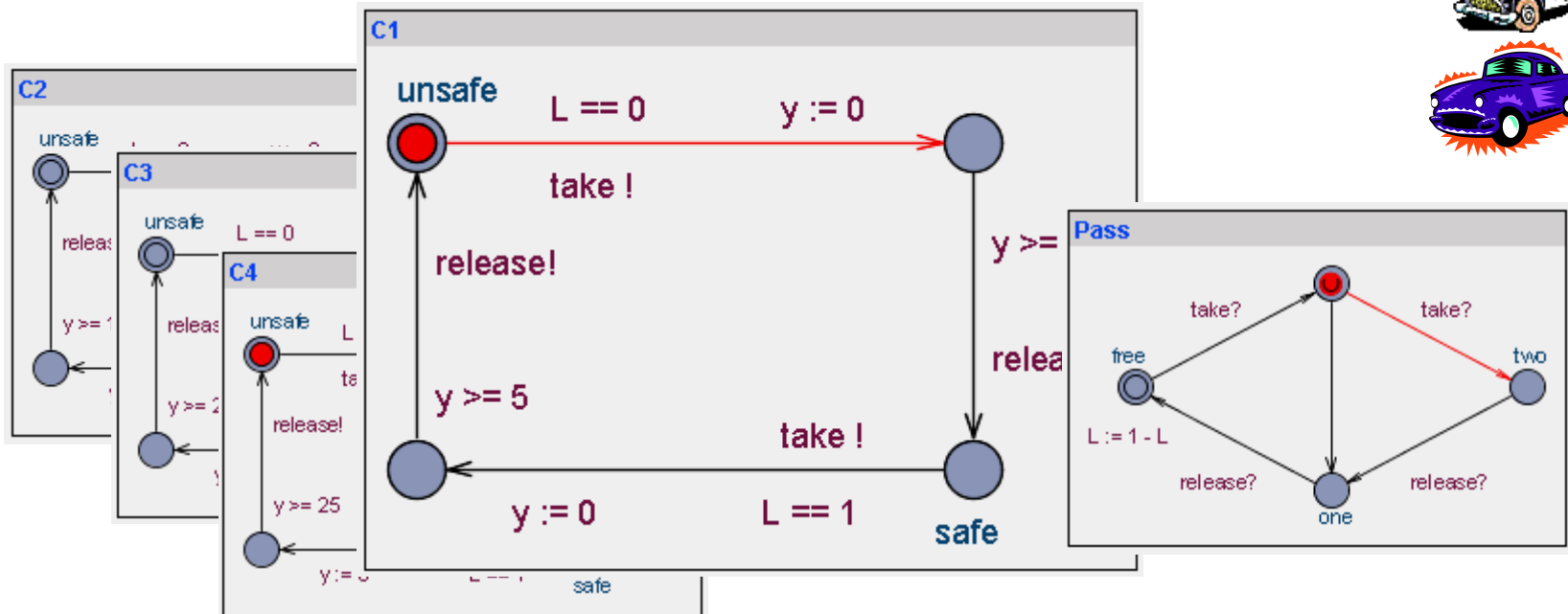
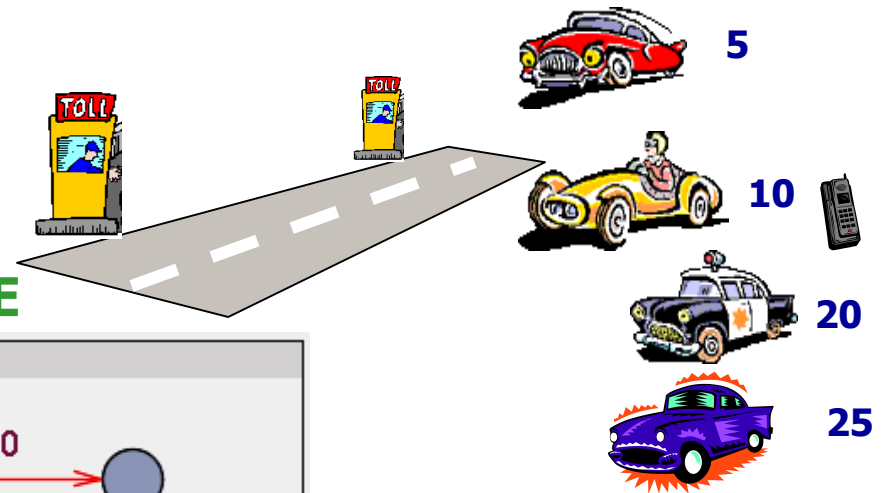
Real Time Scheduling

RIO12/DAY3/

Solve
Scheduling Problem
using **UPPAAL**

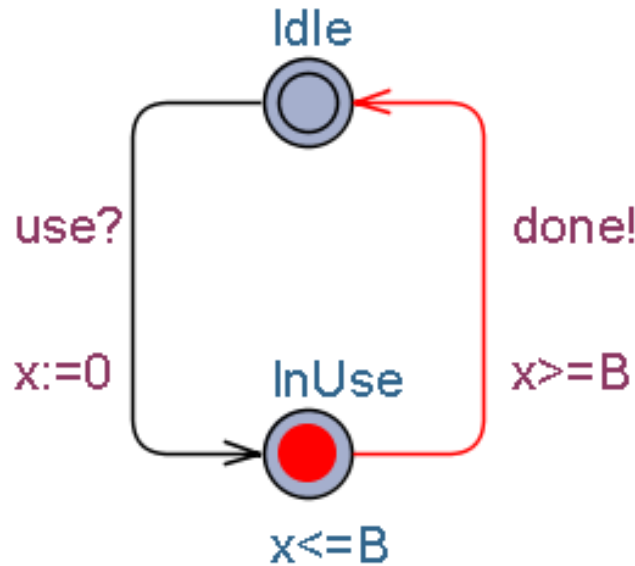
UNSAFE

SAFE



Resources & Tasks

Resource



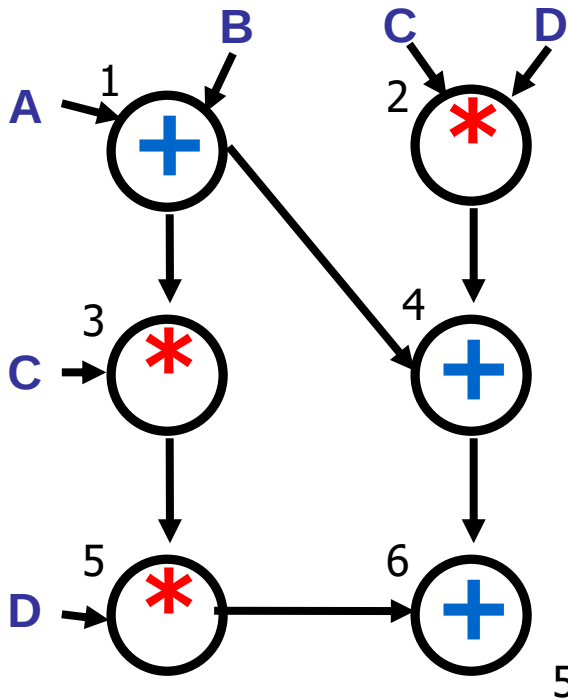
Synchronization

Task



Shared variable

Task Graph Scheduling – Example



Compute :
$$(D * (C * (A + B)) + ((A + B) + (C * D)))$$

4

using 2 processors

P1 (fast)

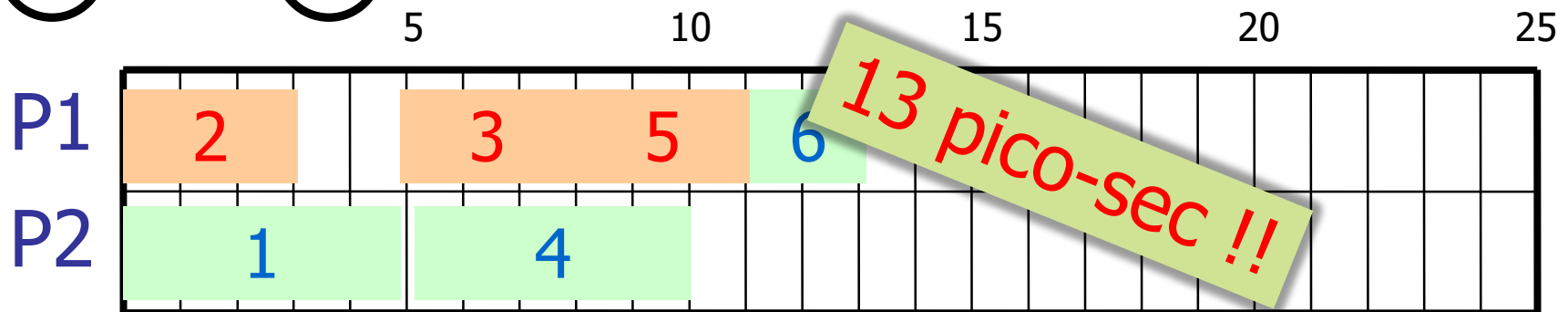


+	2ps
*	3ps

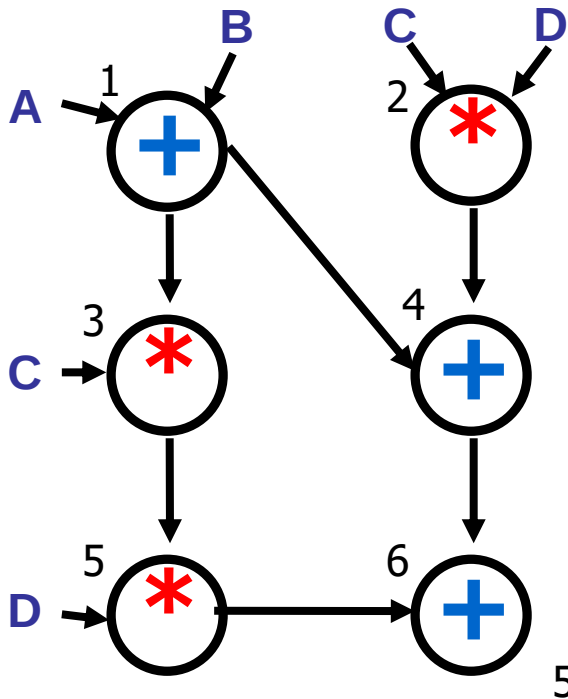
P2 (slow)



+	5ps
*	7ps



Task Graph Scheduling – Example



Compute :
 $(D * (C * (A + B)) + ((A + B) + (C * D)))$

using 2 processors

P1 (fast)

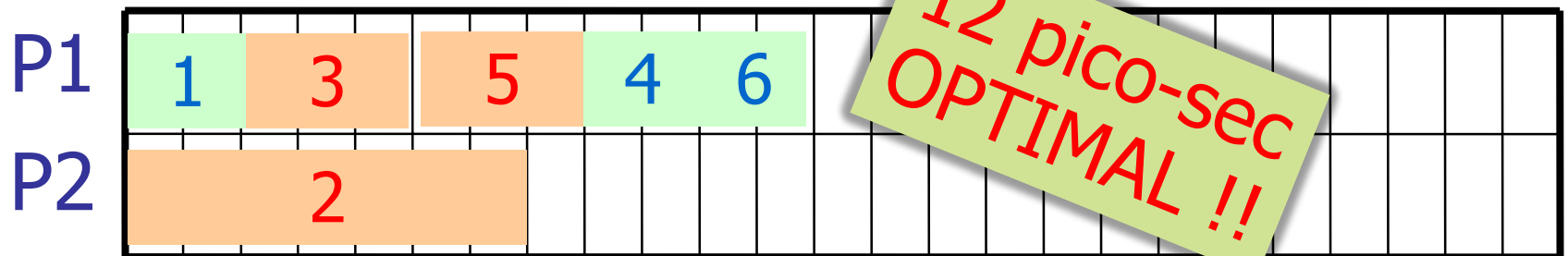


+	2ps
*	3ps

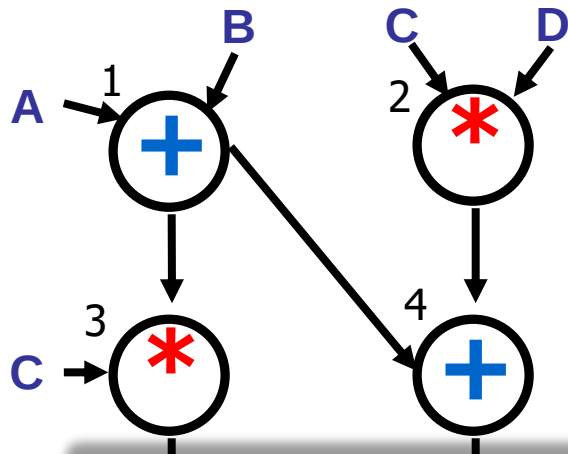
P2 (slow)



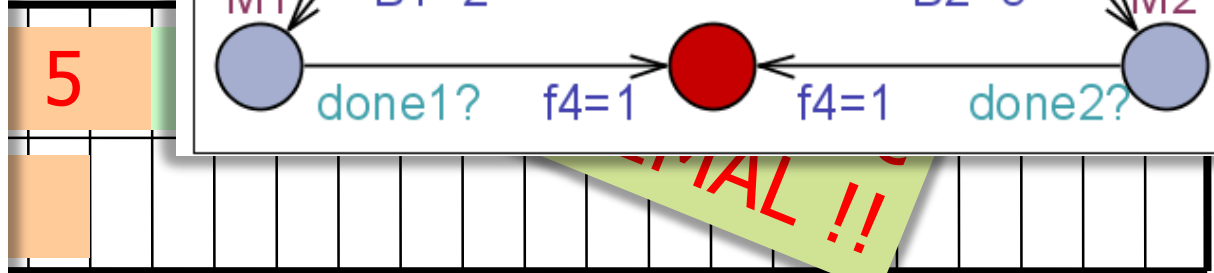
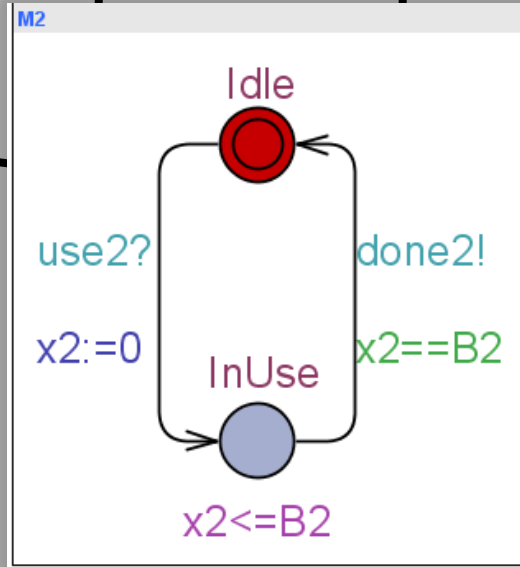
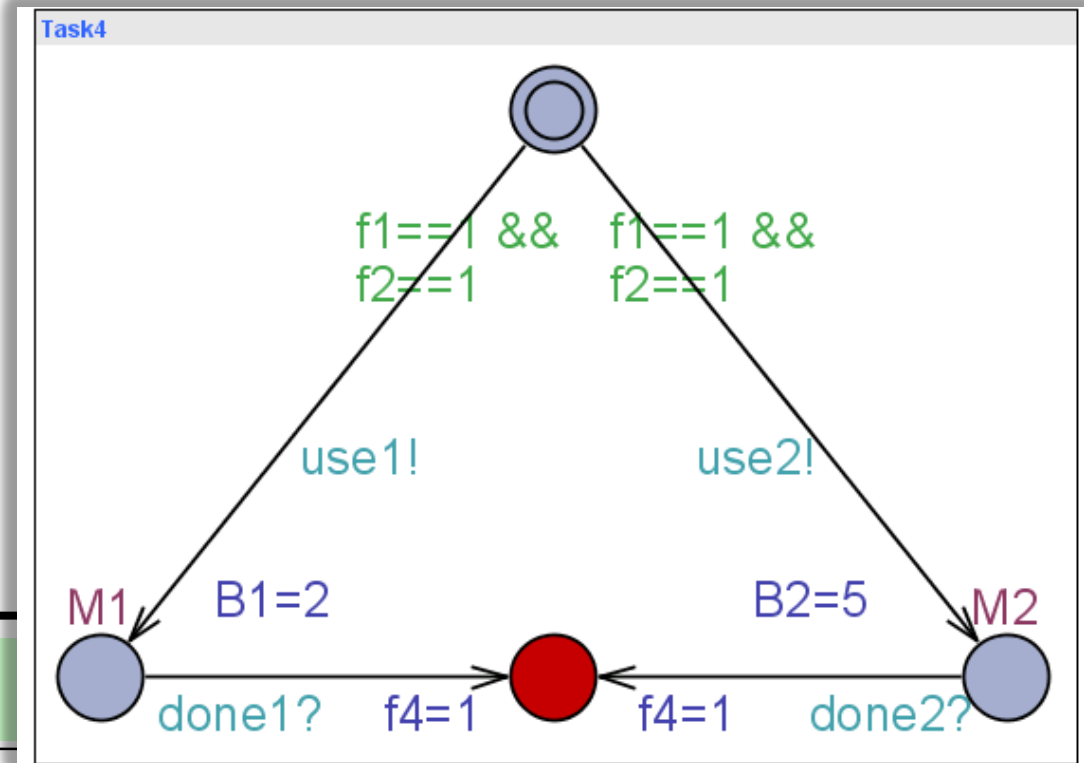
+	5ps
*	7ps



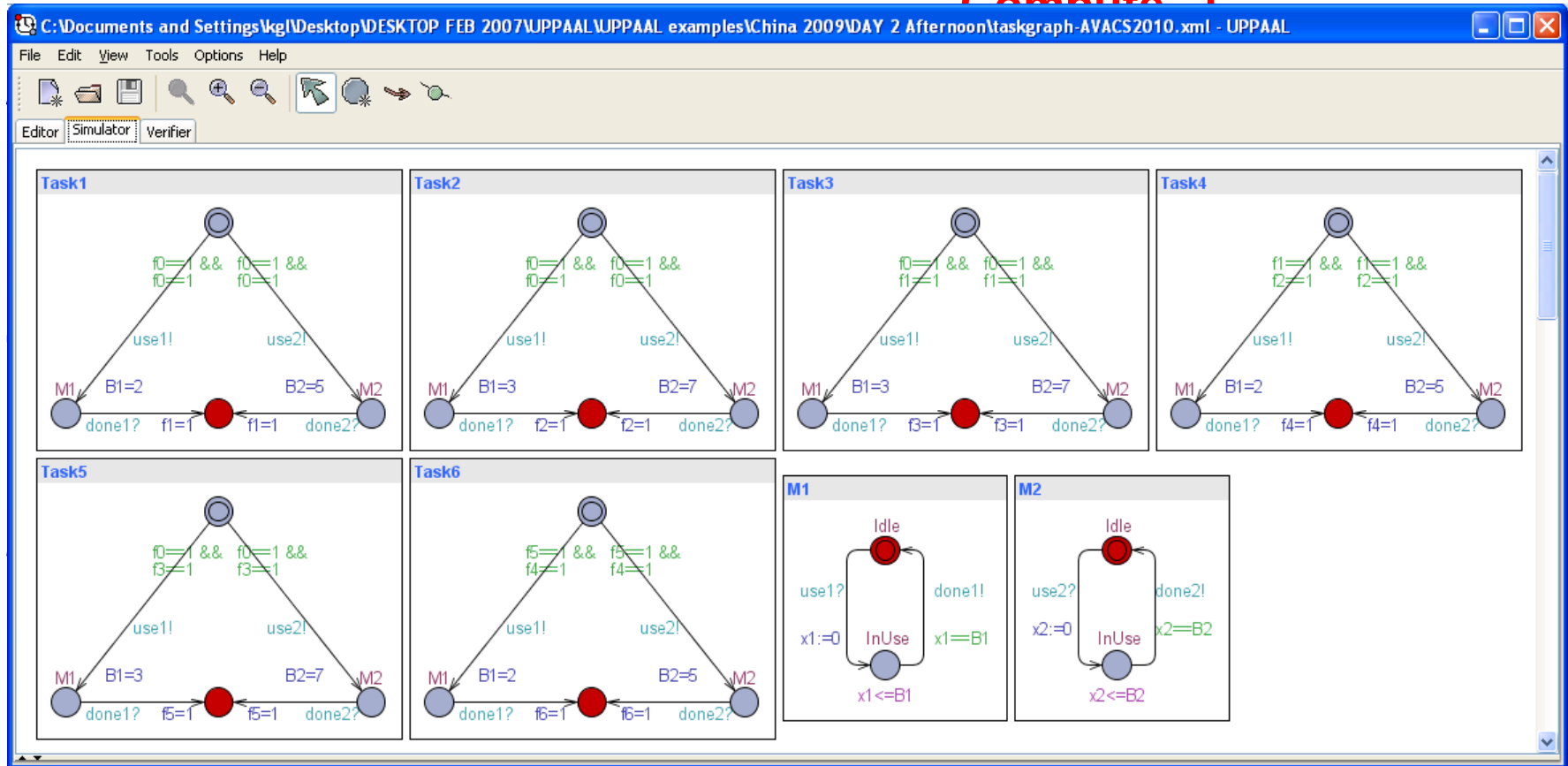
Task Graph Scheduling – Example



Compute :
 $(D * (C * (A + B)) + ((A + B) + (C * D)))$



Task Graph Scheduling – Example



P2

E<> (Task1.End and ... and Task6.End)

time

Experimental Results

name	#tasks	#chains	# machines	optimal	TA
001	437	125	4	1178	1182
000	452	43	20	537	537
018	730	175	10	700	704
074	1007	66	12	891	894
021	1145	88	20	605	612
228	1187	293	8	1570	1574
071	1193	124	20	629	634
271	1348	127	12	1163	1164
237	1566	152	12	1340	1342
231	1664	101	16	t.o.	1137
235	1782	218	16	t.o.	1150
233	1980	207	19	1118	1121
294	2014	141	17	1257	1261
295	2168	965	18	1318	1322
292	2333	318	3	8009	8009
298	2399	303	10	2471	2473



Symbolic A*
Branch-&-Bound
60 sec

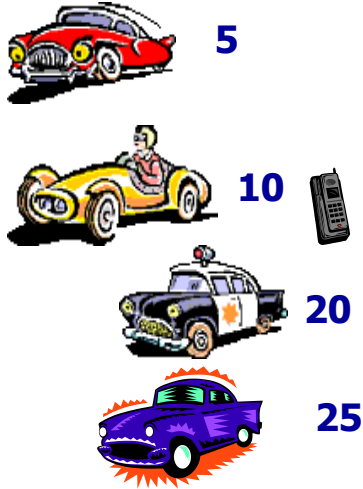
Abdeddaïm, Kerbaa, Maler



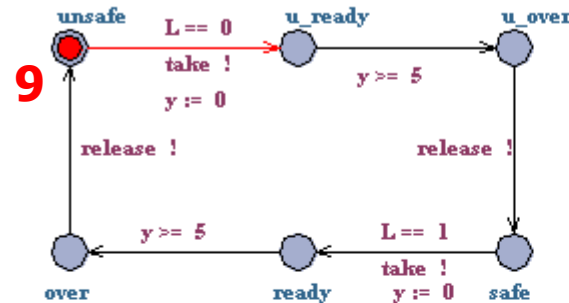
Priced Timed Automata



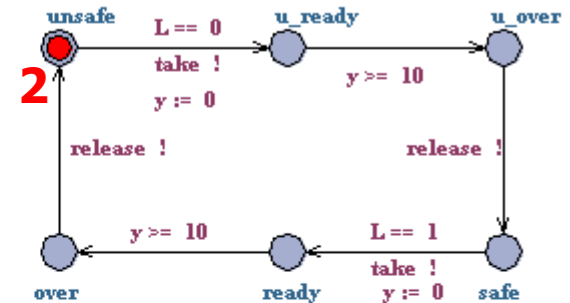
EXAMPLE: Optimal rescue plan for cars with different subscription rates for city driving !



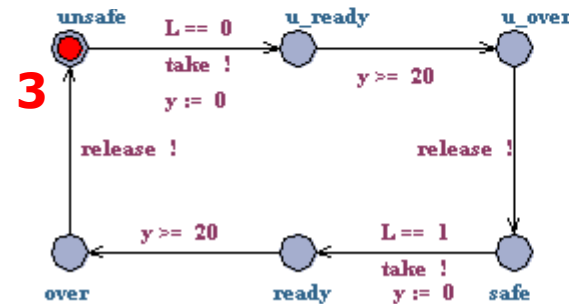
Golf



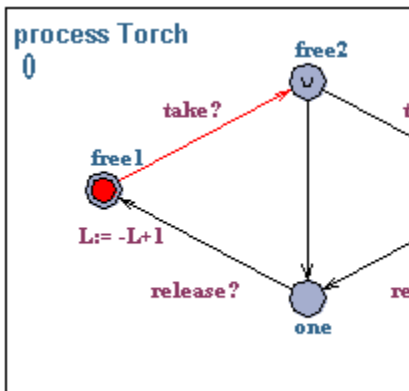
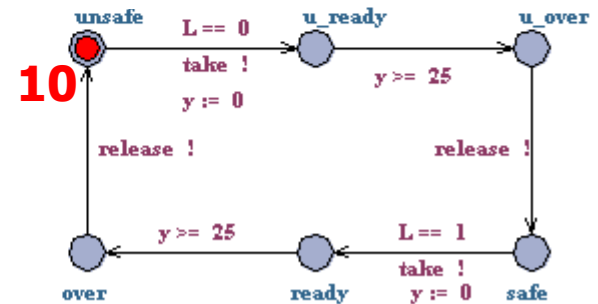
Citroen



BMW



Datsun



OPTIMAL PLAN HAS ACCUMULATED **COST**=195 and **TOTAL TIME**=65!

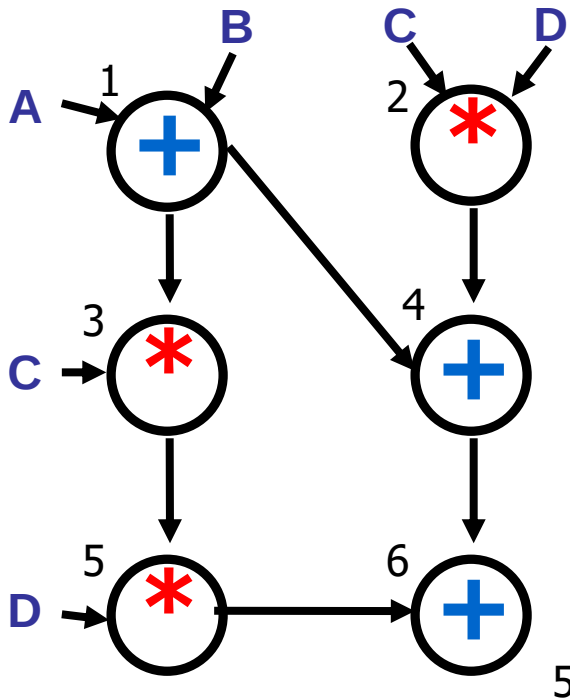


Experiments

COST-rates				SCHEDULE	COST	TIME	#Expl	#Pop'd
G	C	B	D					
Min Time				CG> G< BD> C< CG>		60	1762 1538	2638
1	1	1	1	CG> G< BG> G< GD>	55	65	252	378
9	2	3	10	GD> G< CG> G< BG>	195	65	149	233
1	2	3	4	CG> G< BD> C< CG>	140	60	232	350
1	2	3	10	CD> C< CB> C< CG>	170	65	263	408
1	20	30	40	BD> B< CB> C< CG>	975 1085	85 time<85	-	-
0	0	0	0	-	0	-	406	447



Task Graph Scheduling – Revisited



Compute :
 $(D * (C * (A + B)) + ((A + B) + (C * D)))$

using 2 processors

P1 (fast)



ENERGY:

10

P2 (slow)



20

+	2ps
*	3ps

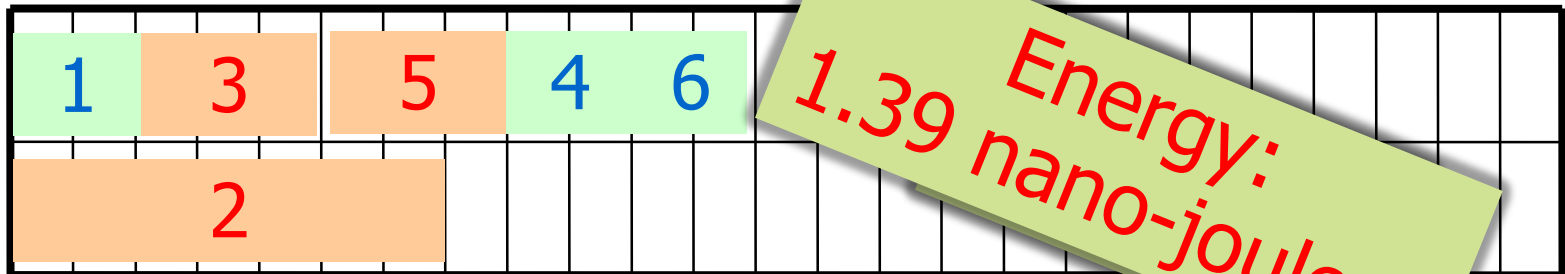
Idle	10W
In use	90W

+	5ps
*	7ps

Idle	20W
In use	30W

P1

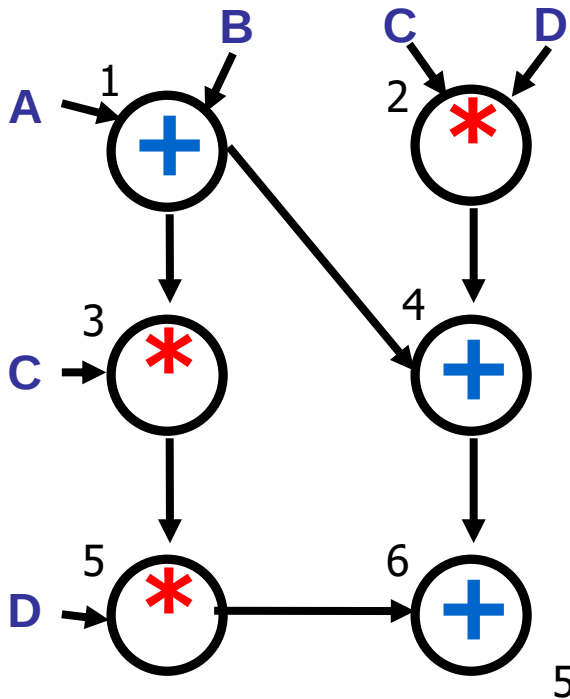
P2



Energy: 1.39 nano-joule !!



Task Graph Scheduling – Revisited



Compute :
 $(D * (C * (A + B)) + ((A + B) + (C * D)))$

using 2 processors

P1 (fast)



ENERGY:
10

P2 (slow)

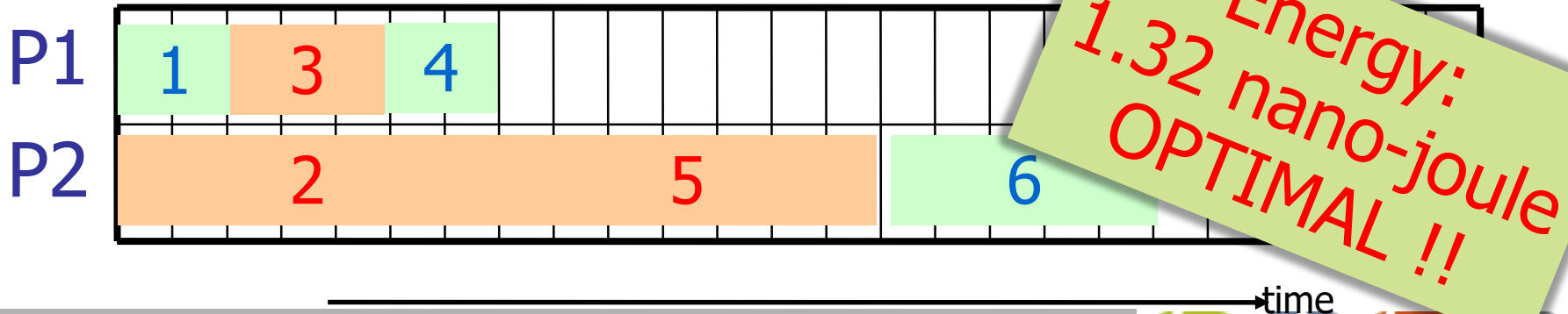


+	2ps
*	3ps

Idle	10W
In use	90W

+	5ps
*	7ps

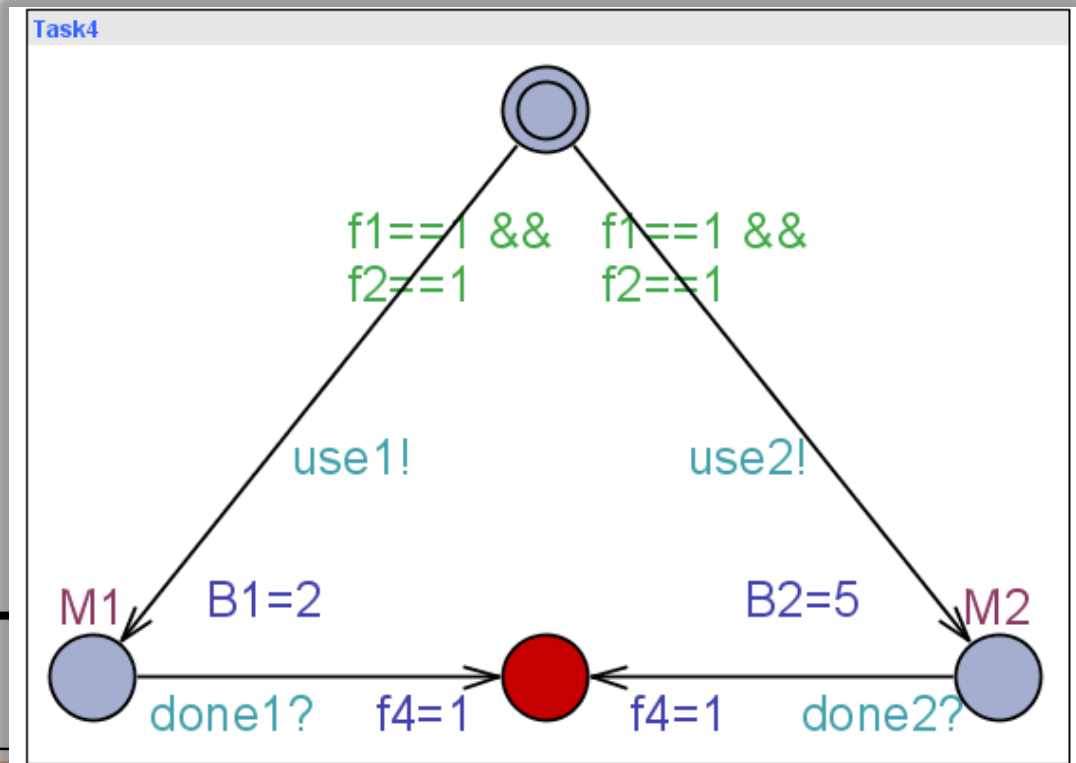
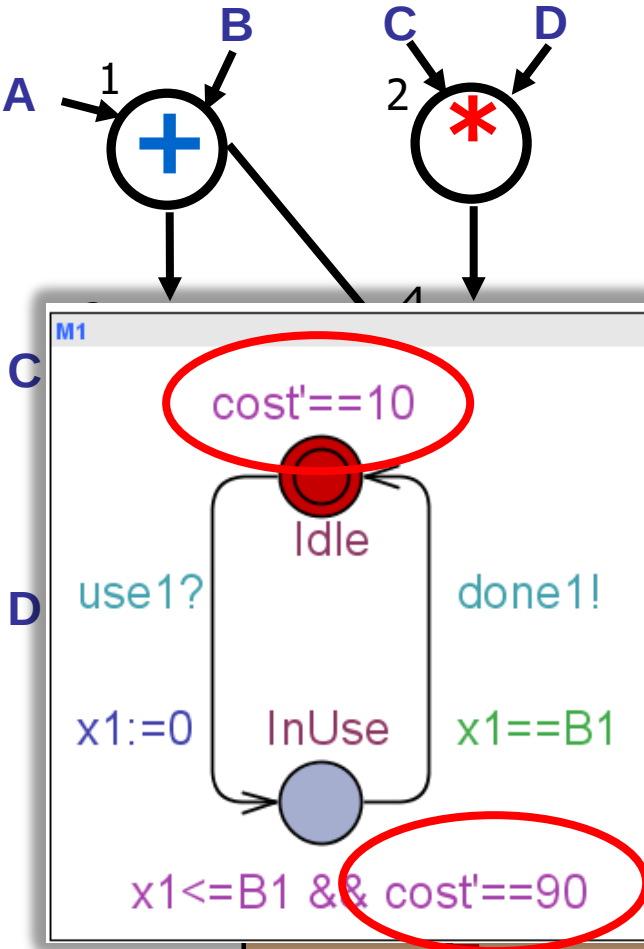
Idle	20W
In use	30W



Task Graph Scheduling – Revisited

Compute :

$$(D * (C * (A + B)) + ((A + B) + (C * D)))$$



5

6

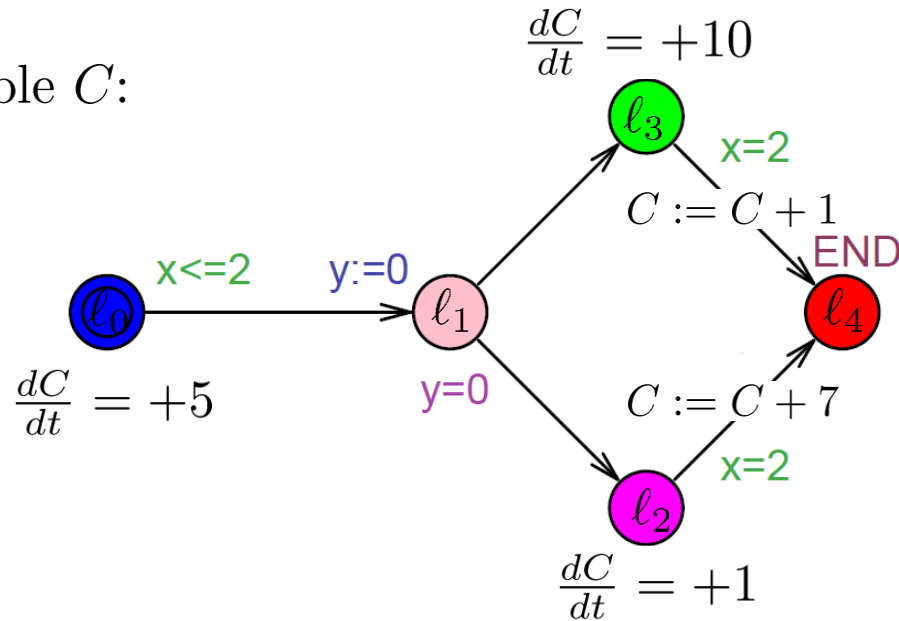
OPTIMAL !!

time



A simple example

Observer variable C :



$$(\ell_0, [0, 0]) \xrightarrow{1.9}_{9.5} (\ell_0, [1.9, 1.9]) \rightarrow_0 (\ell_1, [1.9, 0]) \rightarrow_0$$

$$(\ell_2, [1.9, 0]) \xrightarrow{0.1}_{0.1} (\ell_2, [2, 0.1]) \rightarrow_7 (\ell_4, [2, 0.1])$$

$$\sum C_i = 16.6$$

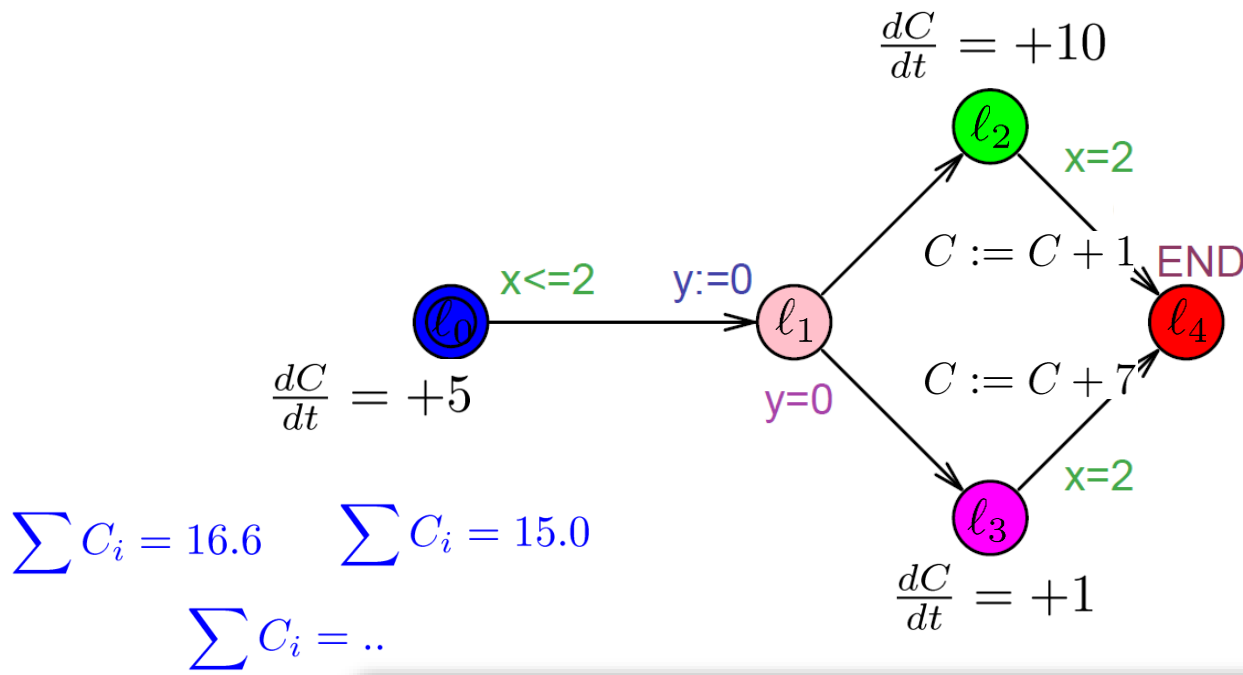
$$(\ell_0, [0, 0]) \xrightarrow{1.2}_{6.0} (\ell_0, [1.2, 1.2]) \rightarrow_0 (\ell_1, [1.2, 0]) \rightarrow_0$$

$$(\ell_3, [1.2, 0]) \xrightarrow{0.8}_{8.0} (\ell_3, [2, 0.8]) \rightarrow_1 (\ell_4, [2, 0.8])$$

$$\sum C_i = 15.0$$



A simple example

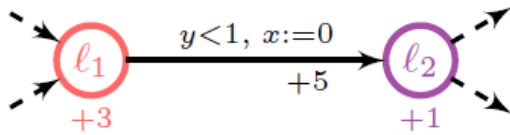


Q: What is cheapest cost for reaching l_4 ?

$$\inf_{0 \leq t \leq 2} \min\{5t + 10(2 - t) + 1, 5t + (2 - t) + 4\} = 9$$

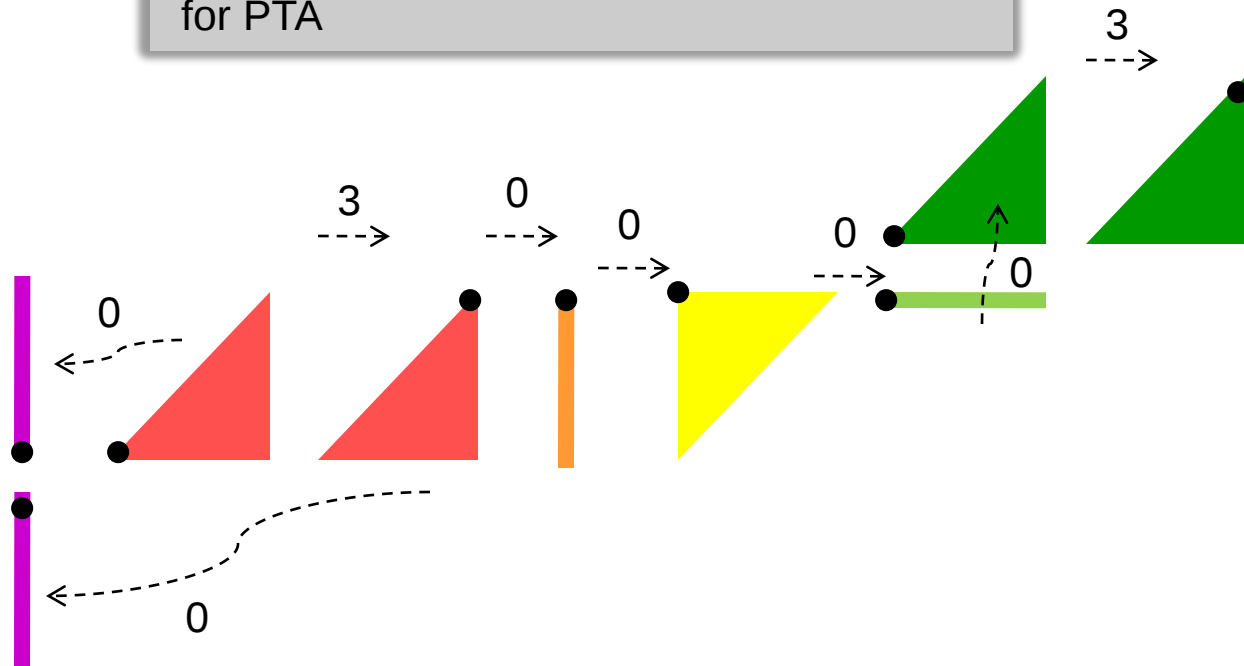
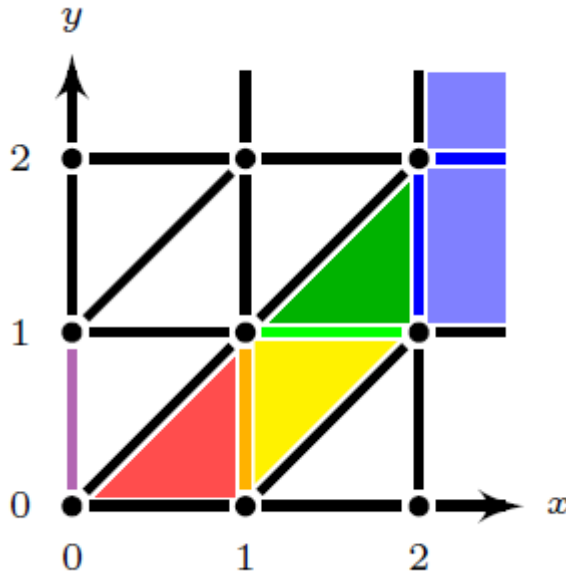
→ **strategy:** leave immediately l_0 , go to l_3 , and wait there 2 t.u.

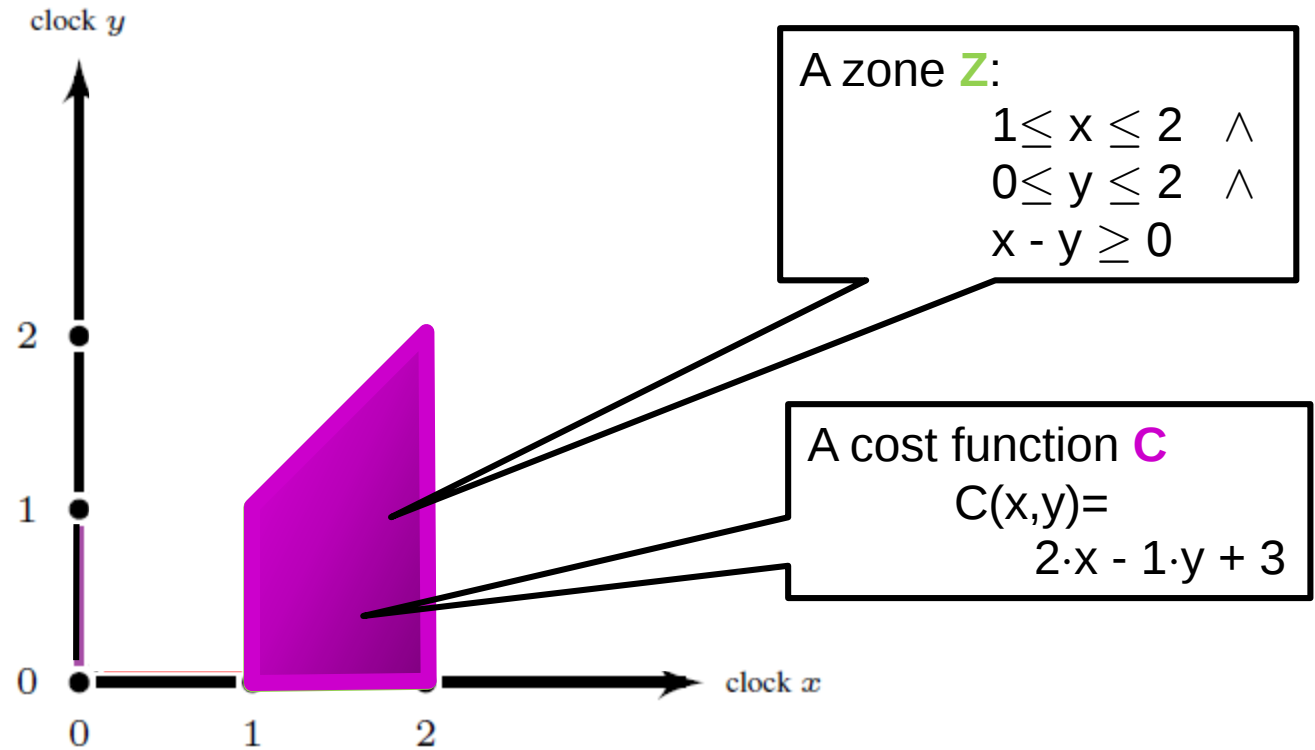
Corner Point Regions



THM [Behrmann, Fehnker ..01] [Alur,Torre,Pappas 01]
Optimal reachability is decidable for PTA

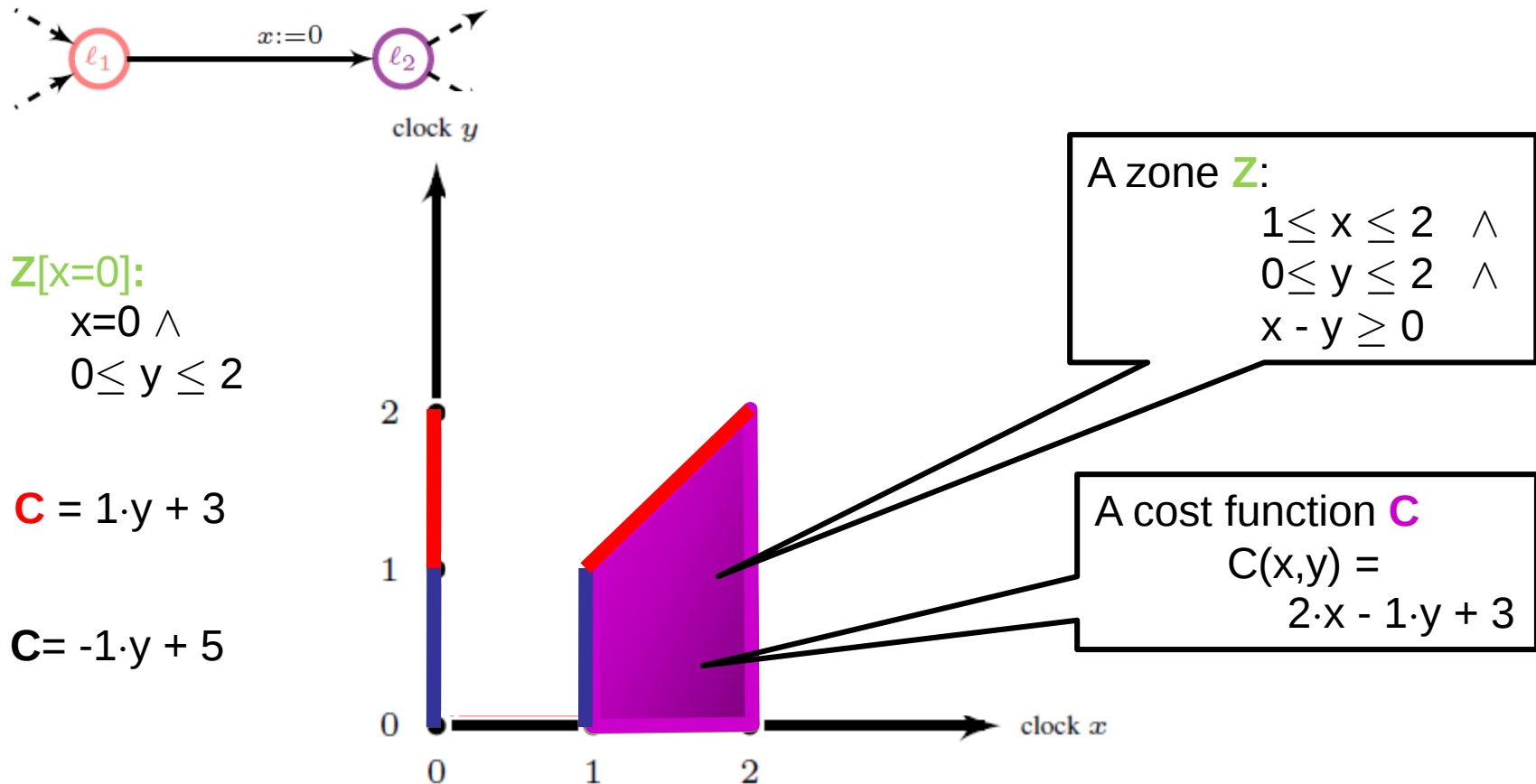
THM [Bouyer, Brojaue, Briuere, Raskin 07]
Optimal reachability is PSPACE-complete
for PTA





Priced Zones – Reset

[CAV01]



Symbolic Branch & Bound Algorithm

Cost := ∞

Passed := $\emptyset$

Waiting := $\{(l_0, Z_0)\}$

while Waiting $\neq \emptyset$ **do**

select (l, Z) from Waiting

if $l = l_g$ and $\text{minCost}(Z) < \text{Cost}$ **then**

 Cost := $\text{minCost}(Z)$

if $\text{minCost}(Z) + \text{Rem}_{(l,Z)} \geq \text{Cost}$ **then**

if for all (l, Z') in Passed: $Z' \not\leq Z$ **then**

add (l, Z) to Passed

add all (l', Z') with $(l, Z) \rightarrow (l', Z')$

return Cost

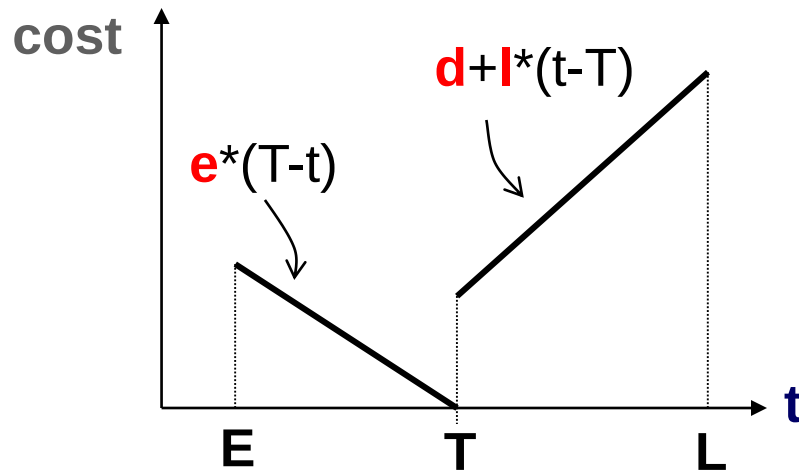
$$Z' \leq Z$$

Z' is bigger & cheaper than Z

$\leq$ is a well-quasi ordering which guarantees termination!

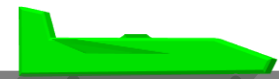


Example: Aircraft Landing

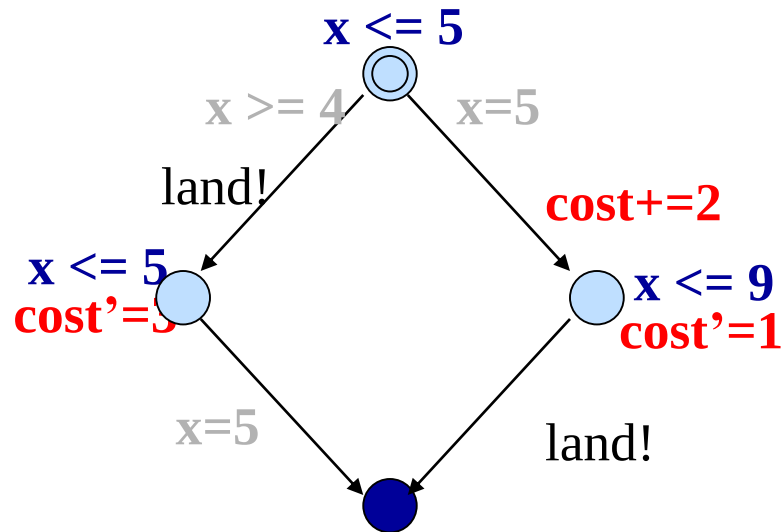


- E** earliest landing time
- T** target time
- L** latest time
- e** cost rate for being early
- l** cost rate for being late
- d** fixed cost for being late

Planes have to keep separation distance to avoid turbulences caused by preceding planes

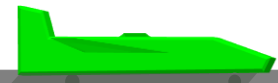


Example: Aircraft Landing



- 4** earliest landing time
- 5** target time
- 9** latest time
- 3** cost rate for being early
- 1** cost rate for being late
- 2** fixed cost for being late

Planes have to keep separation distance to avoid turbulences caused by preceding planes



Aircraft Landing

Source of examples:
Baesley et al'2000

	problem instance	1	2	3	4	5	6	7
	number of planes	10	15	20	20	20	30	44
	number of types	2	2	2	2	2	4	2
1	optimal value	700	1480	820	2520	3100	24442	1550
	explored states	481	2149	920	5693	15069	122	662
	cputime (secs)	4.19	25.30	11.05	87.67	220.22	0.60	4.27
2	optimal value	90	210	60	640	650	554	0
	explored states	1218	1797	669	28821	47993	9035	92
	cputime (secs)	17.87	39.92	11.02	755.84	1085.08	123.72	1.06
3	optimal value	0	0	0	130	170	0	
	explored states	24	46	84	207715	189602	62	N/A
	cputime (secs)	0.36	0.70	1.71	14786.19	12461.47	0.68	
4	optimal value				0	0		
	explored states	N/A	N/A	N/A	65	64	N/A	N/A
	cputime (secs)				1.97	1.53		



Symbolic Branch & Bound Algorithm

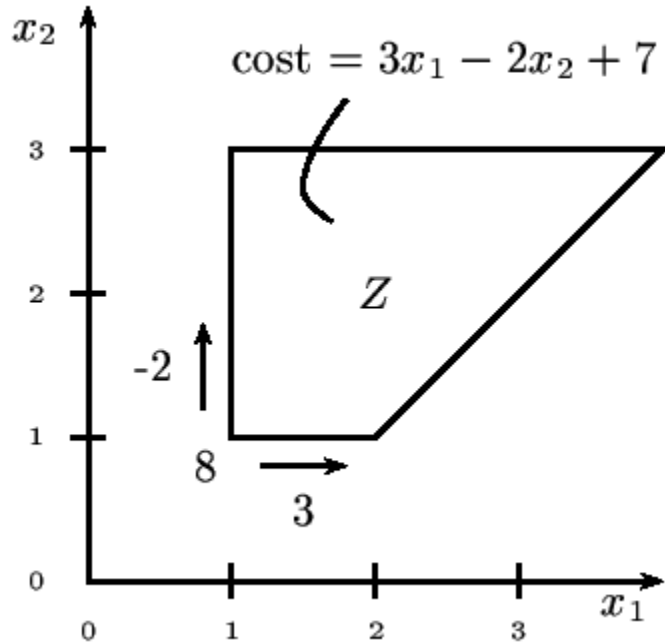
```
Cost :=  $\infty$ 
Passed :=  $\emptyset$ 
Waiting :=  $\{(l_0, Z_0)\}$ 
while Waiting  $\neq \emptyset$  do
    select  $(l, Z)$  from Waiting
    if  $l = l_g$  and  $\text{minCost}(Z) < \text{Cost}$  then
        Cost :=  $\text{minCost}(Z)$ 
    if  $\text{minCost}(Z) + \text{Rem}_{(l, Z)} \geq \text{Cost}$  then break
    if for all  $(l', Z')$  in Passed:  $Z' \not\subseteq Z$  then
        add  $(l, Z)$  to Passed
        add all  $(l', Z')$  with  $(l, Z) \rightarrow (l', Z')$  to Waiting
return Cost
```

Zone based
Linear Programming
Problems
→(dualize)
Min Cost Flow

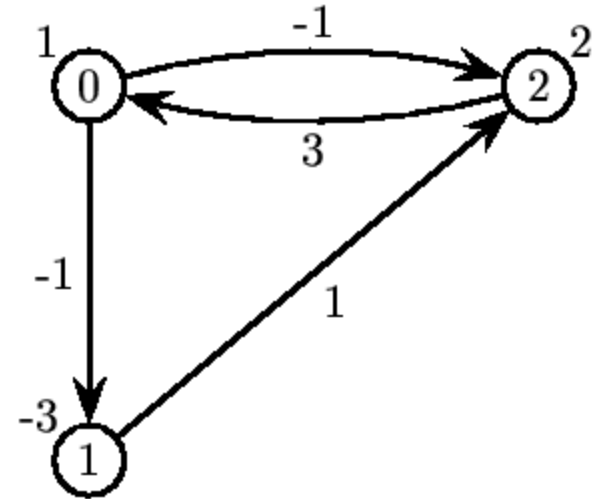


Zone LP $\rightarrow$ Min Cost Flow

Exploiting duality



minimize $3x_1 - 2x_2 + 7$
 when $x_1 - x_2 \leq 1$
 $1 \leq x_2 \leq 3$
 $x_2 \geq 1$

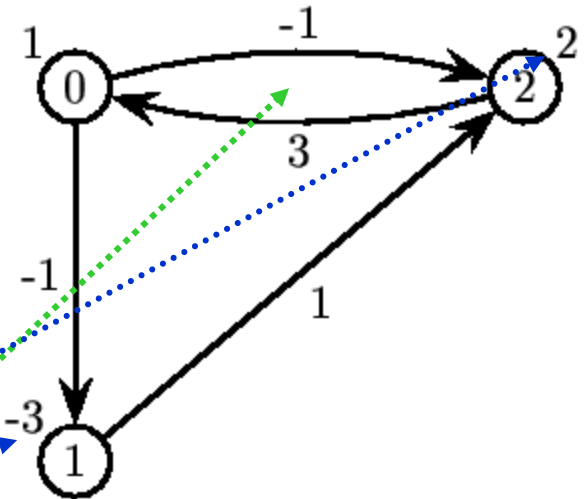
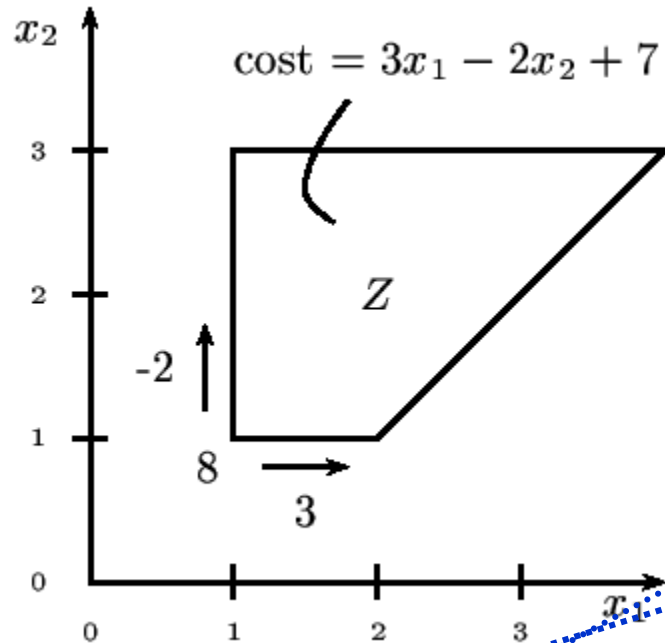


minimize $3y_{2,0} - y_{0,2} + y_{1,2} - y_{0,1}$
 when $y_{2,0} - y_{0,1} - y_{0,2} = 1$
 $y_{0,2} + y_{1,2} = 2$
 $y_{0,1} - y_{1,2} = -3$



Zone LP $\rightarrow$ Min Cost Flow

Exploiting duality



minimize $3x_1 - 2x_2 + 7$
 when $x_1 - x_2 \leq 1$
 $1 \leq x_2 \leq 3$
 $x_2 \geq 1$

minimize $3y_{2,0} - y_{0,2} + y_{1,2} - y_{0,1}$
 when $y_{2,0} - y_{0,1} - y_{0,2} = 1$
 $y_{0,2} + y_{1,2} = 2$
 $y_{0,1} - y_{1,2} = -3$



Aircraft Landing (revisited)

[TACAS04]

RW	Planes	10	15	20	20	20	30	44
	Types	2	2	2	2	2	4	2
1	simplex	0.844s	5.210s	2.135s	17.888s	44.878s	0.451s	0.670s
	netsimplex	0.156s	0.657s	0.369s	2.363s	5.503s	0.127s	0.322s
factor		5.41	7.93	5.79	7.57	8.16	3.55	2.08
2	simplex	2.577s	7.436s	2.175s	94.357s	120.004s	2.322s	0.264s
	netsimplex	0.332s	1.036s	0.436s	13.376s	18.033s	0.600s	0.179s
factor		8.00	7.18	4.99	7.054	6.65	3.87	1.474
3	simplex	0.120s	0.181s	0.357s	740.100s	516.678s	0.166s	N/A
	netsimplex	0.064s	0.104s	0.129s	170.176s	124.805s	0.079s	N/A
factor		1.87	1.74	2.77	4.34	4.14	2.10	
4	simplex	N/A	N/A	N/A	1.603s	0.318s	N/A	N/A
	netsimplex	N/A	N/A	N/A	0.378s	0.093s	N/A	N/A
factor					4.24	3.42		

A. Loebel (2000). MCF Version 1.2 - A network simplex implementation. (<http://www.zib.de>)

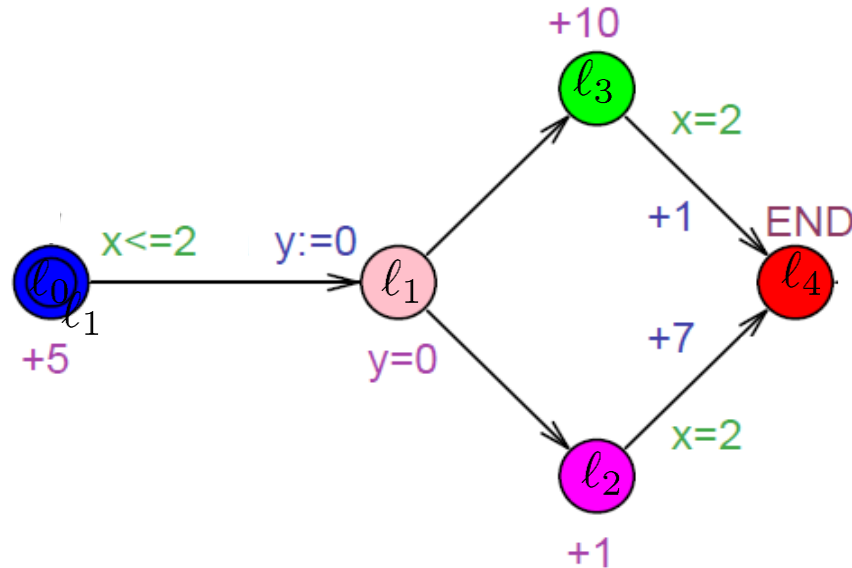


NOT IMPLEMENTED w ZONES



Optimal

Schedule

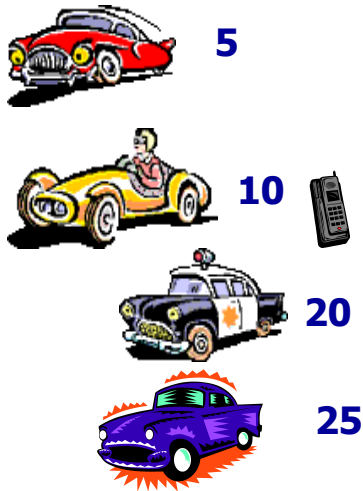


$$\begin{aligned}
 (\ell_0, [0, 0]) &\xrightarrow{1.2}_{6.0} (\ell_0, [1.2, 1.2]) \rightarrow_0 (\ell_1, [1.2, 0]) \rightarrow_0 \\
 &(\ell_3, [1.2, 0]) \xrightarrow{0.8}_{8.0} (\ell_3, [2, 0.8]) \rightarrow_1 (\ell_4, [2, 0.8]) \\
 &\rightarrow_{2.0} (\ell_0, [0, 0])
 \end{aligned}$$

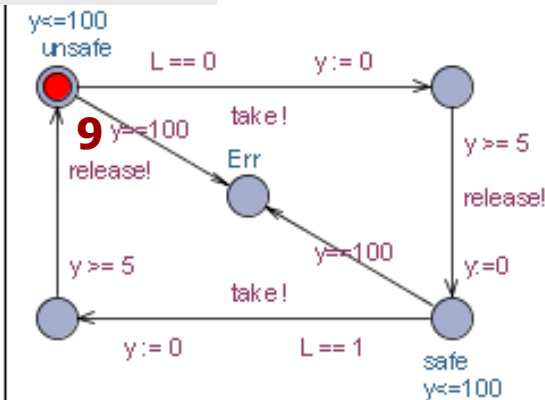
$$\sum_i C_i / \sum_i t_i = 17/2 = 8.5$$

EXAMPLE: Optimal **WORK** plan for cars with different subscription rates for city driving !

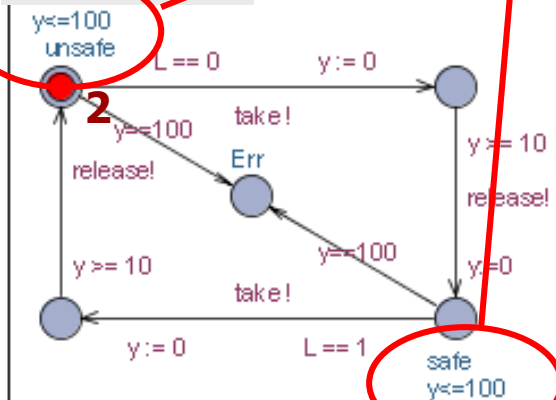
maximal 100 min.
at each location



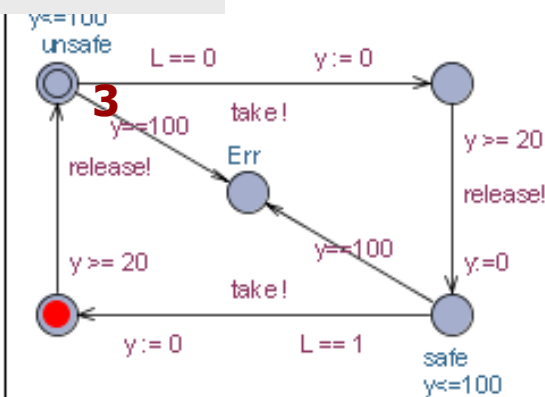
Golf



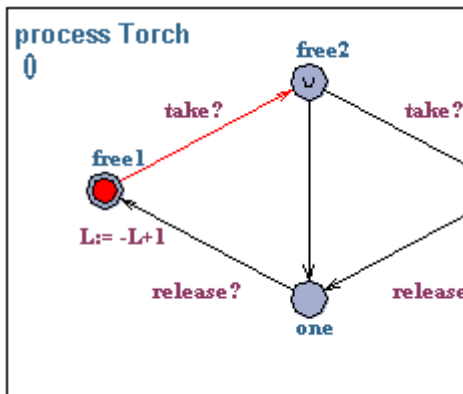
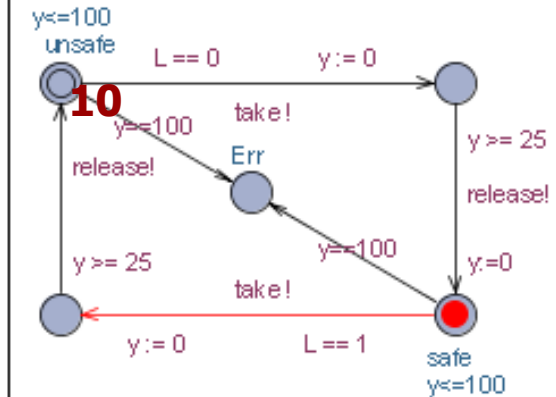
Citroen



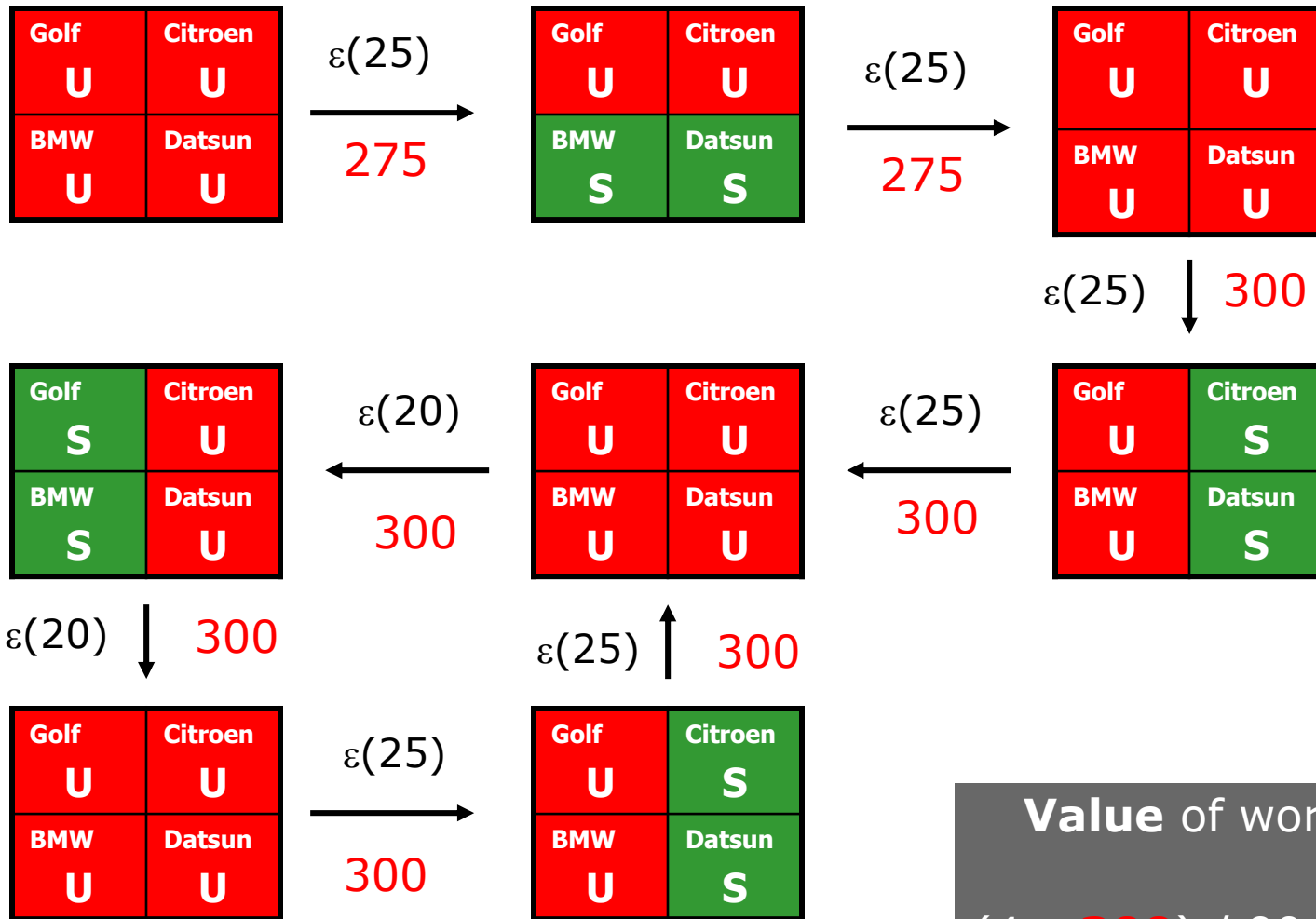
BMW



Datsun



Workplan I



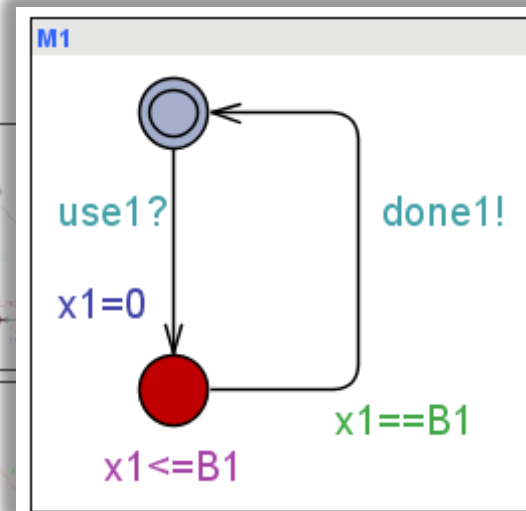
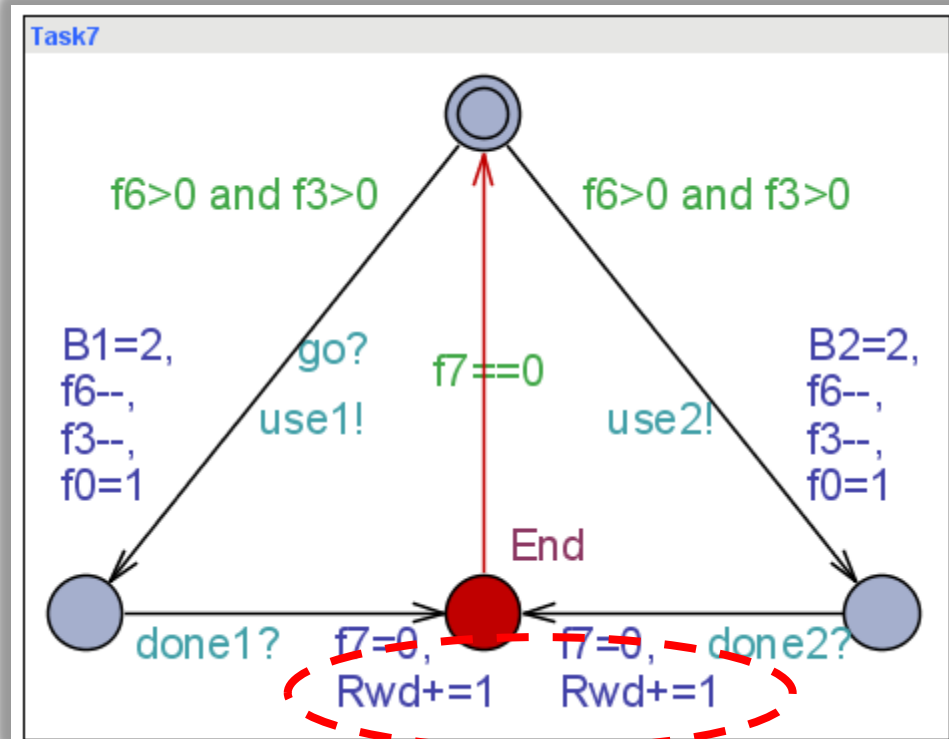
Value of workplan:

$$(4 \times \mathbf{300}) / 90 = \mathbf{13.33}$$

Workplan II

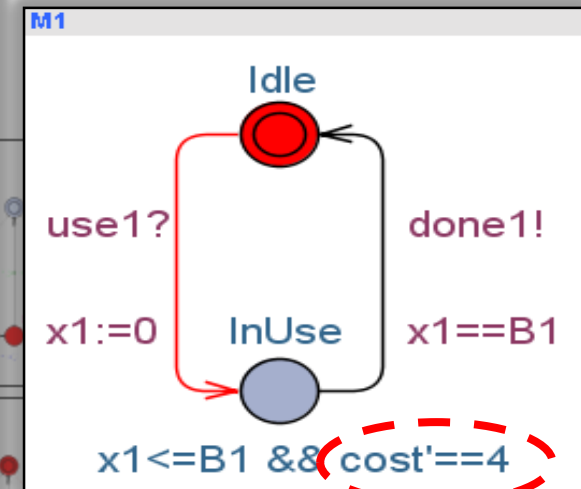
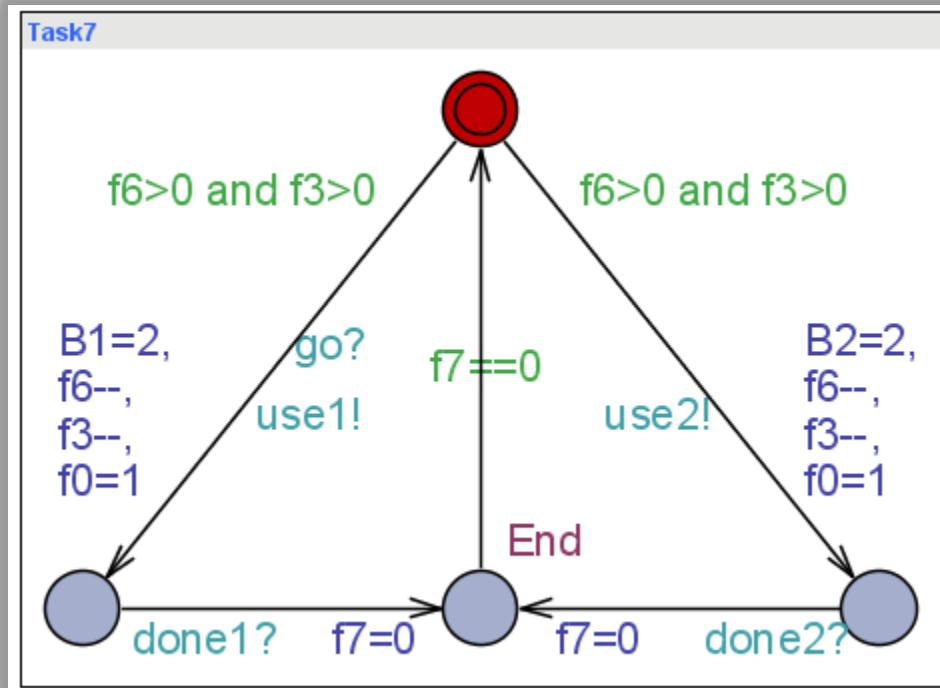


Optimal Infinite Scheduling



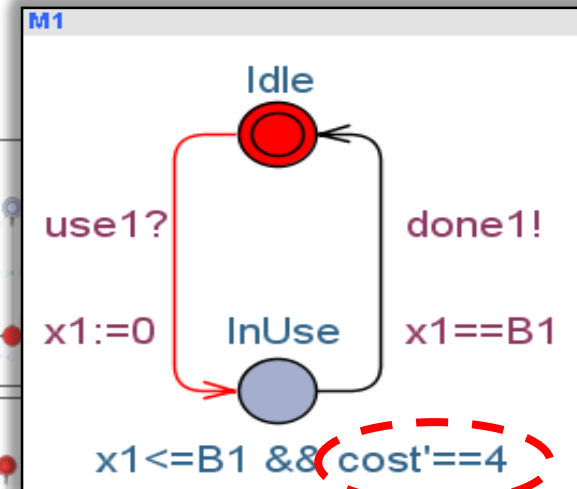
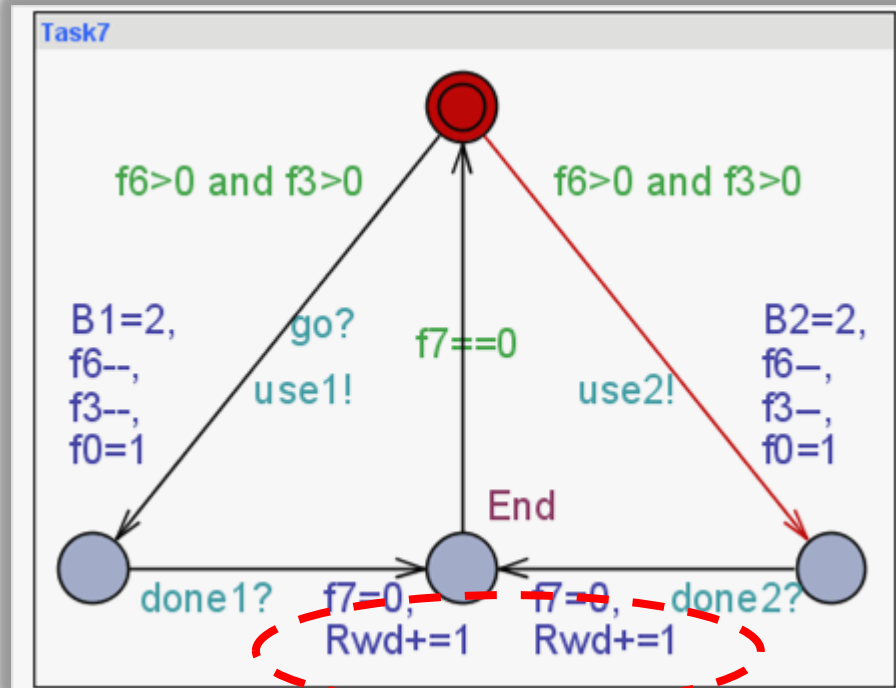
Maximize throughput:
i.e. maximize **Reward** / Time in the long run!

Optimal Infinite Scheduling



Minimize Energy Consumption:
i.e. minimize **Cost** / Time in the long run

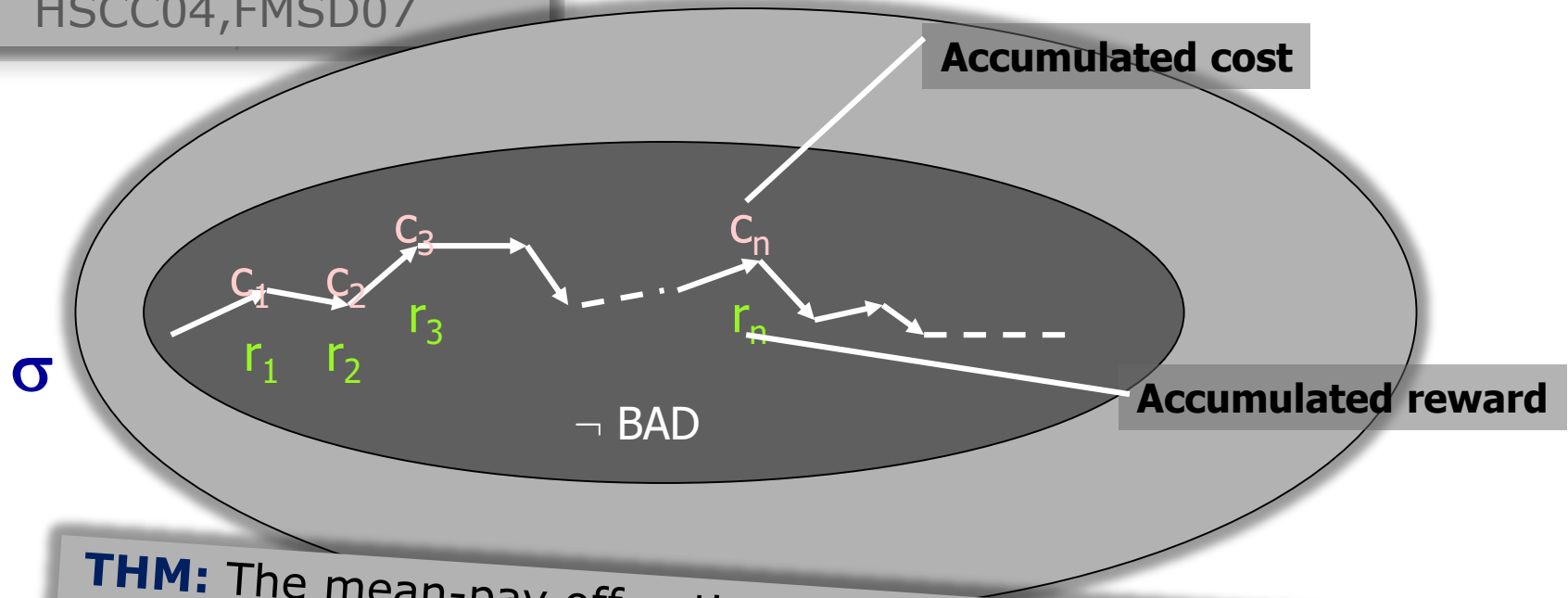
Optimal Infinite Scheduling



Maximize throughput:
i.e. maximize **Reward** / **Cost** in the long run

Mean Pay-Off Optimality

Bouyer, Brinksma, Larsen:
HSCC04, FMSD07



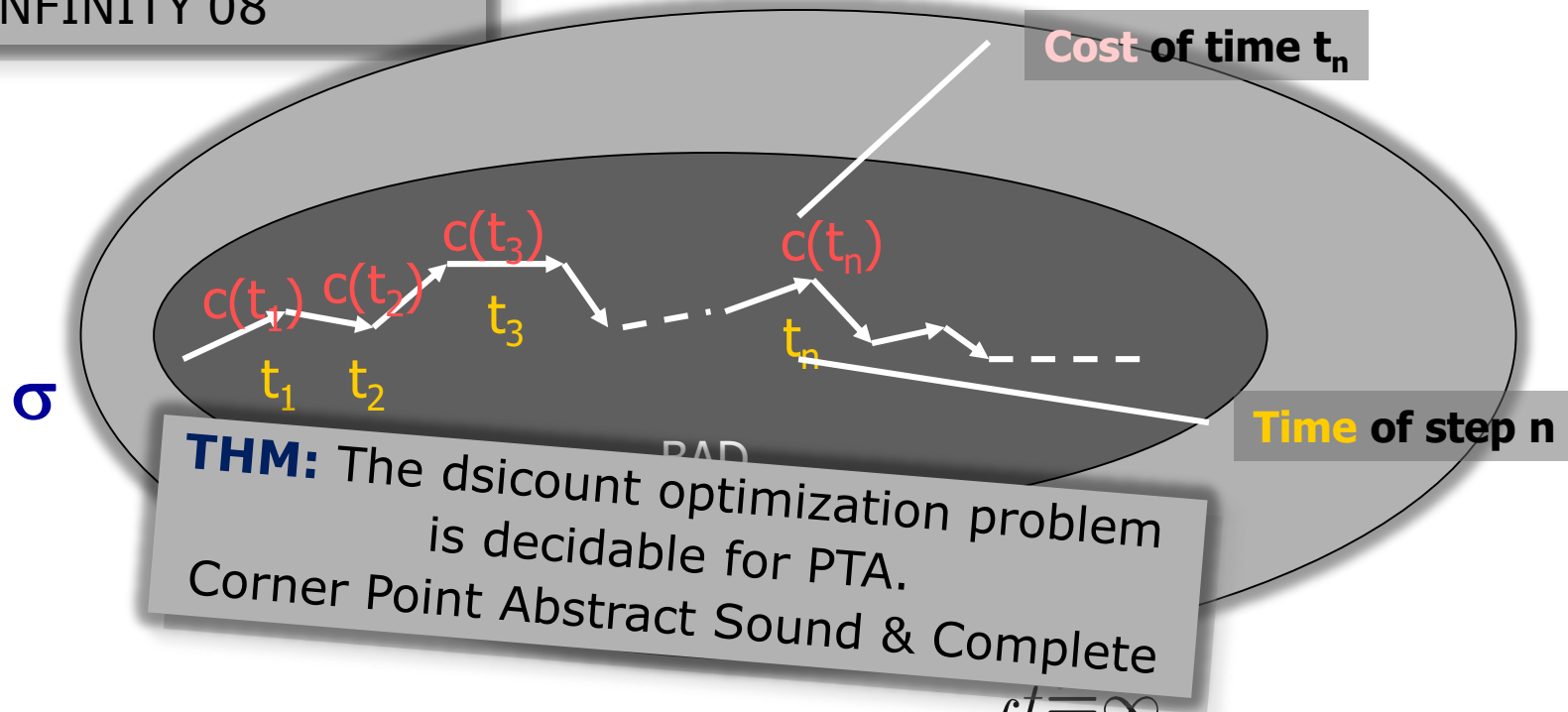
THM: The mean-pay off optimization problem is decidable
(and PSPACE-complete) for PTA.
Corner Point Abstract Sound & Complete

Optimal Schedule σ^* : $\text{val}(\sigma^*) = \inf_{\sigma} \text{val}(\sigma)$

Discount Optimality

$\lambda < 1$: discounting factor

Larsen, Fahrenberg:
INFINITY'08



Value of path σ : $\text{val}(\sigma) = \int_{t=0}^{t=\infty} c(t) \lambda^t dt$

Optimal Schedule σ^* : $\text{val}(\sigma^*) = \inf_{\sigma} \text{val}(\sigma)$

Soundness of Corner Point Abstraction

Lemma

Let Z be a (bounded, closed) zone and let f be a (well-defined) function over Z defined by:

$$f : (t_1, \dots, t_n) \mapsto \frac{a_1 t_1 + \dots + a_n t_n + a}{c_1 t_1 + \dots + c_n t_n + d}$$

then $\inf_Z f$ is obtained at a corner-point of Z (with integer coefficients).

Lemma

Let Z be a (bounded, closed) zone and let f be a function over Z defined by:

$$f : (t_1, \dots, t_n) \mapsto a_1 \lambda^{t_1} + \dots + a_n \lambda^{t_n} + a$$

then $\inf_Z f$ is obtained at a corner-point of Z (with integer coefficients).



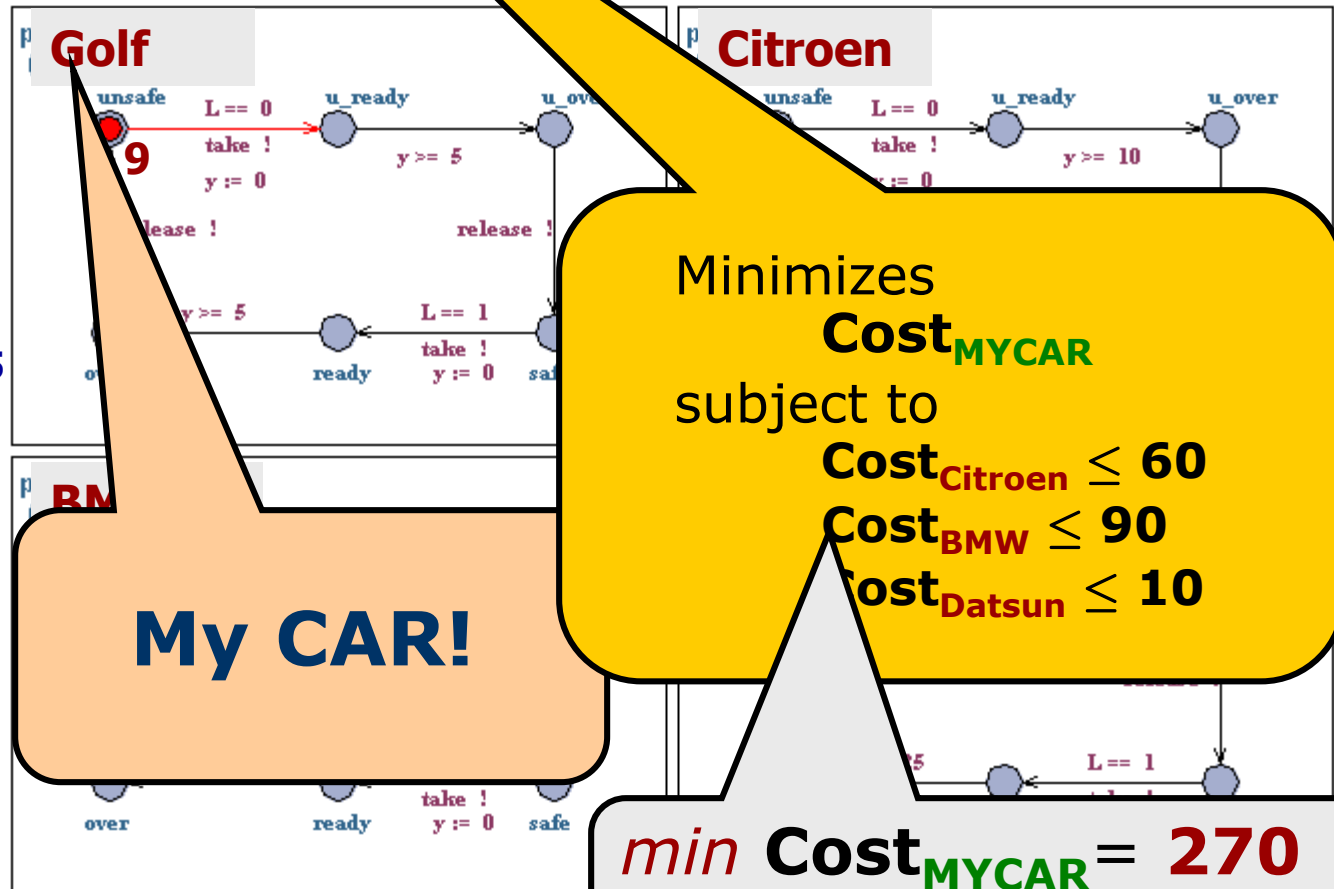
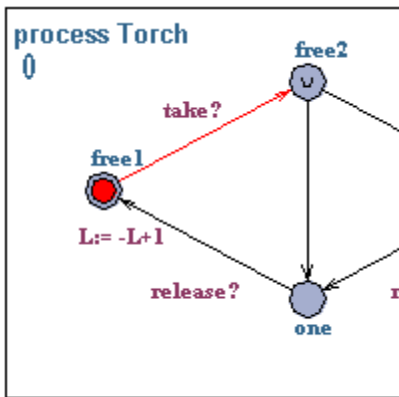
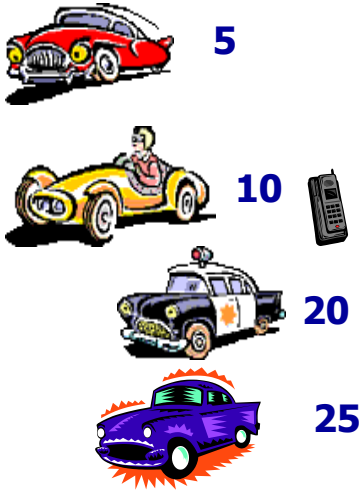
Optimal Conditional Reachability

with Jacob I. Rasmussen

CONDITIONAL

EXAMPLE: Optimal rescue plan for cars with different subscription rates for city driving !

UNSAFE



Minimizes
Cost_{MYCAR}
subject to

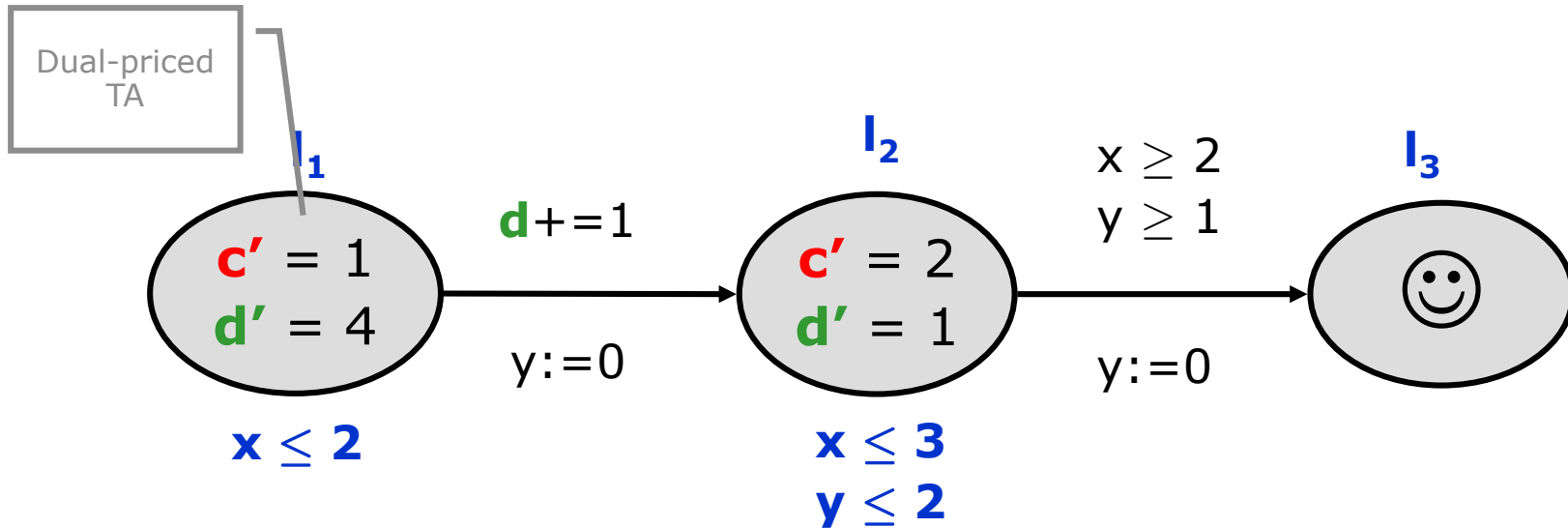
Cost_{Citroen} ≤ 60

Cost_{BMW} ≤ 90

Cost_{Datsun} ≤ 10

min Cost_{MYCAR} = 270
time = 70

Optimal *Conditional* Reachability



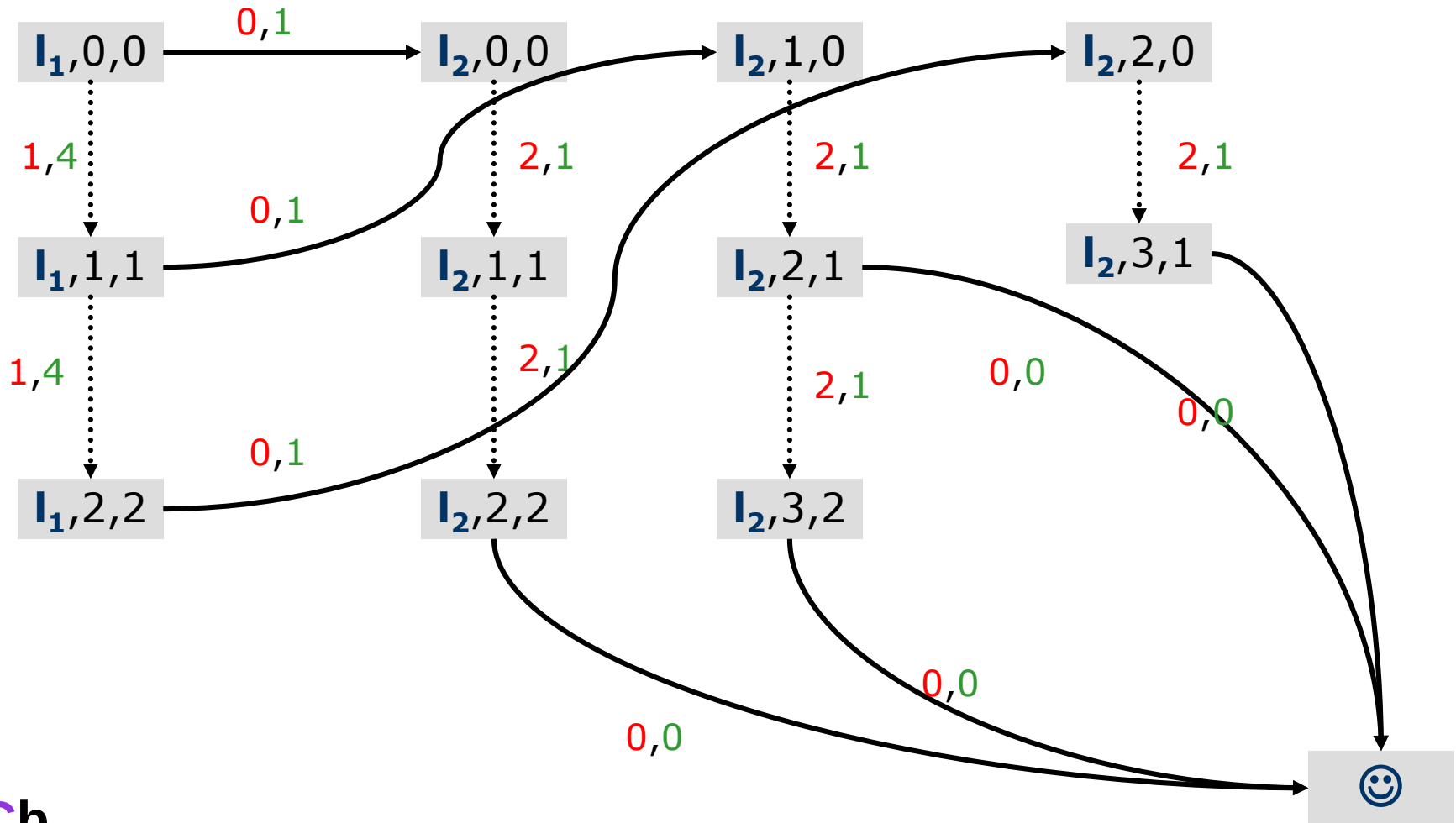
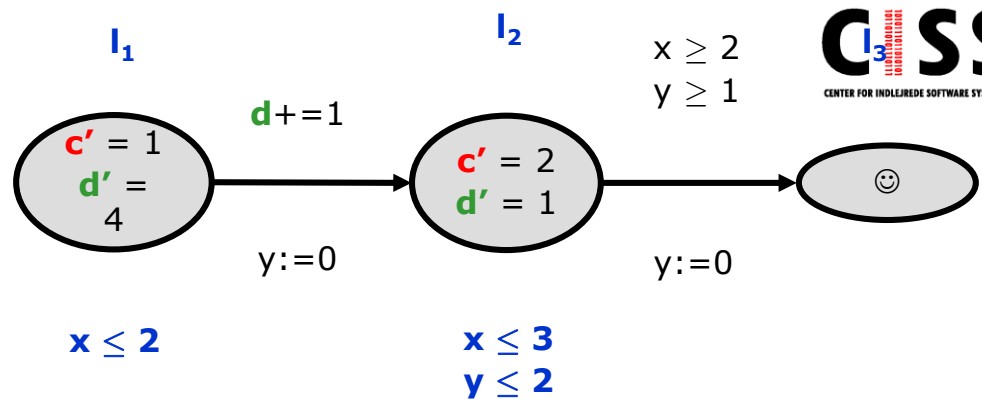
PROBLEM:

Reach l_3 in a way which minimizes c subject to $d \leq 4$

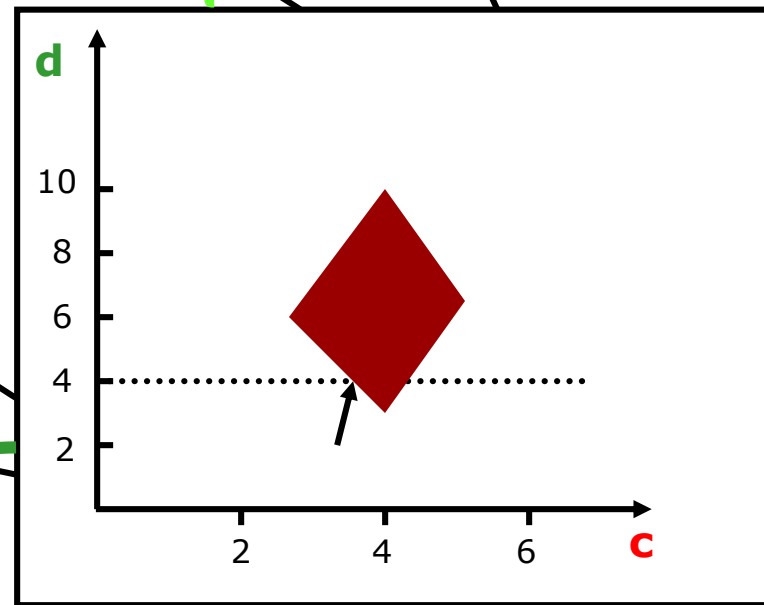
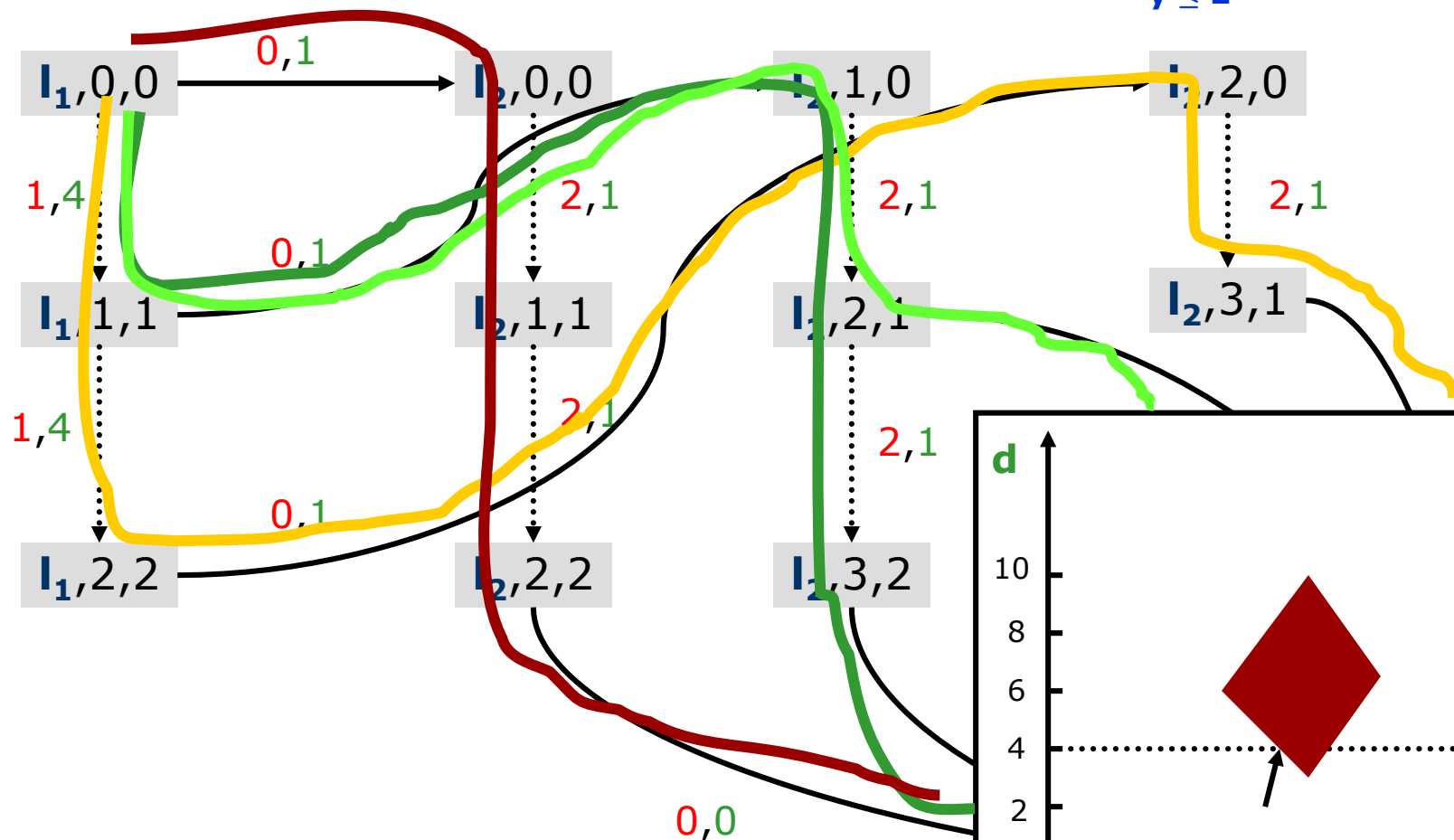
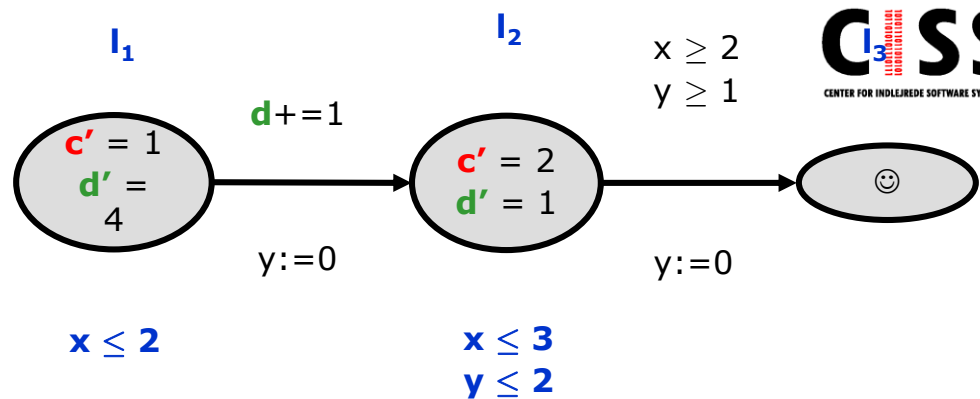
SOLUTION:

$c = 11/3 \rightarrow$
wait $1/3$ in l_1 ; goto l_2 ;
wait $5/3$ in l_2 ; goto l_3

Discrete Trajectories



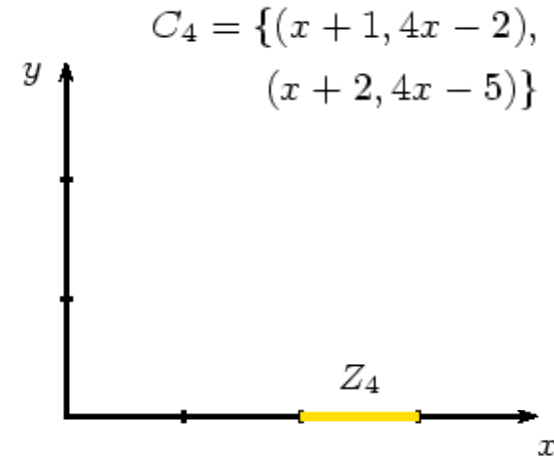
Discrete Trajectories



Dual Priced Zones

Definition

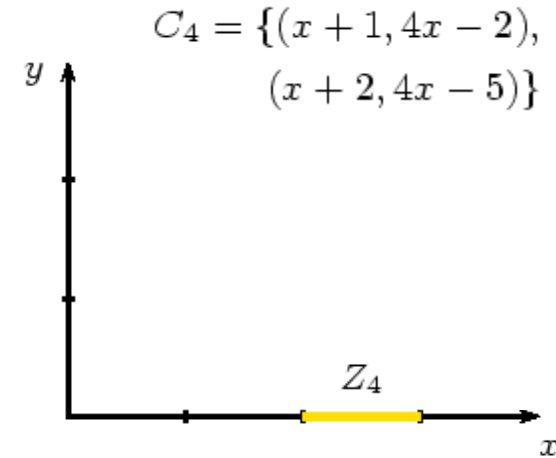
A dual-priced zone is a pair (Z, C) , where Z is a zone and C is a set of pairs of affine cost functions $\{(e_1, d_1), \dots, (e_n, d_n)\}$.



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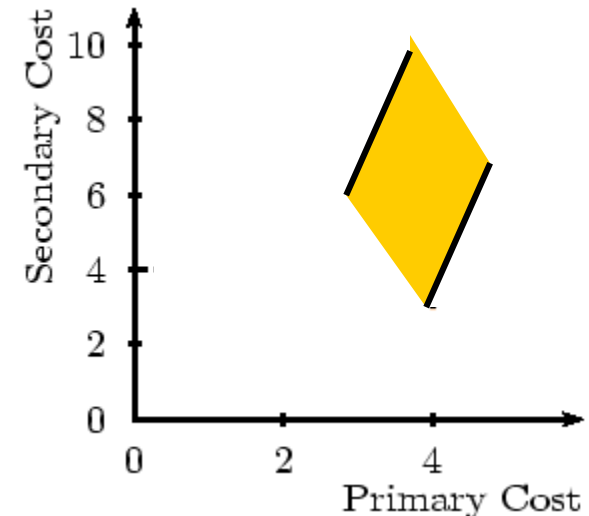


Definition

The dual-priced zone (Z, C) associates with a clock-valuation $u \in Z$ all cost pairs (c_1, c_2) such that:

$$(c_1, c_2) \in \lambda(C(u))$$

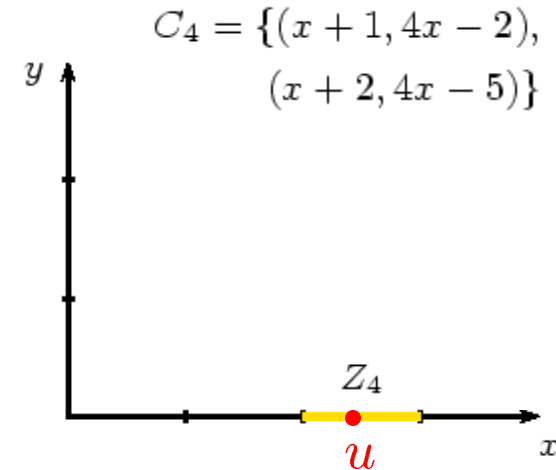
where $C(u) = \{(e_1(u), d_1(u)), \dots, (e_n(u), d_n(u))\}$ and $\lambda()$ is convex-hull closure.



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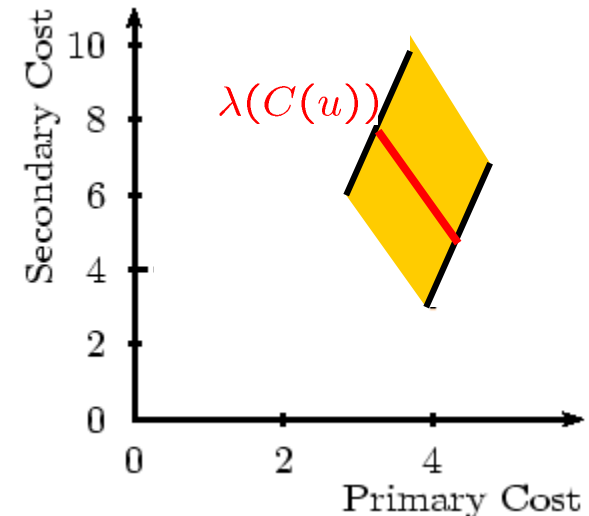


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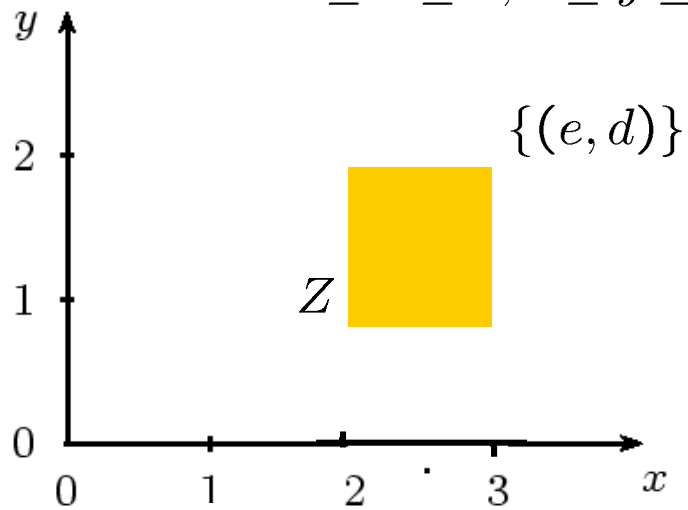


Reset

$$e = x + y$$

$$d = 4x - 3y + 1$$

$$Z : 2 \leq x \leq 3, 1 \leq y \leq 2$$

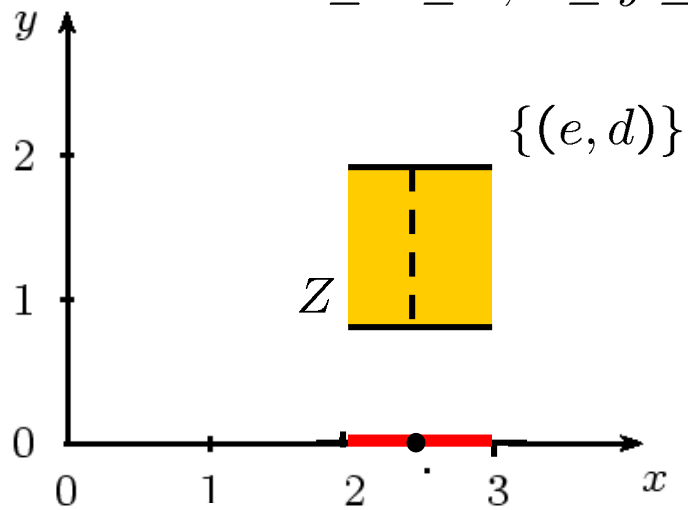


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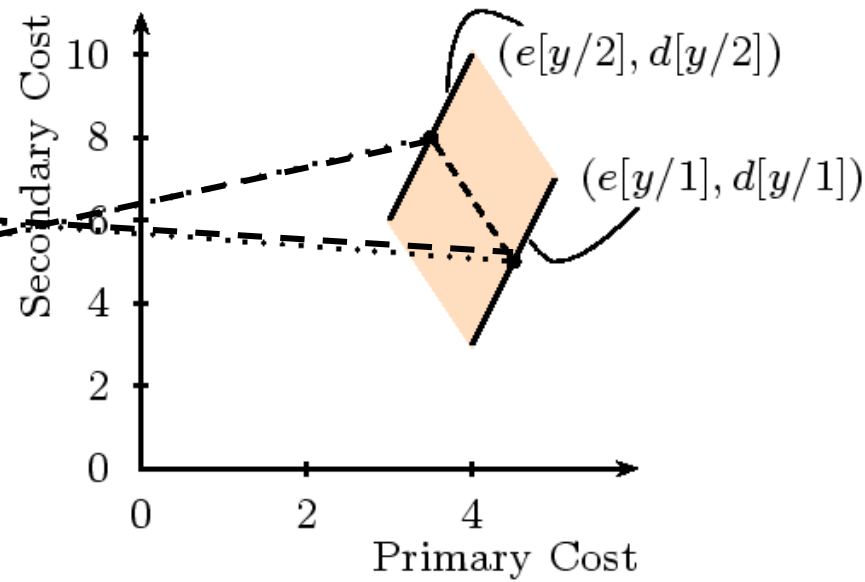
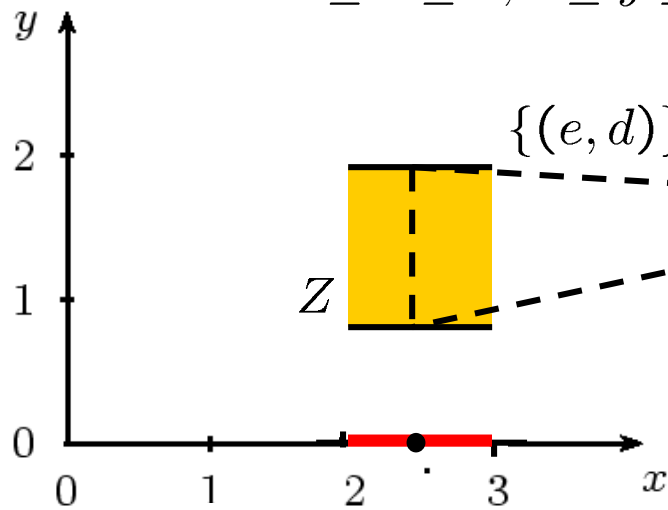


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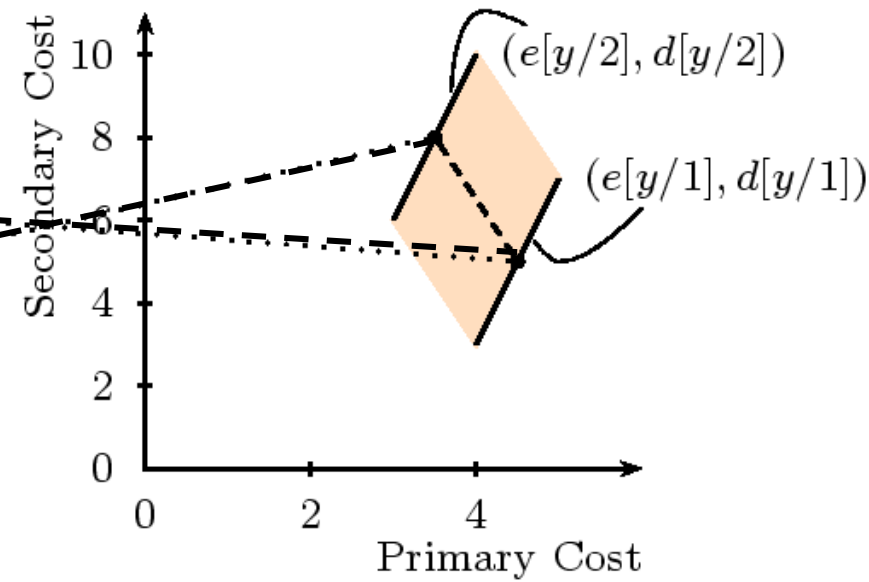
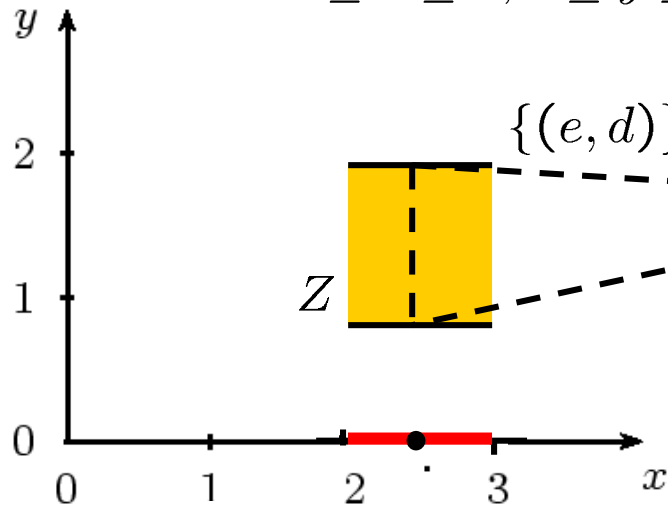


Reset

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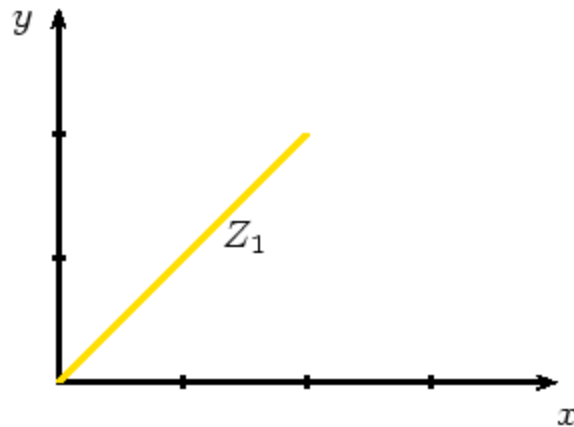
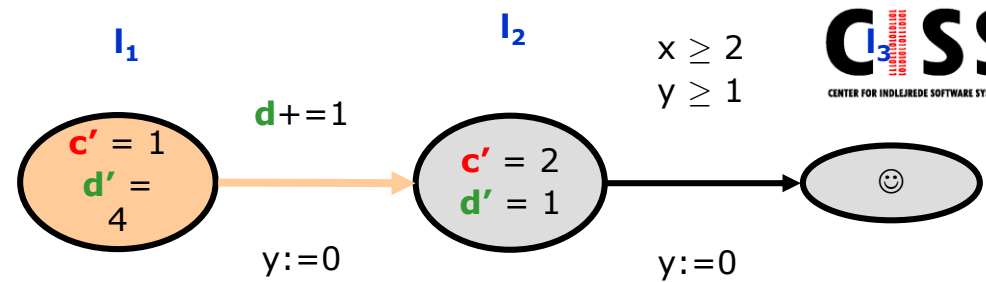
$$d = 4x - 3y + 1$$

$$Z : 2 \leq x \leq 3, 1 \leq y \leq 2$$

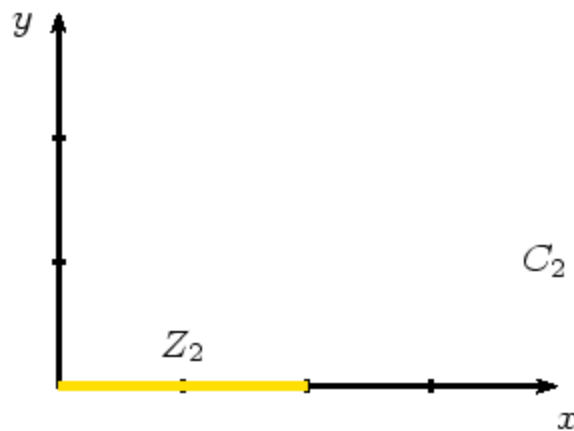
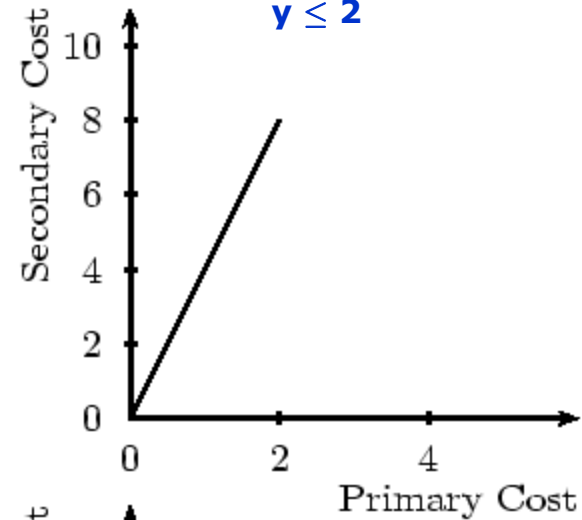


$$C = \{(x + 1, 4x - 2), \\ (x + 2, 4x - 5)\}$$

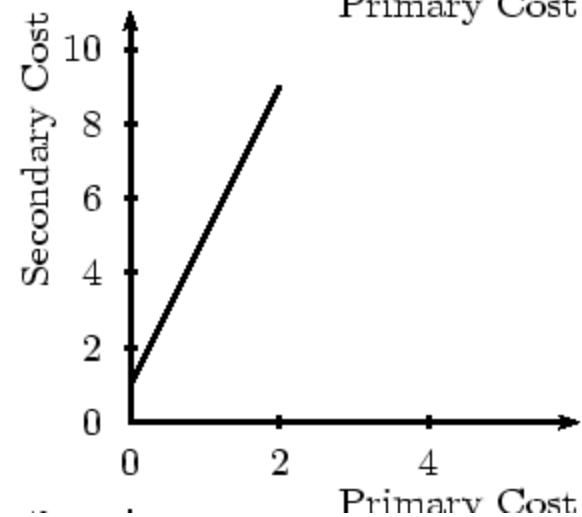
Exploration



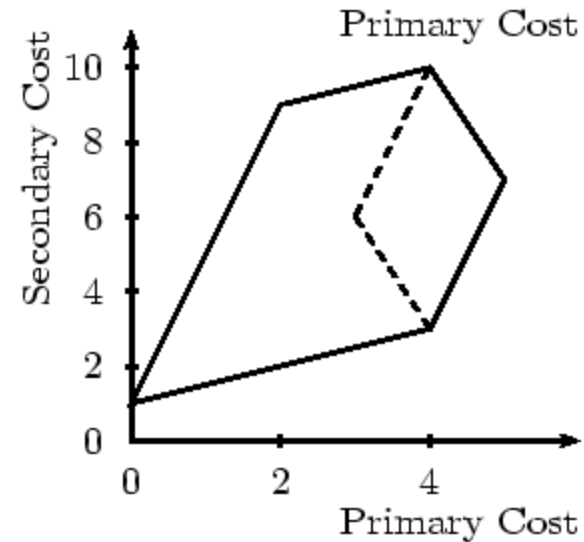
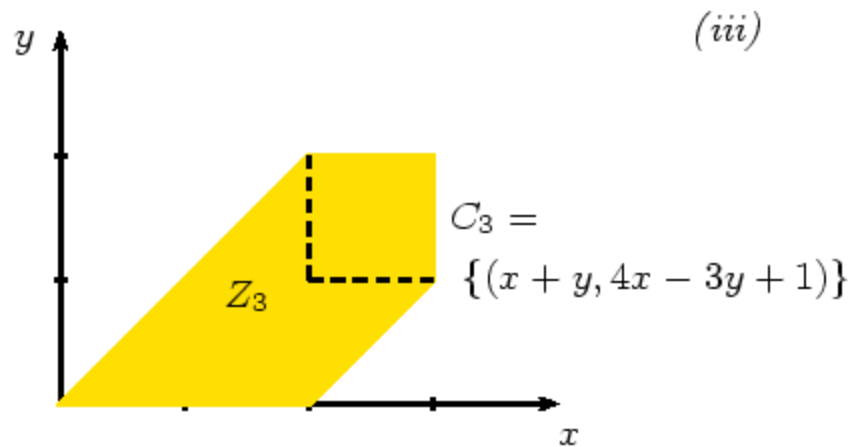
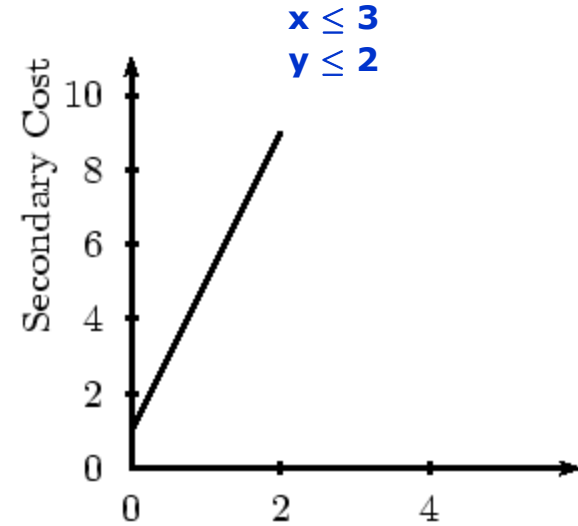
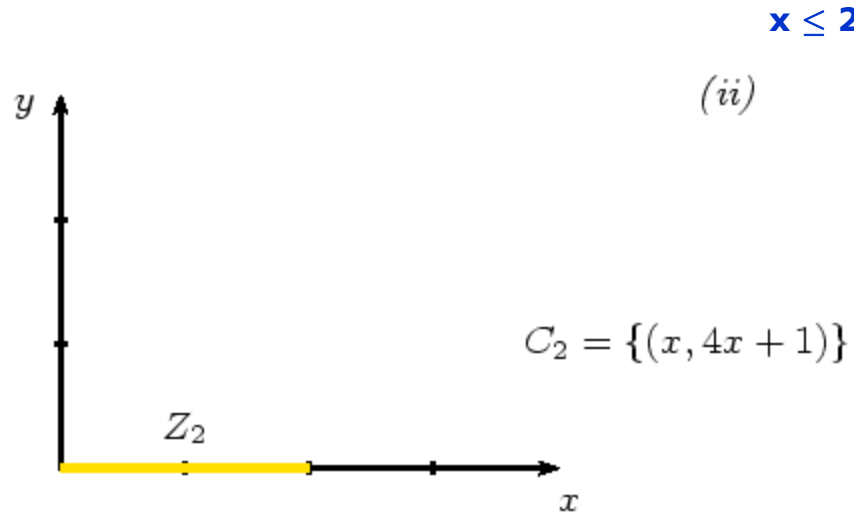
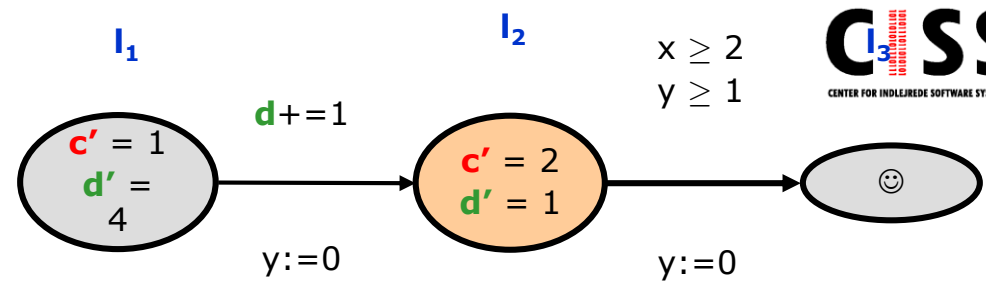
$$C_1 = \{(x, 4x)\}$$



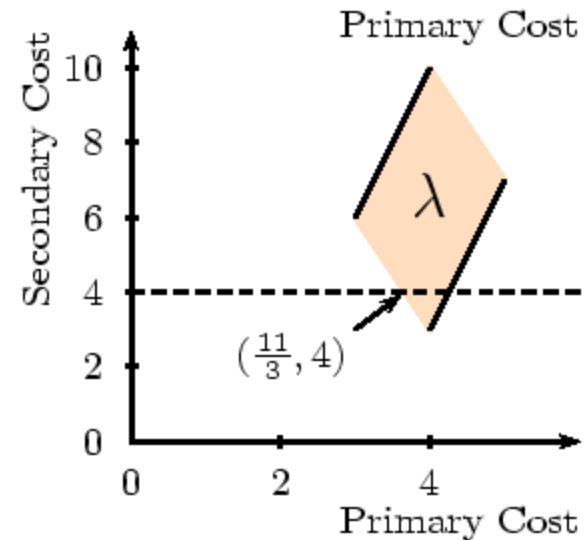
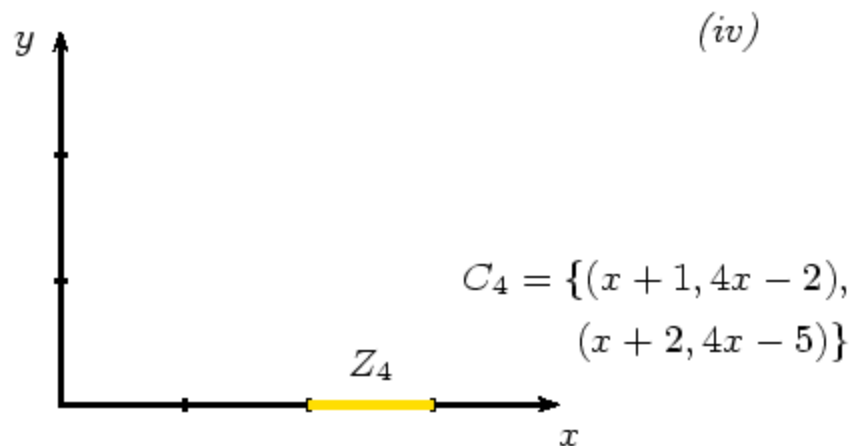
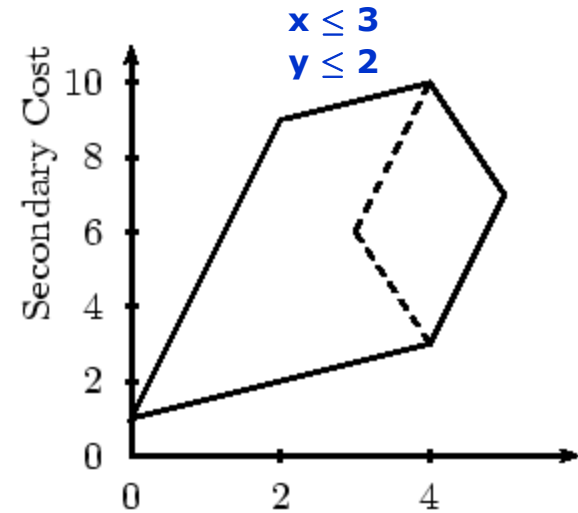
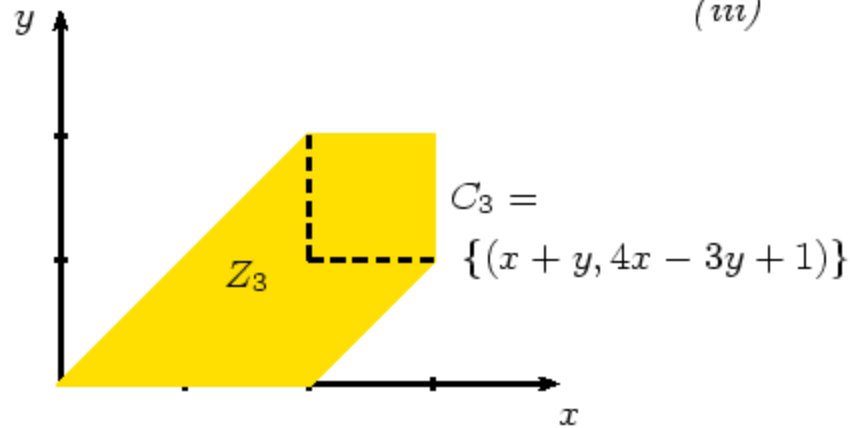
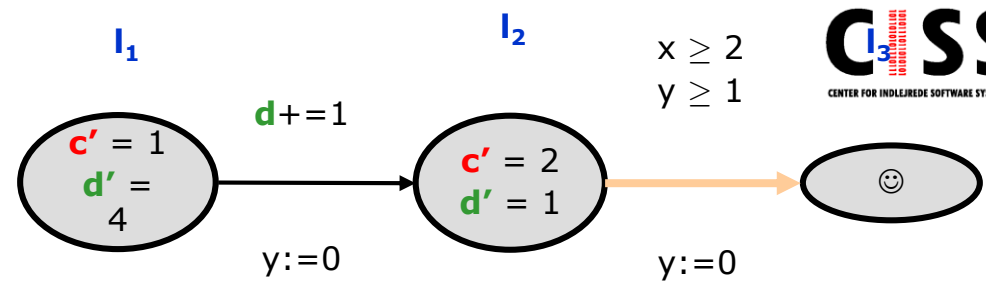
$$C_2 = \{(x, 4x + 1)\}$$



Exploration



Exploration



Termination

Definition

Let (Z, C) and (Z', C') be two dual-priced zones.

Then $(Z, C) \sqsubseteq (Z', C')$ iff

- i) $Z' \subseteq Z$, and
- ii) for any $(e', d') \in C'$ there exists $(e, d) \in C$
s.t. $e \leq_{Z'} e'$ and $d \leq_{Z'} d'$

Theorem

If $(Z,$

$\forall (c'_1,$

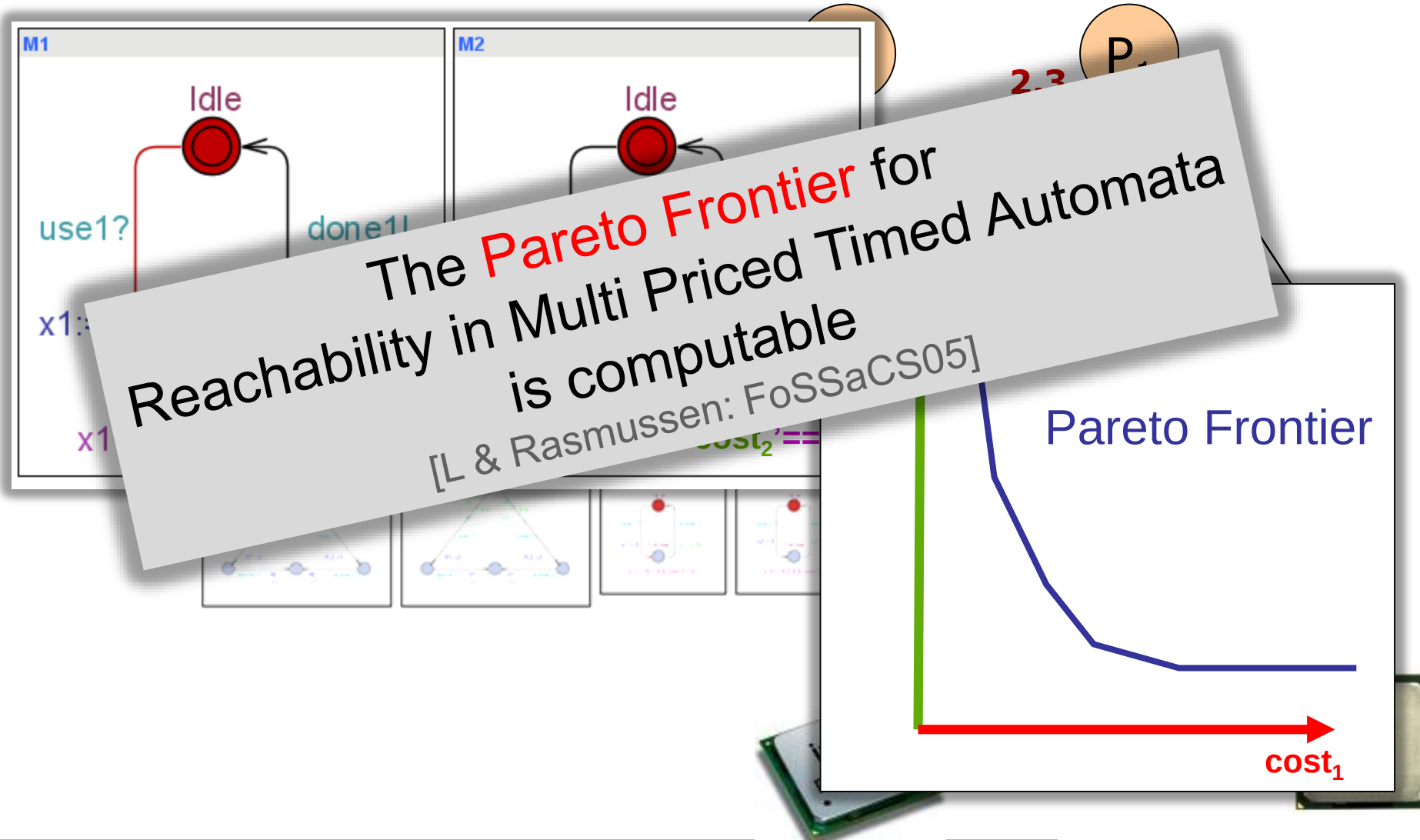
Theorem

$\sqsubseteq$ is a well-quasi ordering.

THEOREM
Optimal conditional
reachability for multi-priced TA
is computable.

$c_1 \sqsubseteq c_2$

Multiple Objective Scheduling



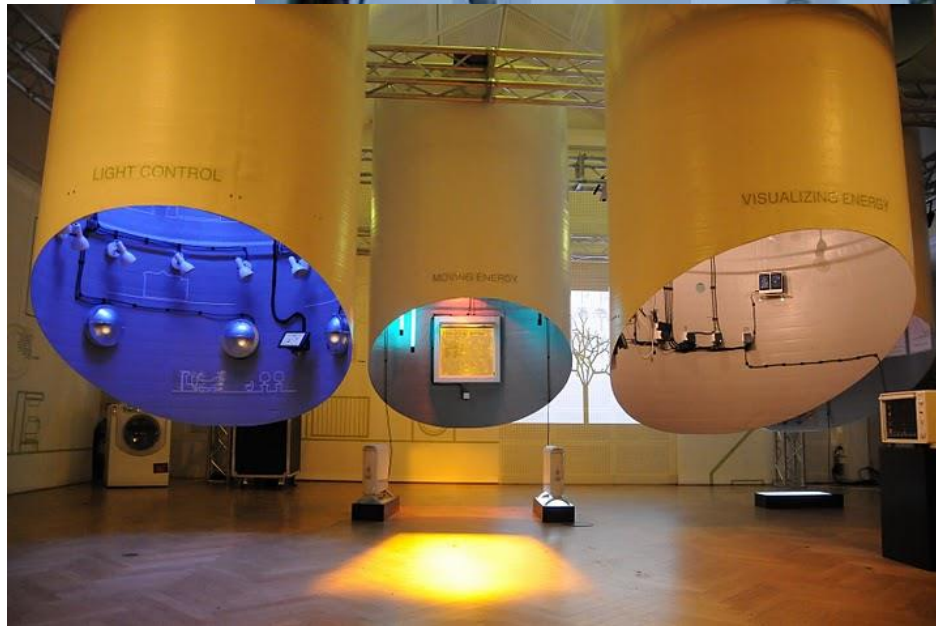
Applications



“Experimental” Results



"Experimental" Results

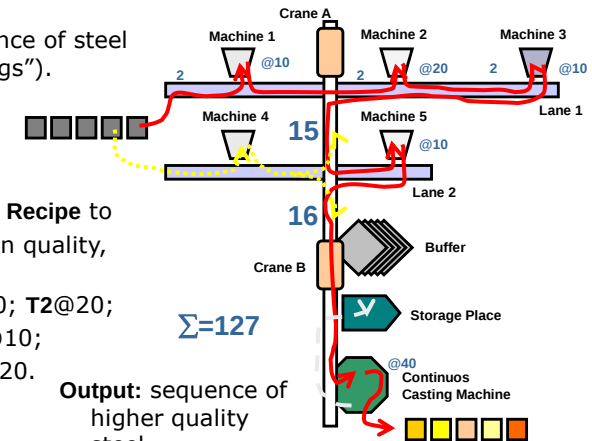
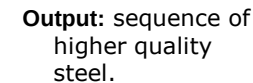
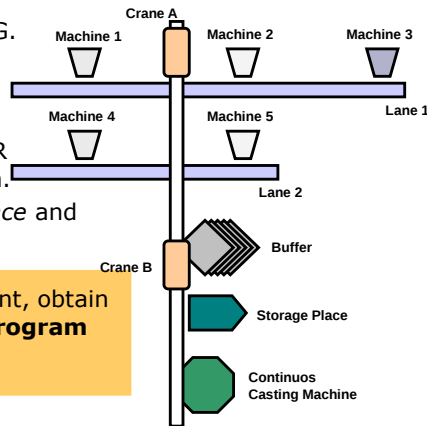


VHS

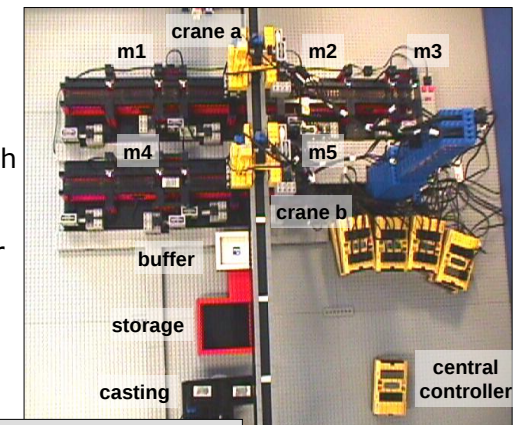


Steel Production Plant

- Objective:** **model** the plant, obtain **schedule** and control **program** for plant.

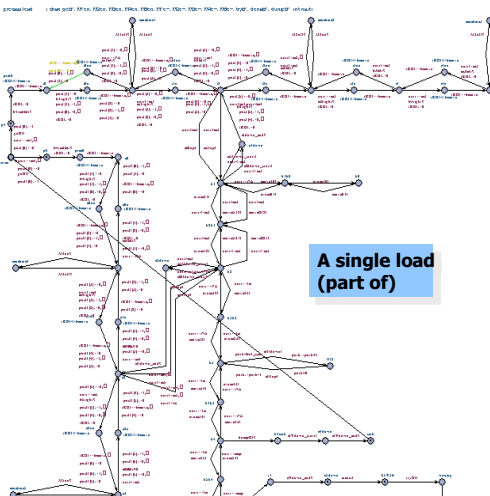


- LEGO RCX Mindstorms.
- Local controllers with control programs.
- IR protocol for remote invocation of programs.
- Central controller.

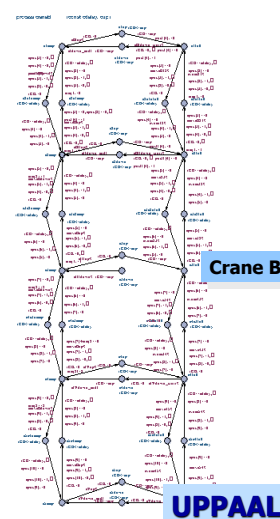


1971 lines of RCX code (n=5),
24860 - " - (n=60).

Synthesis



**A single load
(part of)**

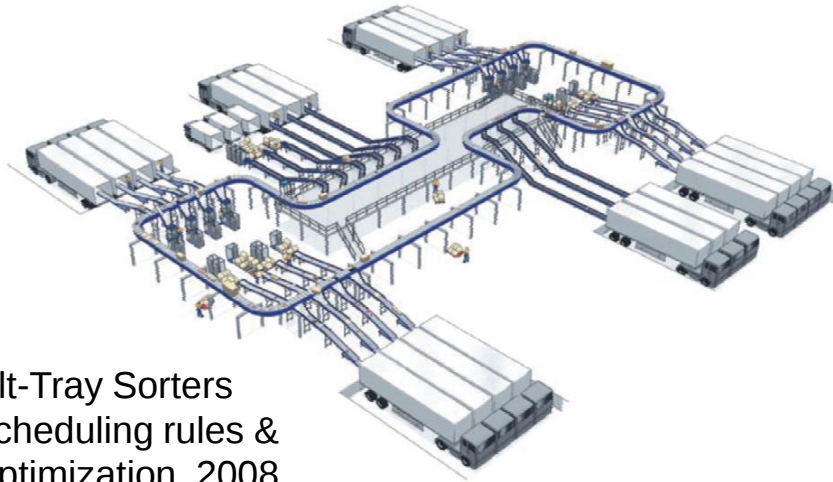


Crane B

UPPAAL

ARTEMIS

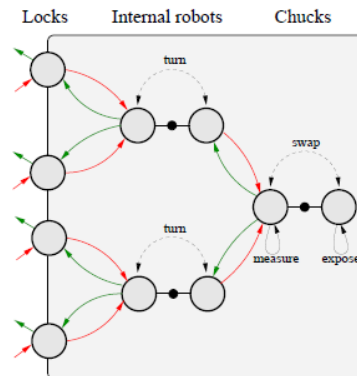




Tilt-Tray Sorters
Scheduling rules &
Optimization, 2008



Océ Datapath, 2012



ASML, 2004:
Wafer Scanners
Optimization of
Throughput

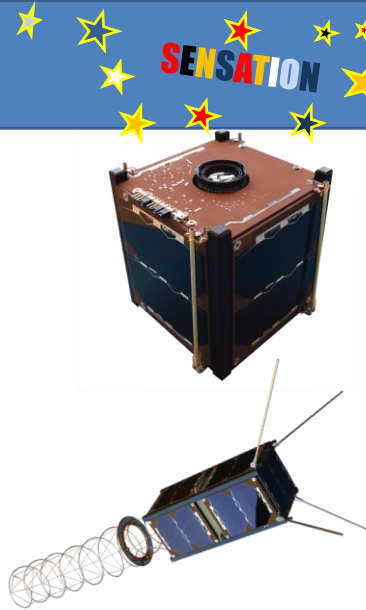
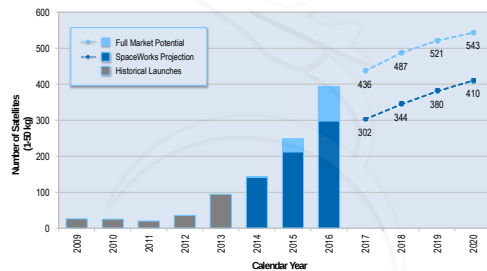
Philips: Indoor Lighting systems, 2014

Within the Prisma project of TNO-ESI and Philips Lighting, research is done into the robustness and reliability of large-scale indoor lighting systems. The focus is on the robustness of the lighting control system. To analyse control system robustness, model checking is used. Timed automata models of lighting control systems have been created and checked with the model checker Uppaal. To validate

NANO Satellites (2015)

NanoSatellites

- Nanosatellite: 1 – 20 kg
 - Smaller
 - Cheaper
 - Faster
 - But also capable!



SENSATION, Review Meeting, April 11, 2016

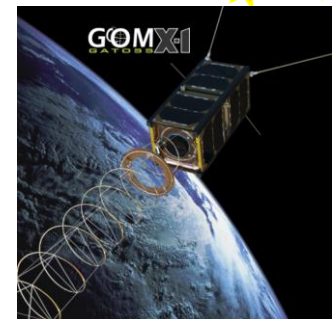
2

W

- Morten Bisgaard, GOMSPACE
- Holger Hermanns, Gilles Nies, Marvin Stenger, Saarbrücken

GomSpace Satellites

- 2U satellite -> 2013
- Tracking airplanes
- “Simple” mission



- 3U satellite -> 2015
- 3 major payloads
- Complex mission

SENSATION, Review Meeting, April 11, 2016

5



May 2017

GOMX-3 Deployment



Scheduling Approach

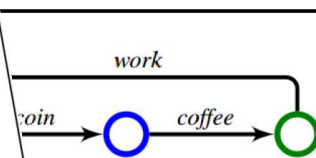
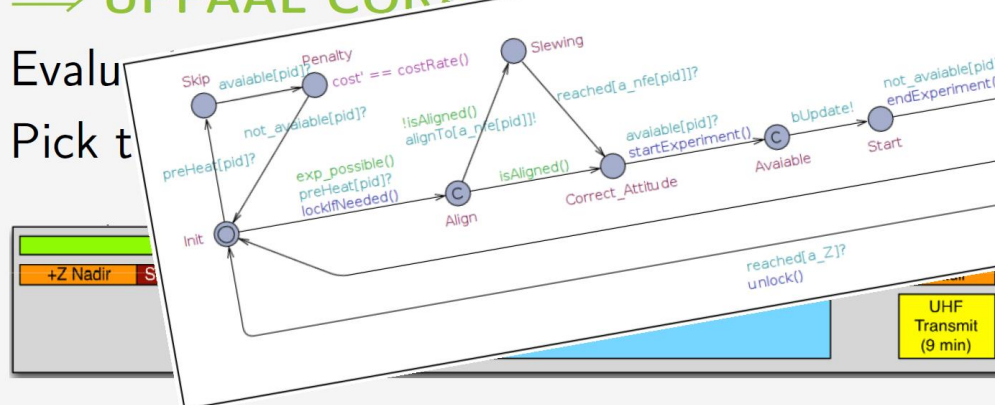
Planning GOMX-3 Behaviour

- Orbit can be accurately predicted
 - ⇒ Experimentation
- Network of **line** (LPTA)
 - Experiments
 - positioning
 - powering
 - Battery (linear)
 - Environment (partially)

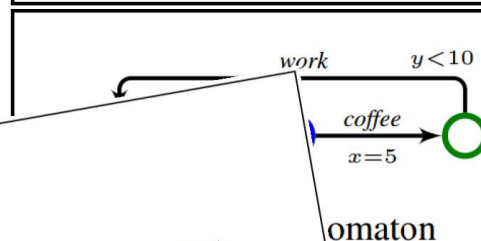
- Generate optimal path candidates for 1

⇒ UPPAAL CORA

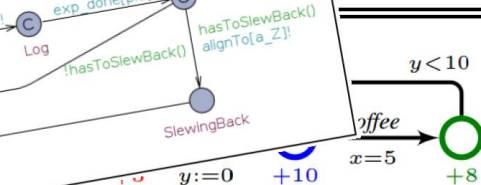
- Evalu
- Pick t



(a) A finite automaton

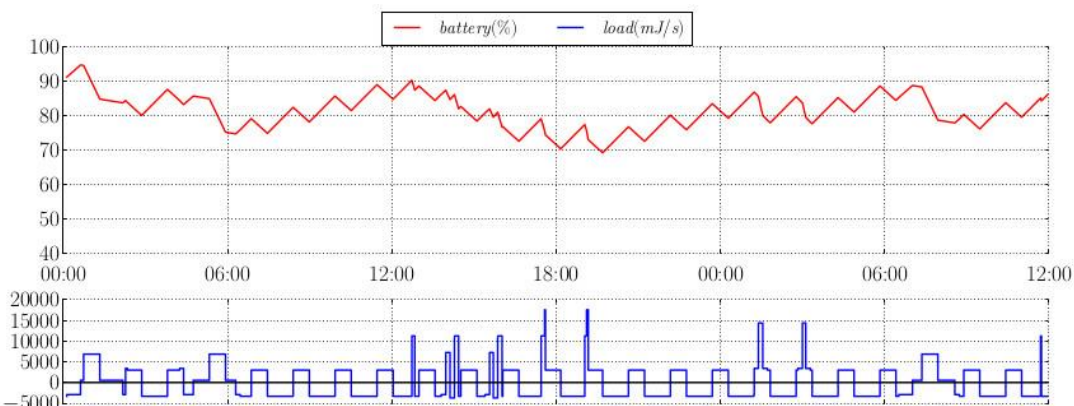
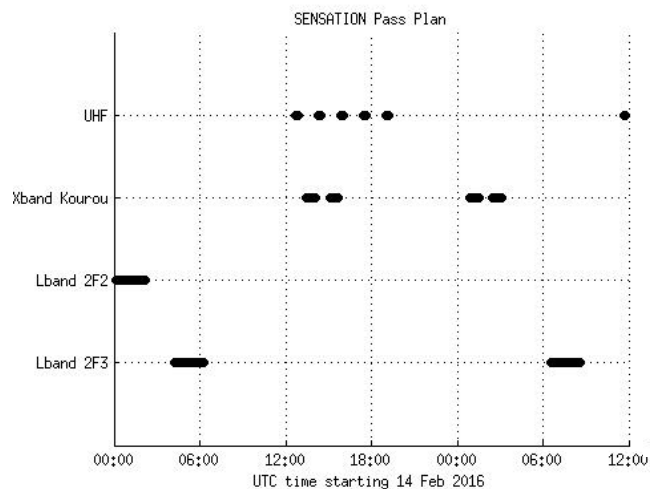


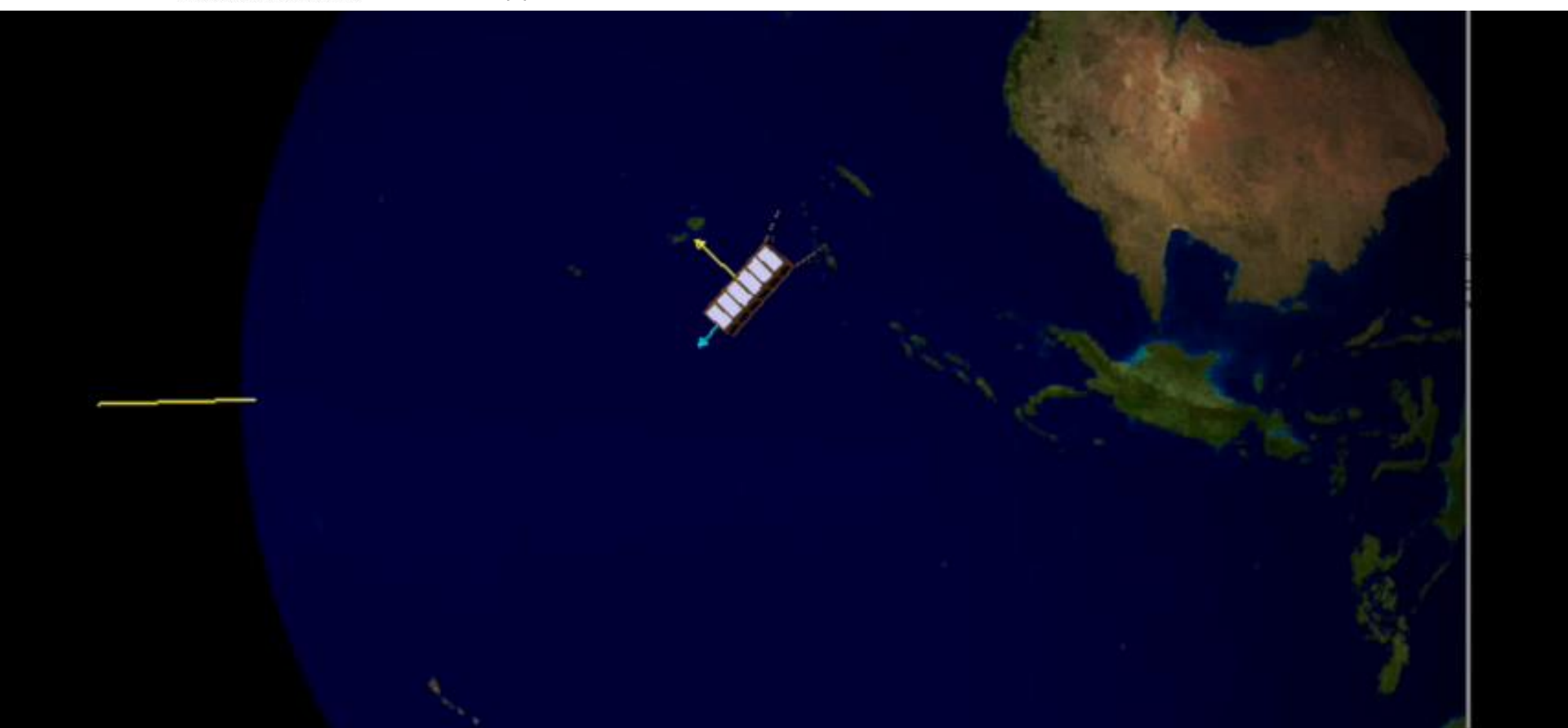
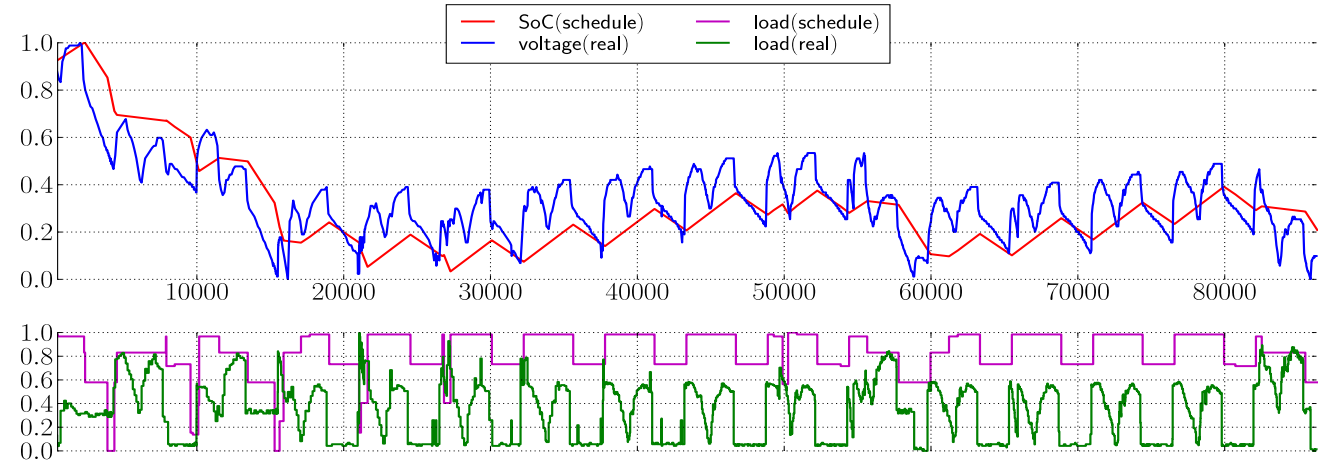
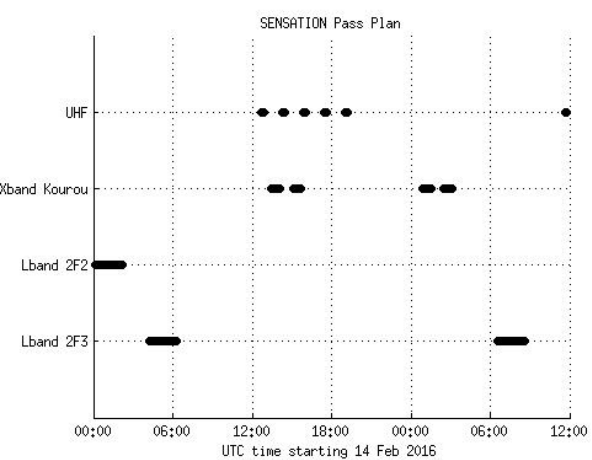
omaton



(c) A priced timed automaton

Derived Schedule





Energy Automata

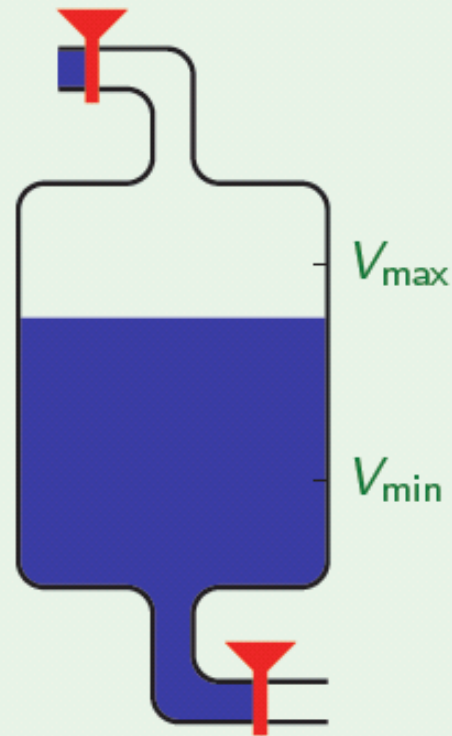


Managing Resources

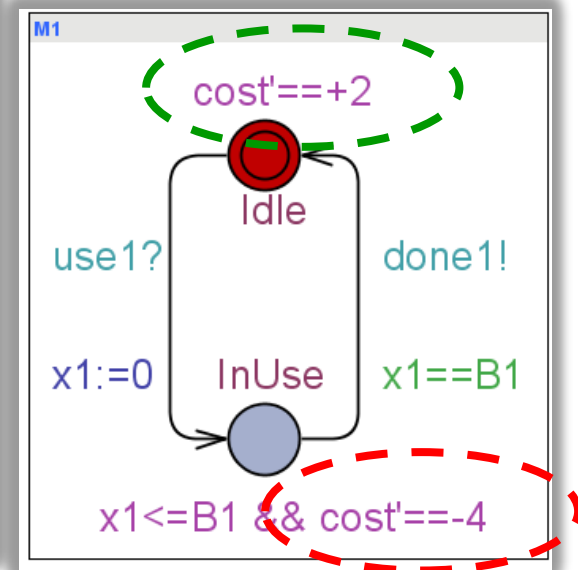
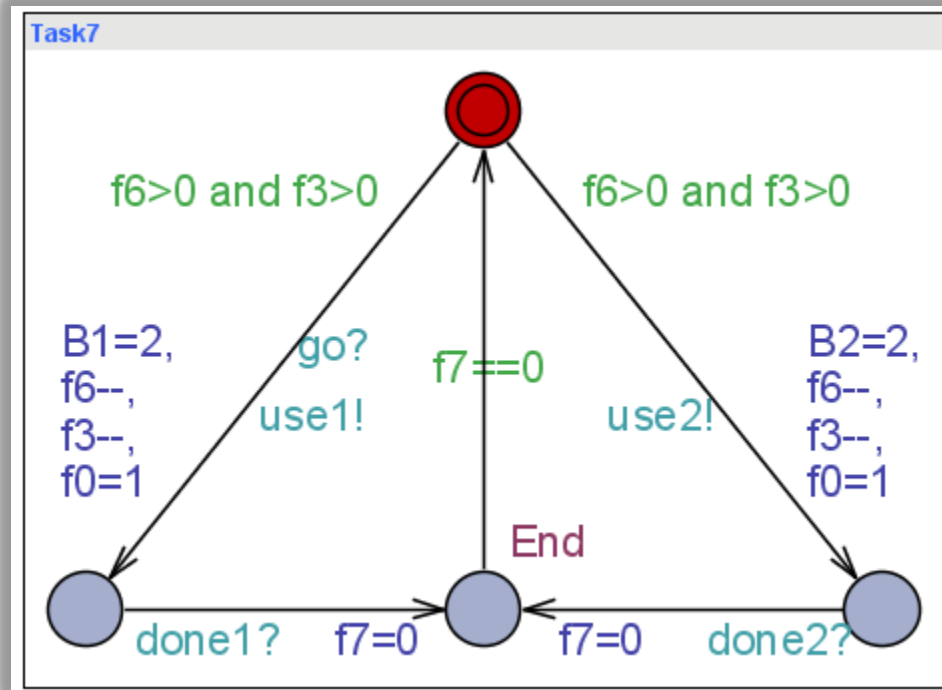
Example

In some cases, **resources** can both be **consumed and regained**.

The aim is then to **keep the level of resources within given bounds**.



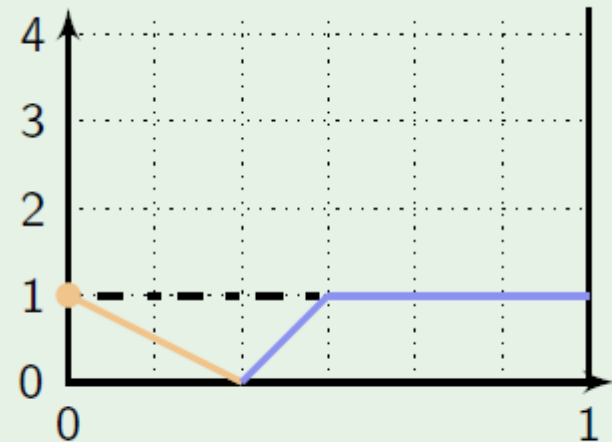
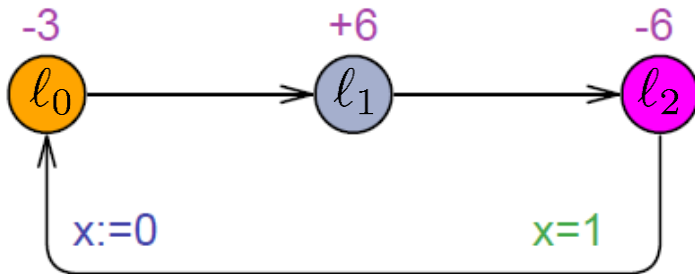
Consuming & Harvesting Energy



Maximize throughput
while respecting: $0 \leq E \leq MAX$

Energy Constrains

- Energy is not only consumed but may also be regained
- The aim is to **continuously** satisfy some energy constraints



lower-weak-upper-bound problem

One Weight Results

1 Clock

	games	existential problem	universal problem
L	?	$\in P$	$\in P$
L+W	?	$\in P$	$\in P$
L+U	undecidable	decidable (flat)	?

1½ Clock

	games	existential problem	universal problem
L	?	decidable	decidable
L+W	?	decidable	decidable

>3 Clocks

	games	existential problem	universal problem
L	undecidable	undecidable	PSPACE-c
L+W	undecidable	undecidable	PSPACE-c

P Bouyer, U Fahrenberg, K Larsen, N Markey,... . Infinite runs in weighted timed automata with energy constraints. 2008.
P. Bouyer, U. Fahrenberg, K. G. Larsen, N. Markey: Timed automata with observers under energy constraints. HSCC 2010
P. Bouyer, K. G. Larsen, and N. Markey. Lower-bound constrained runs in weighted timed automata. QEST 2012



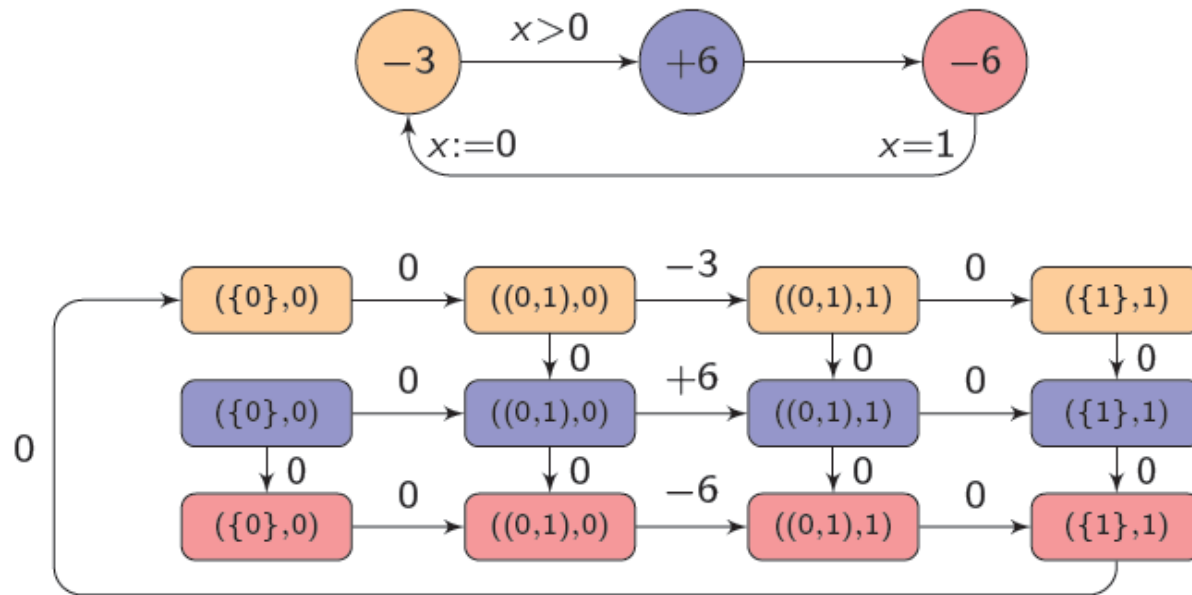
L-Problem for 1-Clock Case

Theorem:

The L-problem is decidable in **PTIME** for **1**-clock PTAs

Proof.

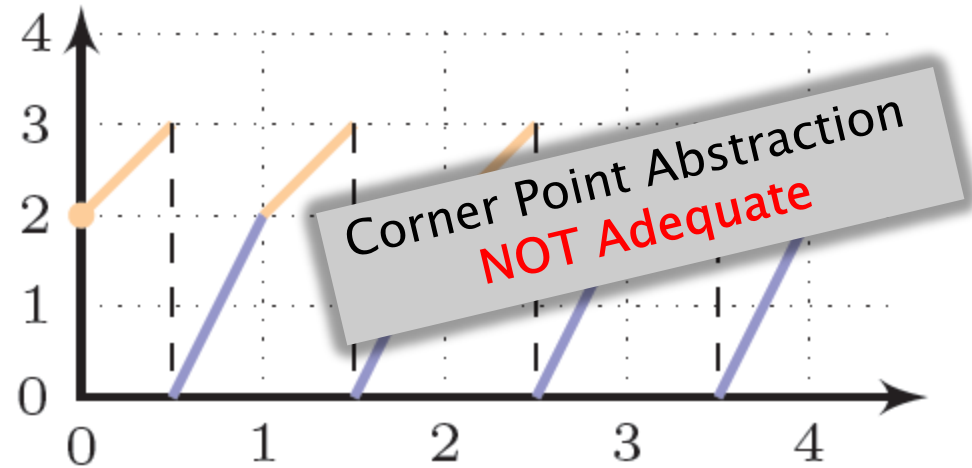
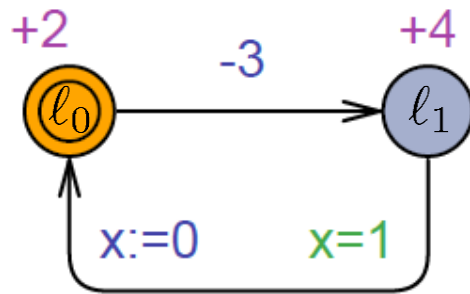
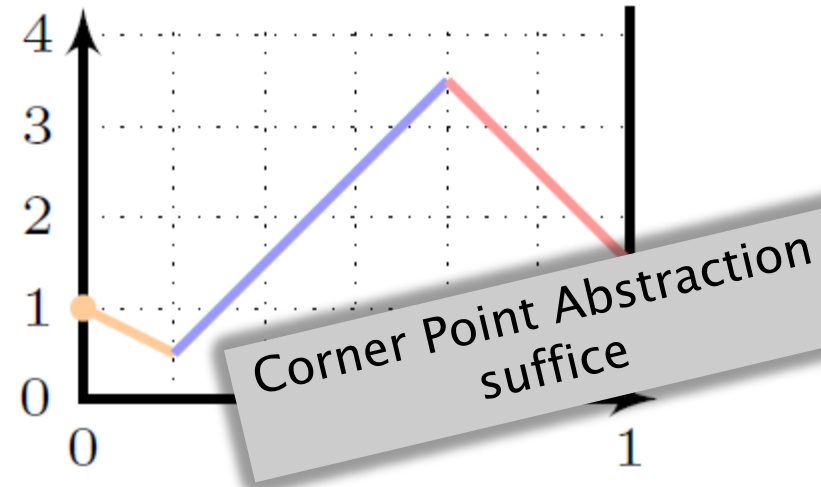
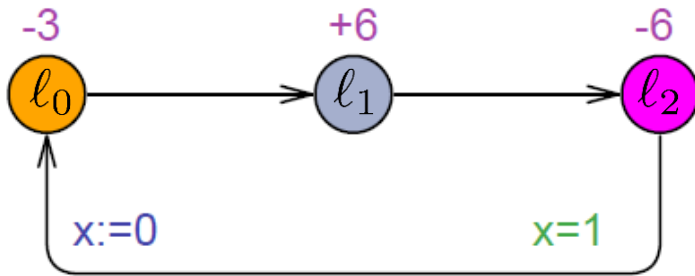
- Corner-point abstraction:



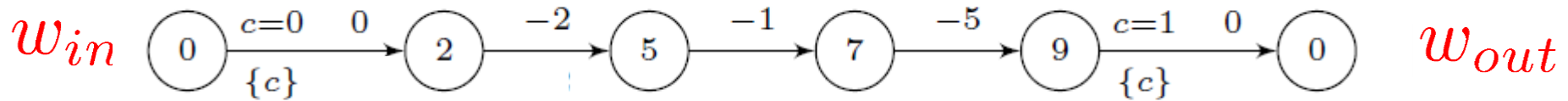
P Bouyer, U Fahrenberg, K Larsen, N Markey, Infinite runs in weighted timed automata with energy constraints. 2008.



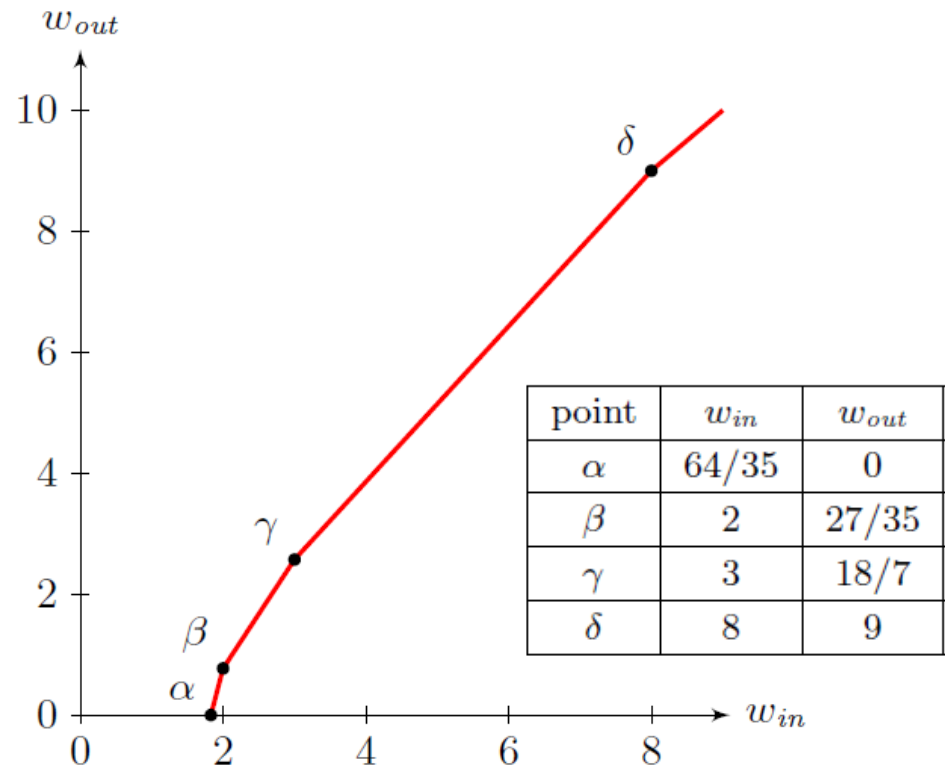
1½ Clocks = Discrete Updates



New Approach: Energy Functions



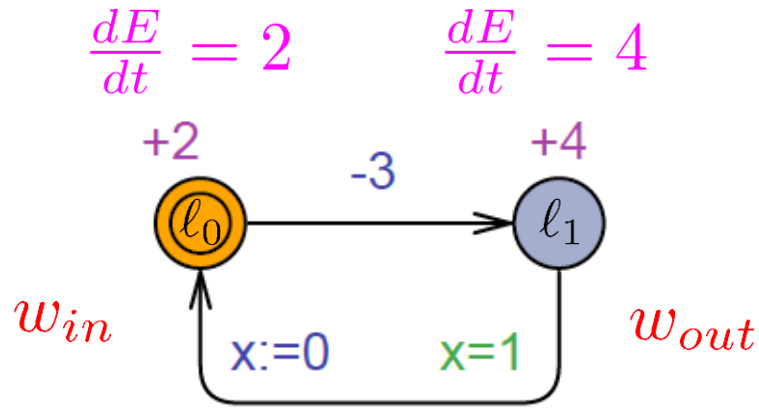
- Maximize energy along paths
- Use this information to solve general problem



P. Bouyer, U. Fahrenberg, K. G. Larsen, N. Markey: Timed automata with observers under energy constraints. HSCC 2010

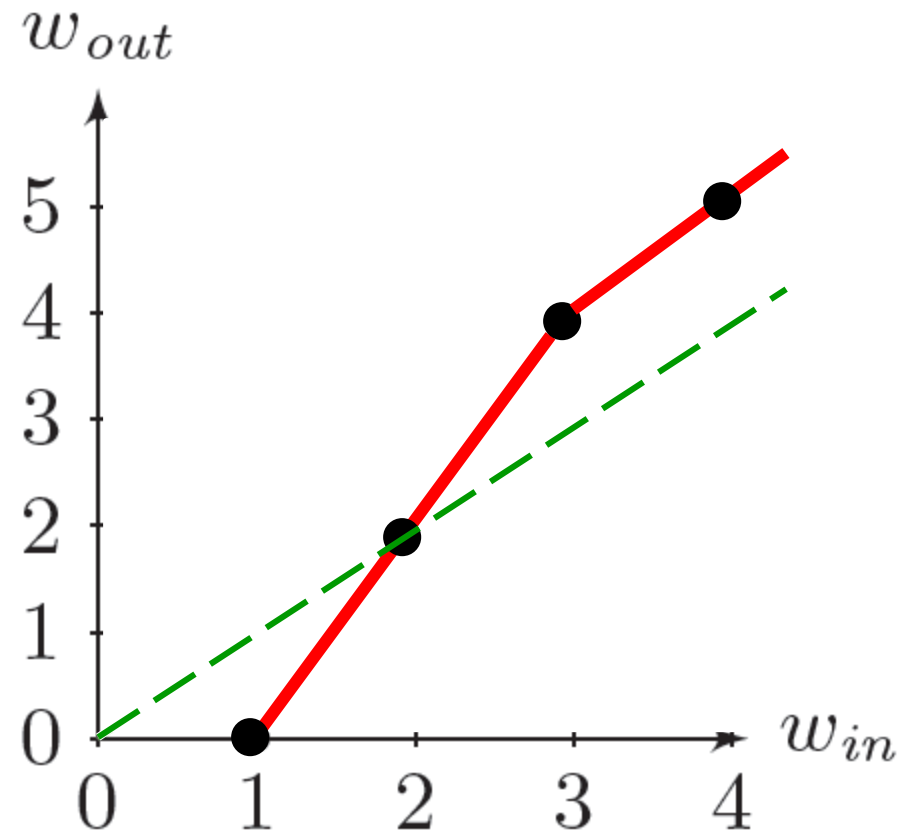


Energy Function

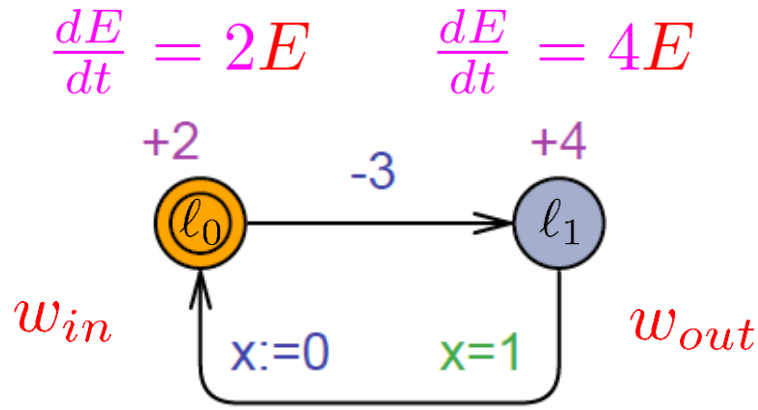


General Strategy

Spend just enough time to survive the next negative update



Exponential PTA

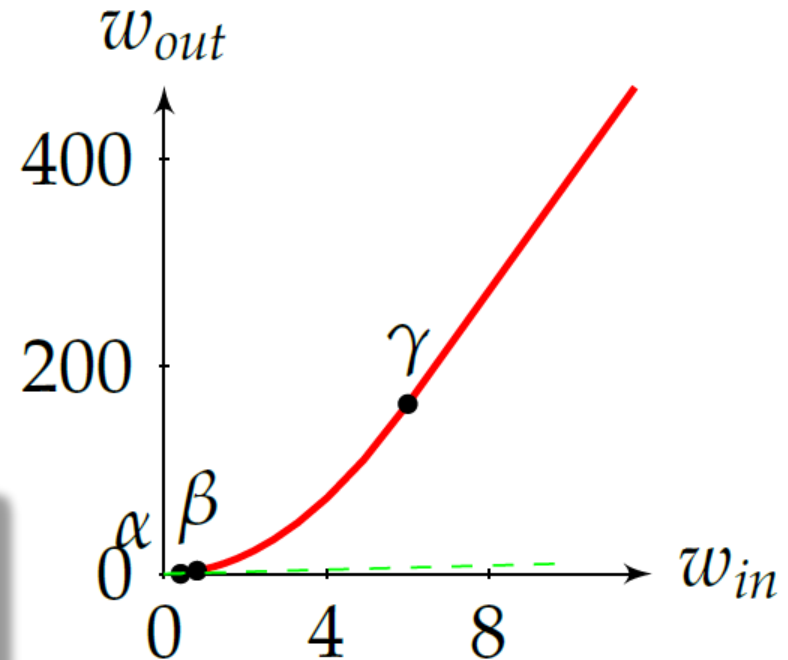


General Strategy

Spend just enough time

~~to survive the next negative update~~

so that after next negative update there is a certain positive amount !

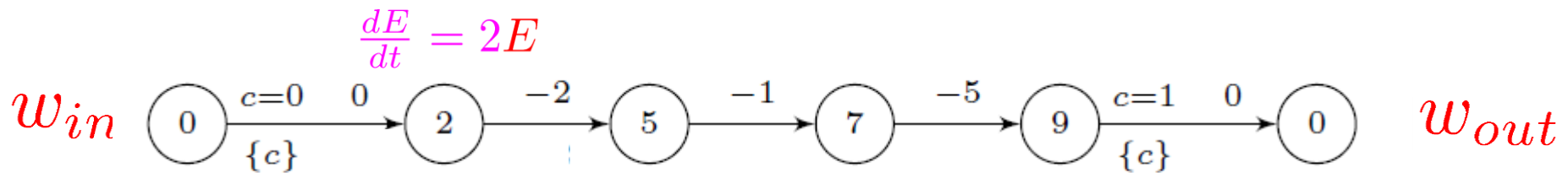


Minimal Fixpoint:

$$\frac{3}{e^2 - 1} \approx 0.47$$



Exponential PTA



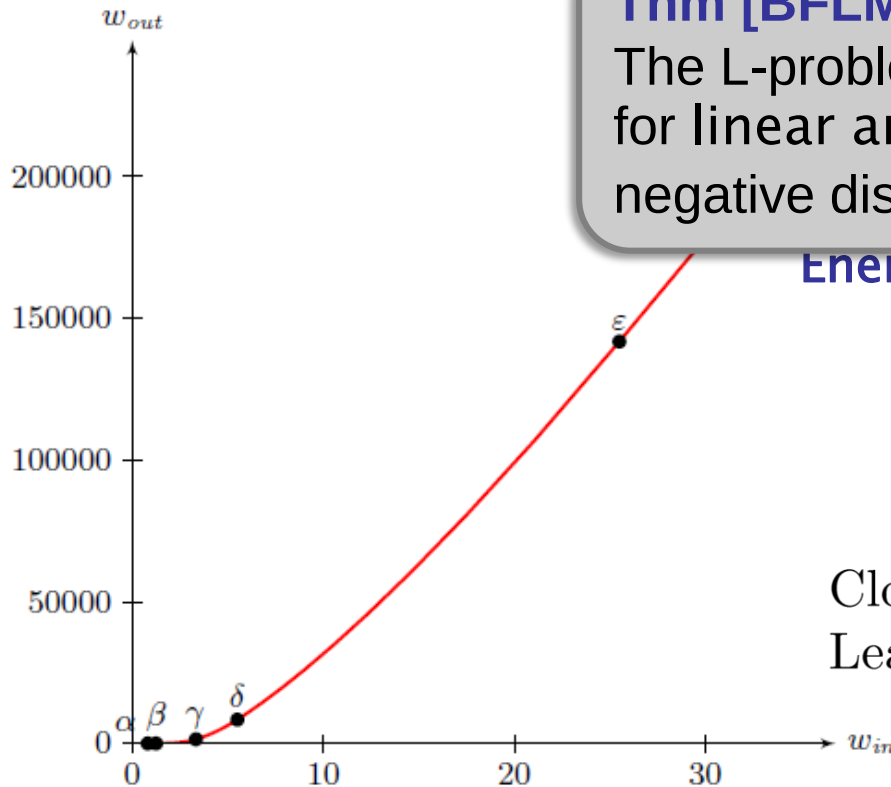
Thm [BFLM09]:

The L-problem is **decidable** for linear and exponential 1-clock PTAs with negative discrete updates.

Energy Function

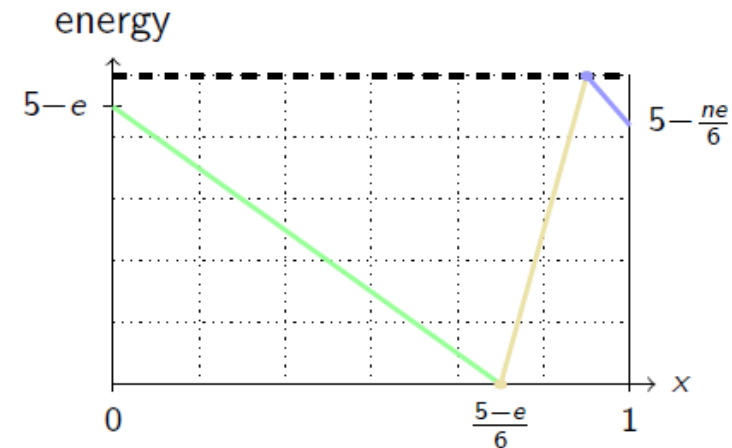
- $f : x \mapsto \alpha \cdot x^r + \beta$ where r is rational
- $\frac{df}{dt} \geq 1$

Closed under max and composition.
Least fixed point computable.



Multiple Costs & Clocks

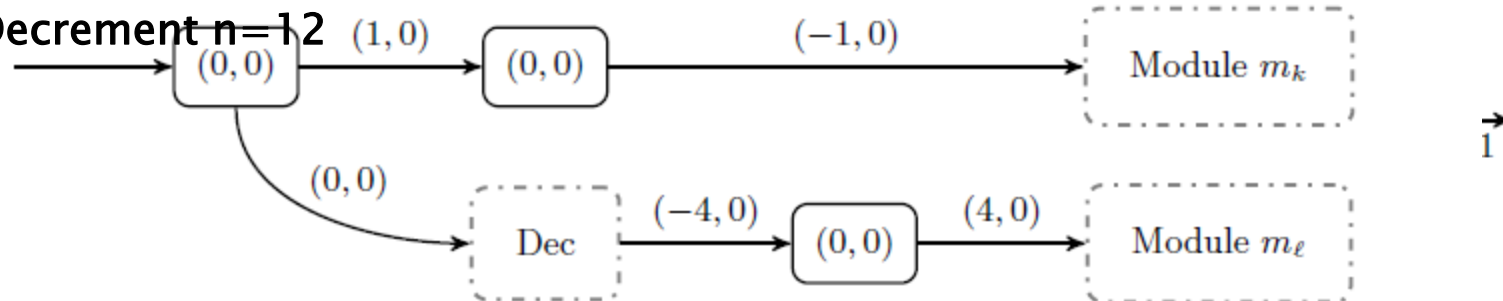
- LU Problem **Undecidable**
 - 2 clocks + 2 costs [1]
 - 1 clock + 2 costs [2]
 - 2 clocks + 1 cost [3]



Update/Decrement

Increment $n=3$

Decrement $n=12$



(1) Karin Quaas. On the interval-bound problem for weighted timed automata. 2011.

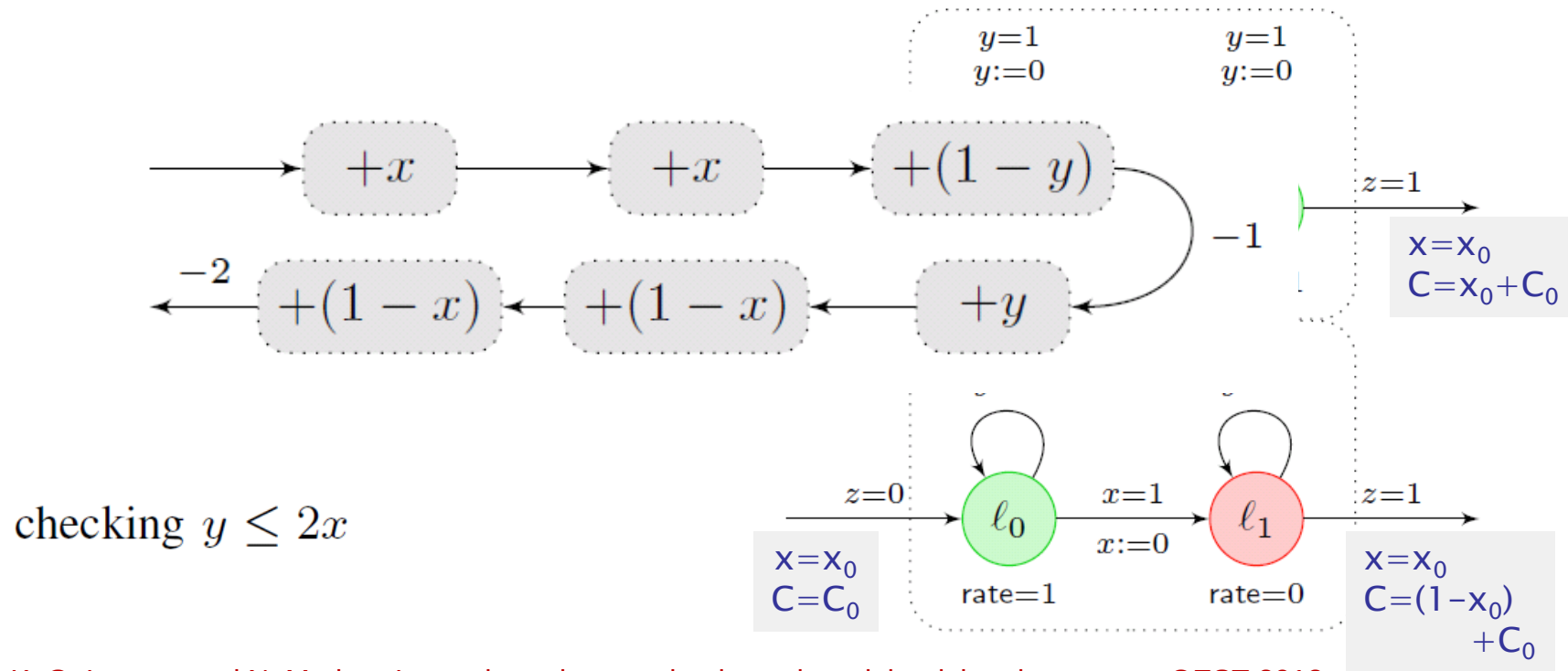
(2) Uli Fahrenberg, Line Juhl, Kim G. Larsen, and Jiri Srba. Energy games in multiweighted automata. 2011

(3) Nicolas Markey. Verification of Embedded Systems – Algorithms and Complexity. 2011.



Multiple Clocks & 1 Cost

- L problem is **undecidable**
 - 4 clocks or more

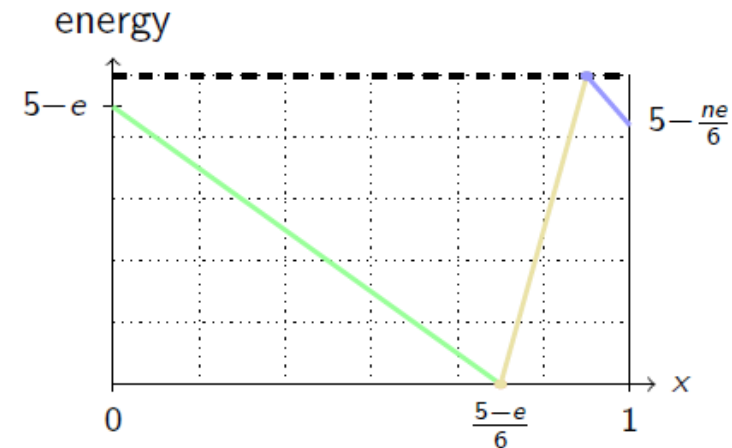


P. Bouyer, K. G. Larsen, and N. Markey. Lower-bound constrained runs in weighted timed automata. QEST 2012



Upper Bounds

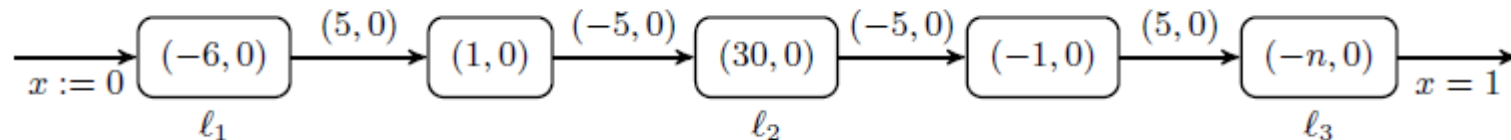
- LU Problem **Undecidable**
 - 2 clocks + 2 costs [1]
 - 1 clock + 2 costs [2]
 - 2 clocks + 1 cost [3]



Update c_1

Increment $n=3$

Decrement $n=12$



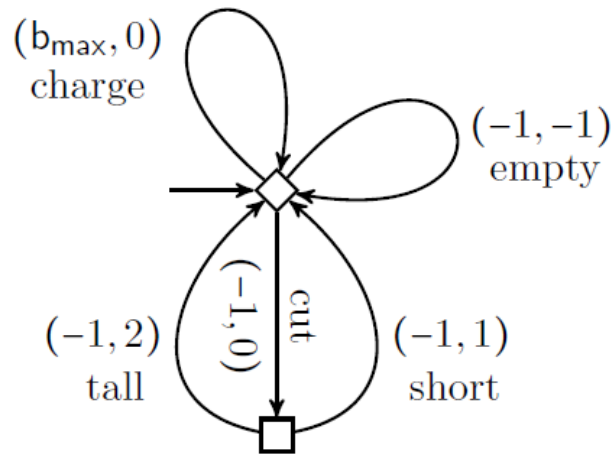
(1) Karin Quaas. On the interval-bound problem for weighted timed automata. 2011.

(2) Uli Fahrenberg, Line Juhl, Kim G. Larsen, and Jiri Srba. Energy games in multiweighted automata. 2011

(3) Nicolas Markey. Verification of Embedded Systems – Algorithms and Complexity. 2011.



Multiple Costs & 0 Clocks

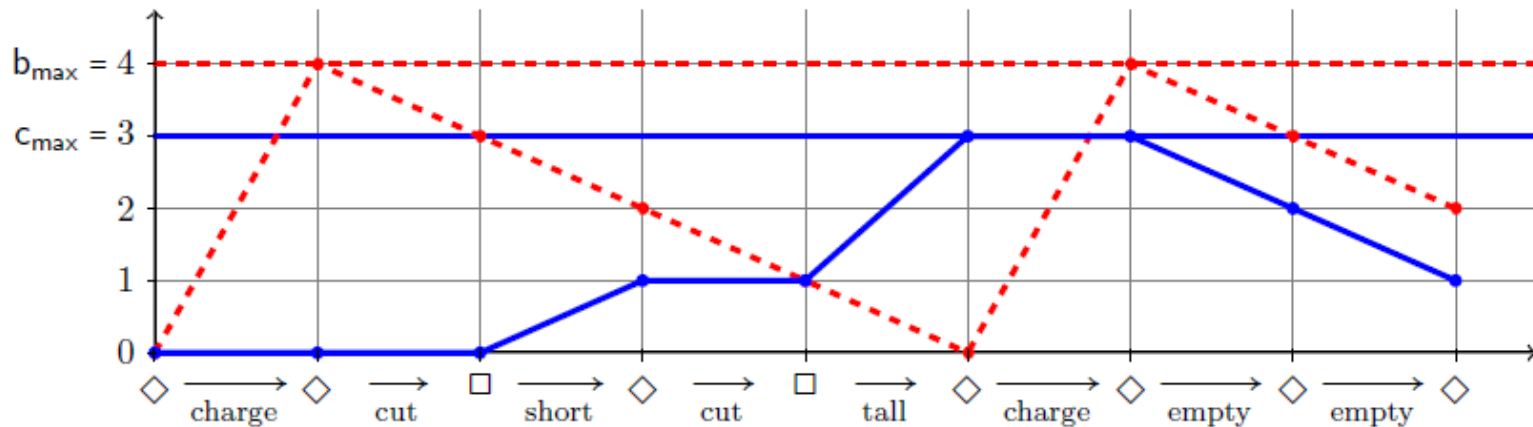


$b_{\max} = 4$ (battery capacity)
 $c_{\max} = 3$ (container capacity)

if *battery* = 0 then charge

else if *battery* ≥ 2 and *container* ≤ 1 then cut

else if *battery* ≥ 1 and *container* ≥ 1 then empty



Uli Fahrenberg, Line Juhl, Kim G. Larsen, and Jiri Srba. Energy games in multiweighted automata. 2011

Multiple Costs & 0 Clocks

# weights	Bound	Existential	Universal	Game
One	L	$\in P$ [4]	$\in P$ [4]	$\in UP \cap coUP$ [4]
	LW	$\in P$ [4]	$\in P$ [4]	$\in NP \cap coNP$ [4]
	LU	NP-hard [4], $\in PSPACE$ [4]	$\in P$ [4]	EXPTIME-complete [4]
Fixed ($k > 1$)	L	NP-hard, $\in k\text{-EXPTIME}$ [3] (Remark 17)	$\in P$ (Remark 18)	EXPTIME-hard, $\in k\text{-EXPTIME}$ [3] (Remark 19)
	LW	NP-hard, $\in PSPACE$ PSPACE-complete for $k \geq 4$ (Remark 20)	$\in P$ (Remark 18)	EXPTIME-complete (Remark 21)
	LU	PSPACE-complete (Remark 20)	$\in P$ (Remark 18)	EXPTIME-complete (Remark 21)
Arbitrary	L	EXPSpace-complete (Theorem 9)	$\in P$ (Remark 18)	EXPSpace-hard (from EL) decidable [3]
	LW	PSPACE-complete (Theorem 9)	$\in P$ (Remark 18)	EXPTIME-complete (Remark 21)
	LU	PSPACE-complete (Theorem 9)	$\in P$ (Remark 18)	EXPTIME-complete (Remark 21)

Uli Fahrenberg, Line Juhl, Kim G. Larsen, and Jiri Srba. Energy games in multiweighted automata. 2011



Conclusion

- Priced Timed Automata a uniform framework for modeling and solving dynamic resource allocation problems!
- Not mentioned here:
 - Model Checking Issues (ext. of CTL and LTL).
- Future work:
 - Zone-based algorithm for optimal infinite runs.
 - Approximate solutions for priced timed games to circumvent undecidability issues.
 - Open problems for Energy Automata.
 - Approximate algorithms for optimal reachability

