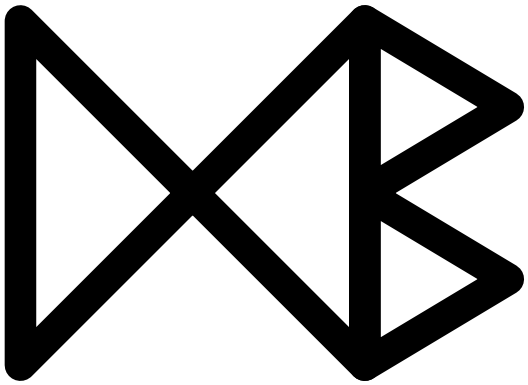


Skyline Ordering: A Flexible Framework for Efficient Resolution of Size Constraints on Skyline Queries

Hua Lu, Christian S. Jensen, Zhenjie Zhang

January 26, 2010

TR-27

A DB Technical Report

Title	Skyline Ordering: A Flexible Framework for Efficient Resolution of Size Constraints on Skyline Queries
Author(s)	Hua Lu, Christian S. Jensen, Zhenjie Zhang
Publication History	"January 2010. A DB Technical Report"

For additional information, see the DB TECH REPORTS homepage: ⟨dbtr.cs.aau.dk⟩.

The DB TECH REPORTS icon is made from two letters in an early version of the Rune alphabet, which was used by the Vikings, among others. Runes have angular shapes and lack horizontal lines because the primary storage medium was wood, although they may also be found on jewelry, tools, and weapons. Runes were perceived as having magic, hidden powers. The first letter in the logo is "Dagaz," the rune for day or daylight and the phonetic equivalent of "d." Its meanings include happiness, activity, and satisfaction. The second letter is "Berkano," which is associated with the birch tree. Its divinatory meanings include health, new beginnings, growth, plenty, and clearance. It is associated with Idun, goddess of Spring, and with fertility. It is the phonetic equivalent of "b."

Abstract

Given a set of multi-dimensional points, a skyline query returns the interesting points that are not dominated by other points. It has been observed that the actual cardinality (s) of a skyline query result may differ substantially from the desired result cardinality (k), which has prompted studies on how to reduce s for the case where $k < s$.

This paper goes further by addressing the general case where the relationship between k and s is not known beforehand. Due to their complexity, the existing pointwise ranking and set-wide maximization techniques are not well suited for this problem. Moreover, the former often incurs too many ties in its ranking, and the latter is inapplicable for $k > s$. Based on these observations, the paper proposes a new approach, called *skyline ordering*, that forms a skyline-based partitioning of a given data set, such that an order exists among the partitions. Then set-wide maximization techniques may be applied within each partition. Efficient algorithms are developed for skyline ordering and for resolving size constraints using the skyline order. The results of extensive experiments show that skyline ordering yields a flexible framework for the efficient and scalable resolution of arbitrary size constraints on skyline queries.

1 Introduction

Given a set of d-dimensional points, a skyline query [7] returns a subset of points that are *interesting* in that they are not dominated by other points. Point p_1 dominates p_2 if p_1 is better than p_2 in at least one dimension and no worse than p_2 in any other dimension. The skyline query is thus fundamental to multi-criteria decision making, where objects must be retrieved according to multiple criteria [7, 15, 21, 26, 28].

Consider the example shown in Figure 1. Each of the hotels has two attributes: room price and distance to the beach. For a tourist who prefers a low-cost hotel close to the beach, hotels A, B, C, and D are interesting—these are exactly the skyline of the hotels in the figure.

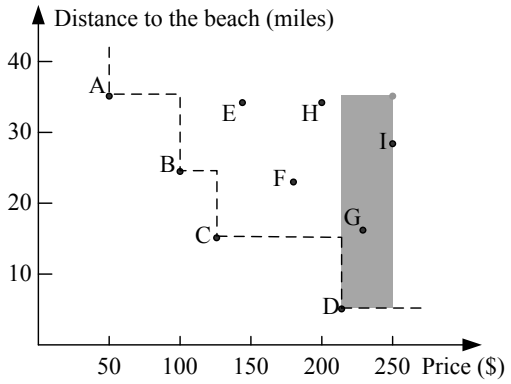


Figure 1: Skyline of Hotels

Although skyline queries are effective in identifying interesting points in multi-dimensional data sets, they also have a well-known weakness—their result cardinalities may vary. The result cardinality can be very large, especially when the dimensionality is high or the data is anti-correlated.

To contend with large skyline results, four categories of approaches have been proposed. In the first category, namely *pointwise ranking*, all d-dimensional points are totally ordered according to some specific scoring function or mapping function, and only the top points are returned [10, 26]. Such approaches face the difficulty of reflecting the multiple criteria in the scoring functions. In the second category, namely *subspace reference*, all subspaces of the full d-dimensional space are investigated, and those points preferred in subspaces are favored in the result [9, 10, 31]. Such approaches incur high computation costs in the traversal of all subspaces. In the third category of approaches, namely *set-wide maximization*, a subset of the skyline is deliberately selected such that a collective quantity based objective, e.g., the number of points

dominated by those in that subset, is maximized [25]. The problem has NP-hard computational complexity, so only approximate results can be computed efficiently. In the fourth category, namely *approximate selection*, points are compared approximately with a predefined threshold such that more points are identified as being dominated [20]. The approach in this category is not able to fully control the result cardinality.

It is meaningful for an approach to belong to different categories. For example, the subspace skyline frequency method [10] belongs to the first two categories. Section 2.2 offers a more detailed review of previous approaches.

The above approaches focus on reducing large skylines; however, small skylines can also be a problem. Consider again the classical example in Figure 1. The tourist now specifies a parameter k—the number of interesting hotels to be returned in the query result. A conventional skyline query usually, if not always, fails to return precisely k hotels. If k is 3, the skyline is too big. On the other hand, a large k value can offer practical convenience. Suppose a large number of hotel rooms are needed to accommodate all participants to a popular event to take place. To make sure to get enough rooms, the event organizer may specify k to be 5 or even larger, if all hotels in the skyline (namely A, B, C and D) together provide fewer vacant rooms. In such cases where skylines may be too small, the use of large k values is well motivated.

Another real-life example comes from online shopping. Many online shopping sites, such as eBay, allow users to specify multiple product dimensions (e.g., price, quality, guarantee, etc.) as criteria for good bargains. The resulting user requests are well captured by skyline queries against the background product database. However, if a user only specifies a few dimensions as being important, too few products will be returned as the skyline query result, which leaves the user with too little choice. In such cases, it makes practical sense to extend the small skyline to a larger size as expected by the user.

Given an arbitrary d-dimensional data set, its skyline cardinality s can be considerably smaller than a user-specified number k, and there is no good, straightforward way to increase s to k. We conclude that an approach is needed that supports skyline querying for arbitrary k.

This paper studies *size constrained skyline queries*, which takes an arbitrary size constraint k as a parameter and retrieves k desirable points from a d-dimensional data set. In contrast to traditional top-k queries, a size constrained skyline query must be insensitive to dimension scaling and shifting, which is an important property of a skyline query [7].

The conventional techniques for size constrained skyline queries, pointwise ranking and set-wide maximization, are not well suited for size constraints. Pointwise ranking approaches sometimes prefer non-skyline points to skyline points. Also, pointwise ranking approaches usually prefer points with similar features, since similar points are likely to get similar ranking scores, which limits the variety found in query results. With set-wide maximization it is possible to return results with larger variety, but set-wide maximization is not applicable to arbitrary query size constraints—cases where $k > s$ cannot be handled. Set-wide maximization is also computationally expensive due to its counting and set-wide optimization characteristics.

Motivated by these shortcomings, we propose a new approach, called *skyline ordering*, that supports arbitrary size constraint over the conventional skyline query. Skyline ordering introduces a skyline-based partitioning of a given data set, it provides an ordering among the partitions, and it reserves room for the use of various set-wide maximization techniques within partitions.

Skyline ordering aims to overcome the limitations of pointwise ranking and set-wide maximization by combining them into a uniform framework. In particular, given the partitions in skyline order, the following hold: (1) no point can dominate any other point in the same partition or in a previous partition; (2) any point in a partition, except in the first partition, must be dominated by some point(s) in the previous partition.

When answering a size constrained skyline query with skyline ordering, we start from the first partition and continue to output partitions until at least k points have been output; the last partition is pruned to guarantee that the total result size is exactly k. This way, arbitrary size constraints on skyline queries can be computed efficiently within a flexible framework built upon skyline ordering.

Our approach differs from skyband-based ranking[29]. The latter first retrieves from the database a K-skyband [27], a set of K points dominated by at most $K-1$ other ones, that has the smallest $K \geq k$. Then it returns the k best points from the K-skyband. In contrast, the partitioning in our approach neither counts nor minimizes the numbers of dominators. When selecting k points as the result, our approach may involve several consecutive partitions rather than a single K-skyband.

The paper makes the following contributions. First, its proposal supports arbitrary size constraints on skyline queries. Second, it includes a comprehensive review of previous approaches that modify the cardinality of a skyline, it analyzes their application to arbitrary skyline size constraints, and it proposes simple yet efficient heuristics for selecting representative skyline points. Third, it proposes the skyline ordering concept, together with algorithms for computing and maintaining skyline orders. Fourth, it defines size constrained skyline queries based on skyline order and develops various query processing algorithms. Fifth, the paper offers results of empirical evaluations of skyline order computation and size constrained skyline query processing.

The rest of this paper is organized as follows. Section 2 gives a general problem definition of size constrained skyline queries and briefly reviews related work. Section 3 discusses different approaches for tuning skyline cardinality. Section 4 covers the skyline order definition and computation. Section 5 applies skyline ordering to size constrained skyline queries. Section 6 presents the empirical studies. Section 7 concludes the paper.

2 Preliminaries

2.1 Problem Definition

Given a d-dimensional data set P with cardinality N, each data point p represents a choice of interest to a user. Without loss of generality, we assume that the user interests cover all d dimensions and that smaller values are preferred in each dimension. For two points p and q in P, we use $p \prec q$ ($p \succ q$) to represent that p dominates (is dominated by) q under the conventional skyline definition [7].

We may define a *size constrained skyline query* $Q_k^{scs}(P)$ informally as a subset S of P consisting of k points that are *good* in terms of user interest. In this informal definition, the query parameter k explicitly specifies the cardinality of the result to be returned. But what it means for a k-point subset of P to be *good*, and in what sense one subset of points is *better* than another, remain to be formalized. Put differently, we need criteria to determine which skyline points to discard or which non-skyline points to include when necessary. By applying such precise criteria to the result point selection, we are able to give formal definitions. These issues are covered in Sections 3 and 5.

Table 1 lists the notation used in this paper.

2.2 Related Work

2.2.1 Skyline Algorithms

Algorithms for computing conventional skyline queries can be divided into two categories. One category contains those algorithms that do not require any indexes on the data set. The theoretical algorithms for maximal vector computation [3, 5, 22] fall into this category. Borzonyi et al. [7] introduce the skyline query into database systems, defining *Block Nested Loop* (BNL) and *Divide-and-Conquer* (D&C) algorithms. Chomicki et al. [13] propose a variant of BNL called the *Sort-Filter-Skyline* (SFS) algorithm. Godfrey et al. [15] provide a comprehensive analysis of these non-index-based algorithms and propose a hybrid method with improvements. Bartolini et al. [4] propose a presorting based algorithm that is able to stop dominance

Table 1: Table of Notations

Notation	Description
P	Argument data set
N	Cardinality of P
d	Dimensionality of P
$\mathcal{R}^d$	d-dimensional space
S_P	Skyline of P
s	Size of S_P
$Q_k^{scs}(P)$	A k-size constrained skyline query on P
$\mathcal{S}_P$	Skyline order of P
S_i	i-th skyline order subset of $\mathcal{S}_P$
n	Skyline order length
$D_P^{\prec}(p)$	All points in P that are dominated by p
$D_P^{\prec}(S)$	All points in $P \setminus S$ dominated by point(s) in S

Table 2: Previous Approaches to Dominance Based Skyline Query Derivatives

Approach Nature	Approach Name	$k < s$	$k > s$	Remarks
pointwise ranking	Top-k dominating query [26]	$+$	$+$	
Subspace reference	*Skyline frequency* [10]	$+$	$+$	Also a pointwise ranking.
	Strong skyline point [31]	$-$	$\times$	
	K-dominant skyline [9]	$-$	$\times$	
Set-wide maximization	Top-k RSP [25]	$+$	$\times$	
Approximate selection	ε-ADR dominance [20]	$-$	$\times$	
	Thick skyline [19]	$\times$	$-$	

$+$: Result of exact k points; $-$: Result of uncontrolled size; $\times$: Inapplicable.

tests after accessing specific points. Zhang et al. [32] propose a dynamic indexing tree for skyline points that helps reduce the CPU costs in dominance tests in skyline computation.

The other category contains algorithms that require the presence of specific indexes. Tan et al. [28] propose two progressive algorithms: *Bitmap* and *Index*. The former represents points in bit vectors and employs bit-wise operations, while the latter utilizes data transformation and B$^+$-tree indexing. Kossmann et al. [21] propose a *Nearest Neighbor* (NN) method that identifies skyline points by recursively invoking R*-tree based depth-first NN search over different data portions. Papadias et al. [26] propose a *Branch-and-Bound Skyline* (BBS) method based on the best-first nearest neighbor algorithm [16].

2.2.2 Dominance Based Skyline Query Derivatives

Several approaches have been proposed as skyline query derivatives based on the dominance definition. All approaches known to the authors are listed in Table 2. For each approach, we consider its methodological nature, its applicability to the case where the number of points expected (k) is less than the skyline size (s) and the opposite case ($k > s$), and its relevant algorithm. For the nature of an approach, we consider four types: *pointwise ranking*, *subspace reference*, *set-wide maximization*, and *approximate selection*.

Papadias et al. [26] propose the top-k dominating query that retrieves points that dominate the largest number of points. Yiu and Mamoulis [30] have recently proposed efficient processing algorithms for this problem. The top-k dominating query belongs to the pointwise ranking category.

By utilizing the frequency of points' membership in subspace skylines, Chan et al. [10] formalize a top-k ranking problem that gives priority to points that appear more frequently in subspace skylines. This skyline frequency approach is able to return a fixed number of points for both cases. Zhang et al. [31]

propose the concept of strong skyline points, which frequently appear in small-sized subspace skylines in high-dimensional spaces. This method does not return a fixed number of points in the case of $k < s$, and it is not applicable to the case of $k > s$. Chan et al. [9] propose K-dominant[1] skyline for high dimensional space. The strict dominance covering all dimensions is relaxed to only K dimensions in any subspace. A K-dominant skyline does not return a fixed number of points in the case of $k < s$, and it is not applicable to the case of $k > s$. All the three approaches just covered use subspaces for result determination, while the skyline frequency approach can also be regarded as a total pointwise ranking.

Lin et al. [25] propose the k most representative skyline point problem (top-k RSP for short), which selects a portion of points from the traditional skyline that maximizes the total number of dominated points. This approach is an instance of set-wide maximization.

Koltun and Papadimitriou [20] introduce approximately dominating representatives, which produces a smaller, but not fixed-size, so-called ε-ADR skyline. The approximation lies in that before a point is compared with others, it is first enlarged by ϵ in all dimensions (if larger values are assumed to be preferred). This approach is not applicable to the case of $k > s$. Rather than evaluating single data points, Jin et al. [19] extend the conventional skyline, which they term a thin skyline, to a so-called thick skyline, by including points in the proximity of the conventional skyline points. This approach is only applicable to the case of $k > s$, and it cannot return a fixed number of points. Both of the above approaches perform approximate selections, as they employ approximate measures when evaluating point candidates.

Balke et al. [1, 2] propose to allow user interference or feedback to reduce skyline sizes. Lee et al. [24] propose to rank skyline points according to user-specific preference that prioritizes dimensions and requires search in preferred subspaces. In contrast, our skyline order approach ranks full-space skylines without any user-specific preference.

3 Skyline Size Tuning

3.1 Pointwise Ranking

To support size constraints on skyline queries, we need criteria for point candidates that select satisfactory ones and rule out undesirable ones. In this section, we discuss options to resolve the query size constraint by ranking all points with a dominance-based scoring function.

To enable pointwise ranking of all points in P, a mapping function $f : P \longrightarrow C$, is needed that maps each individual point p in P to a single value $f(p)$ in a totally ordered domain C. As a result, all points in P can be ranked in terms of their $f(p)$ values.

The pointwise ranking approach essentially converts the multiple criteria optimization that underlies a size constrained skyline query to a top-k ranking problem. It is capable of fully controlling the result size according to arbitrary k from 0 to $|P|$, as C is totally ordered. However, this approach needs a careful design of the mapping function f, which is expected to be dominance based, and invariant to the scaling and shifting with any constant factor of dimensions.

Next, we discuss two classes of mapping functions that can be used for supporting pointwise ranking.

3.1.1 Dominating Points Counting

One option is to take dominating capabilities into account when defining a mapping function f. A straightforward way is to count for a point p the number of points it dominates [26]. As a result, the mapping function is $f_{DN}(p) = |D_P^{\prec}(p)|$, where $D_P^{\prec}(p) = \{q | q \in P \wedge p \prec q\}$ is the set of points from P that are dominated by p. This definition is simple, but it sometimes prefers non-skyline points over skyline points.

[1] This K carries a different meaning than does k in our problem definition.

Referring to Figure 1, we have $f_{DN}(A) = 0$ because point A dominates nobody, $f_{DN}(F) = 2$ because F dominates both H and I. Following the same line of reasoning, we have $f_{DN}(B) = 3$, $f_{DN}(C) = 5$, and $f_{DN}(D) = 2$. If a size constrained skyline query based on f_{DN} asks for $k = 4$ points, points C, B, D and F will be returned but the skyline point A will be suppressed. However, since F is apparently dominated by C, F is unlikely interesting to the user. In this sense, the result actually offers fewer options compared to the result that includes A instead.

3.1.2 Subspace Skyline Frequency

Another way to rank all points in P is to consider for each point p its frequency of appearance in the subspace skylines. The mapping function is defined as follows.

$$f_{SSF}(p) = \left| \mathcal{R}' \in 2^{\mathcal{R}^d} \mid p^{\mathcal{R}'} \in S_{P^{\mathcal{R}'}} \right| \tag{1}$$

Here $p^{\mathcal{R}'}$ is the projection of the point p on the subspace $\mathcal{R}'$, and function $P^{\mathcal{R}'}$ is the projection of all points in P onto $\mathcal{R}'$. Intuitively, $f_{SSF}(p)$ counts the number of subspaces on which p locates on the skyline of the data set P.

However, this method requires a traversal of all subspaces of $\mathcal{R}^d$, computing the skyline of P's projection on each subspace, and counting subspace frequency for each point in P. Therefore its computation calls for approximate algorithms [10]. The time complexity of the approximate counting algorithm in [10] is upper bounded by $\mathcal{O}(2d^2 \ln(2/\delta)/\epsilon^2)$, when the approximate result is within the error of ϵ with a confidence level $\geq 1 - \delta$.

3.2 Set-Wide Maximization

Set-wide maximization is also an alternative for evaluating size constrained skyline queries. Instead of considering each single point separately, this method considers a set of points collectively to maximize a target value.

3.2.1 Count Based Maximization

The top-k representative skyline points problem [25] is limited to the case where k is smaller than the skyline size $|S_P|$. This definition can be extended by modifying the outcome for the case where $k \geq |S_P|$. A direct extension is given as follows.

Definition 3.1 (**Extended Top-k Representative Skyline Points Query**) *The* extended top-k representative skyline points query $Q_k^{ersp}(P)$ *retrieves a subset S of k points from a d-dimensional point set P that maximizes $|D_P^{\prec}(S)|$, where $D_P^{\prec}(S)$ is the set of points from $P \setminus S$ dominated by some point in S.*

For the extended definition above, we have the following lemma that explicitly states the relationship between the usual skyline and the result of query $Q_k^{ersp}(P)$.

Lemma 3.1 *Given the skyline S_P of a d-dimensional point set P, and the result S_{ersp} of an extended top-k representative skyline points query $Q_k^{ersp}(P)$, the following properties hold:*

1. $S_P \subset S_{ersp}$, *if* $k > |S_P|$;
2. $S_{ersp} \subset S_P$, *if* $k < |S_P|$.

Proof. 1. As $|S_{ersp}| = k > |S_P|$, there must exist at least one non-skyline point p s.t. $p \in S_{ersp}$ but $p \notin S_P$. Suppose there exists a skyline point s s.t. $s \in S_P$ but $s \notin S_{ersp}$. If s dominates p, $(S_{ersp} \setminus \{p\}) \cup \{s\}$ is a better result than S_{ersp}, in terms of the number of points dominated. If s does not dominate p, there must exist another skyline point s' that dominates p. If $s' \notin S_{ersp}$, $(S_{ersp} \setminus \{p\}) \cup \{s'\}$ is a better result than S_{ersp}; otherwise, $(S_{ersp} \setminus \{p\}) \cup \{s\}$ is better since $|D^{\prec}_P((S_{ersp} \setminus \{p\}) \cup \{s\})| \geq |D^{\prec}_P(S_{ersp})| + 1 > |D^{\prec}_P(S_{ersp})|$. Each case leads to a contradiction, which proves that $S_P \subset S_{ersp}$ if $k > |S_P|$.
2. Suppose there is a $p \in S_{ersp}$ but $p \notin S_P$. There must be $s \in S_P$ that dominates p. If $s \notin S_{ersp}$, $(S_{ersp} \setminus \{p\}) \cup \{s\}$ is a better result than S_{ersp}, since $|D^{\prec}_P((S_{ersp} \setminus \{p\}) \cup \{s\})| \geq |D^{\prec}_P(S_{ersp})| + 1 > |D^{\prec}_P(S_{ersp})|$. If $s \in S_{ersp}$, for any skyline point $s' \notin S_{ersp}$ (such a s' must exist as $|S_{ersp}| < |S_P|$), $(S_{ersp} \setminus \{p\}) \cup \{s'\}$ is a better result than S_{ersp} since $|D^{\prec}_P((S_{ersp} \setminus \{p\}) \cup \{s'\})| \geq |D^{\prec}_P(S_{ersp})| + 1 > |D^{\prec}_P(S_{ersp})|$. Both cases lead to a contradiction, which proves that $S_{ersp} \subset S_P$ if $k < |S_P|$. ■

Lemma 3.1 gives indications on how to answer an extended top-k representative skyline points query: its result is contained in the skyline if less points are expected, or it contains the skyline if more points are expected. However, the lemma also indicates that the direct extension given in Definition 3.1 is problematic when k is larger than the original skyline size s. For that case, $D^{\prec}_P(S)$ actually can never exceed $D^{\prec}_P(S_P)$ because every point in $P \setminus S_{ersp}$ must be dominated by some skyline point in S_P. It is shown by Lemma 3.1 that S_P is always contained in the query result S_{ersp}, but it makes no difference which non-skyline points are included into S_{ersp}, as none of them increases $|D^{\prec}_P(S_{ersp})|$. Thus, it is attractive to modify Definition 3.1 so that it becomes more meaningful for an arbitrary k. This will be discussed in Section 4.

3.2.2 Estimation Based Maximization

It is expensive to count exactly how many points are dominated by a point or a set of points [25]. In this section, we propose another result criterion based on dominating capability estimation. Such estimations take into account the dominating region [17] of each point and can be used for queries with $k < s$.

The dominating region of a point p is a hyper-rectangle, whose main diagonal is the line segment from p to the maximum corner of P. As any point in this dominating region is definitely dominated by p, its volume, termed as $VDR(p)$, is used to estimate p's capability of dominating other points.

To illustrate, consider hotel D in Figure 1. Its dominating region is a rectangle as there are only two dimensions. For all hotels, \$250 is the maximum price and 35 miles is the longest distance to the beach. These two maximum values together correspond to the (virtual) maximum corner that together with D defines the main diagonal of D's dominating region. Hotels G and I are within that region and are thus dominated by hotel D.

For a set of points, we consider their overall dominating capability which is indicated by the union of all their dominating regions. By applying the Inclusion-Exclusion principle, we can compute the volume of such a union of all points in a set $S = \{s_1, s_2, \ldots, s_k\}$, with U_l denoting the maximum value on dimension l, as follows.

$$VDR(S) = \sum_{i=1}^{k} VDR(s_i) + \sum_{j=2}^{k}((-1)^{j-1} \sum_{1 \leq i_1 < \ldots < i_j \leq k} \prod_{l=1}^{d}(U_l - max(s_{i_1}.a_l, \ldots, s_{i_j}.a_l))) \quad (2)$$

Formula 2 only refers to k points in the set S, without any other global information like the number of dominated points and the subspace skyline frequency. However, an optimization based on Formula 2, i.e., selection of a subset S from P with maximum $VDR(S)$, is still not cheap when k is very large, whose time complexity is $\mathcal{O}((\frac{2e|P|}{k})^k)$ [14]. Specific heuristics can be used to approximately compute the $VDR(S)$.

One straightforward option is to simply sum up all VDR values of k candidate points, and select as S the combination with the maximum sum. Another option greedily maximizes the sum of point distances, as intuitively the farther points are apart, the less their dominated regions overlap. First two points with the

largest distance are selected into S. Then in each step to increase S, a new point is added such that the sum of its distances to all points currently in S is maximized. As the straightforward method usually obtains approximations with better precision [14], it is used in the experimental studies.

3.3 Discussion

The existing approaches exhibit limitations when attempting to use them for resolving arbitrary size constraints on skyline queries.

The use of pointwise ranking approaches causes three difficulties. First, a total ranking tends to deconstruct the essence of multi-criteria optimization when it converts a complex problem into a simple one. As pointed out in Section 3.1.1, if we count dominating points only, non-skyline points may be preferred over skyline points. This also happens in subspace skyline frequency based ranking, where a non-skyline point can have an advantage in more subspaces than has a skyline point.

Second, a total ranking can incur too many ties, which invalidates the ranking. For example in Figure 1, we cannot differentiate D and F (or E and G) by counting dominating points. For an arbitrary data set, such ties are very likely to exist. Ties also occur in the subspace skyline frequency based ranking, especially when the dimensionality is not large enough.

Third, a total ranking is a time-consuming task especially when the data set is large. Therefore, complex index structures [30] or approximate algorithms [10] are employed.

Next, the count based set-wide maximization approach is not applicable to arbitrary size constraints on skyline queries because it is unable to handle the case where more points than the skyline cardinality are expected. Moreover, the count based set-wide maximization approach is computationally very expensive due to its counting and set-wide optimization characteristics.

It makes sense to apply pointwise ranking approaches and set-wide maximization approaches in combination. The former is focused on judging whether any individual point is good enough to be included in the result. The latter is concerned with maximizing the collective advantage of a number of result points. The two approaches thus tackle the same problem from opposite directions: bottom-up and top-down, respectively. An interesting question thus arises: Can we find some novel way in-between the two to overcome the aforementioned obstacles in supporting arbitrary size constrained skyline queries? Our answer is positive. We will elaborate our skyline order based framework in the next two sections.

We proceed to present the concept *skyline order* which ranks points partially and in batch rather than fully, one by one.

4 Skyline Order

4.1 Definitions and Properties

Definition 4.1 (**Skyline Order**) *The* skyline order *of a set of d-dimensional points P is a sequence $\mathcal{S} = \langle S_1, S_2, \ldots, S_n \rangle$ defined as follows:*

1. *S_1 is the skyline of P, i.e., S_P;*
2. *$\forall i, 1 < i \leq n$, S_i is the skyline of $P \setminus \bigcup_{j=1}^{i-1} S_j$.*
3. *$\bigcup_{i=1}^{n} S_i = P$*

We call each S_i a *skyline subset* in the skyline order or a *skyline order subset*, and we call n the *skyline order length*, i.e., the number of subsets in $\mathcal{S}$. The definitions iteratively removes the current skyline starting

with P until all points in P belong to a skyline subset, resulting in n skyline subsets are obtained. By construction, the points in S_i are incomparable and $S_i \cap S_j = \varnothing$ for $i \neq j$.

For convenience, we define *skyline order index* for any point p in P as follows.

Definition 4.2 (**Skyline Order Index**) *Given the skyline order $S_1, S_2, \ldots, S_n$ of a d-dimensional point set P, the* skyline order index *of a point $p \in P$ is i if $p \in S_i$.*

The skyline order has some interesting properties as described in the following lemma. We omit its proof here as the reasoning is quite direct from the definition.

Lemma 4.1 *Given the skyline order $S_1, S_2, \ldots, S_n$ of a d-dimensional point set P, the following properties hold:*

1. $\forall i > 1\ \forall p_1 \in S_i\ \exists p_2 \in S_{i-1}\ (p_2 \prec p_1)$.
2. $\forall i > 1\ \forall p_1 \in S_i\ \forall j < i\ \exists p_2 \in S_j\ (p_2 \prec p_1)$.
3. $\forall p_1 \in S_i\ \forall j > i\ \nexists p_2 \in S_j\ (p_2 \prec p_1)$.

An example of skyline order is shown in Figure 2. Here, we have S_1 = {A, B, C, D}, which is also the skyline of all hotels. Furthermore, we have S_2 = {E, F, G}, the skyline of subset {E, F, G, H, I}, and S_3 = {H, I}. Further, the skyline order index is 1 for any point in {A, B, C, D}, 2 for any point in {E, F, G}, and 3 for any point in {H, I}, as indicated by the corresponding numeric subscripts in the figure.

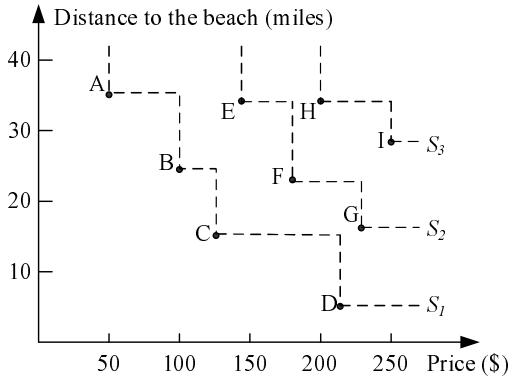


Figure 2: Skyline Order

Skyline order has practical applications. Referring to Figure 2, assume that a large number of attendees are to be accommodated by a conference. A skyline query will return the best hotels A, B, C, and D. They together, however, may provide fewer vacancies than needed by the attendees. To provide enough rooms, hotels E, F and G can be easily included in the result. The skyline order is thus a systematic approach to provide results with enough vacancies.

A notion of ranking through iterated preference is proposed in [12], which is focused on preference modeling without specific skyline order definitions or algorithms. In-place algorithms for computing layers of maxima in [6] are focused on 2-dimensional data sets only. In contrast, this paper's setting poses no in-place requirement, and our algorithms in Section 4.2 are not limited to two dimensions. The idea of layered skyline has also been used in the literature as part of an indexing method for top-k queries [18]. In this paper, we focus on computing the skyline layers efficiently and on applying them in size-constrained skyline queries, which has not been studied in the literature.

Our skyline order also differs from the Onion technique [11]. First, we partition a given data set based on the skyline concept, whereas the Onion technique is based on the convex hull definition. Second, the

skyline order supports size constrained skyline queries well, as we will see in the rest of this paper, whereas the onion technique is focused on indexing for linear optimization queries. Third, the skyline order is able to support *local queries*, in our setting, skyline queries with size constraints, without any specific hierarchical structures as needed by the Onion technique.

4.2 Skyline Order Computation

In this section, we introduce algorithms requiring no index to compute skyline order. In Section 6.2.3, we will adapt index based skyline algorithm for skyline order.

4.2.1 Basic Algorithm

We adapt the BNL algorithm [7] to compute the skyline order. The pseudo code is described in Algorithm 1. It loops on each point p in the input data set P: It checks p against each subset in the current skyline order and put it into a specific subset or creates a new subset for it. Instead of keeping one list of skyline candidates only, as does BNL, the algorithm maintains all subsets of the current skyline order. While determining the membership for p, some points in the skyline subset being checked currently may also change their membership, thus producing a new subset in the skyline order.

Algorithm 1 SkylineOrderScan (data set P)

```
1:  S = ⟨⟩
2:  for each point p in P do
3:    for each S_i from S_1 to S_n in S do
4:      isSky=TRUE; S_tmp = ∅
5:      for each point q in S_i do
6:        if q ≺ p then
7:          isSky=FALSE; break
8:        else
9:          if p ≺ q then
10:           move q from S_i to S_tmp
11:     if isSky then
12:       if S_i = ∅ then
13:         S_i = {p}; insert S_tmp into S after S_i
14:       else
15:         if S_tmp ≠ ∅ then
16:           AdjustSkyOrd(S_tmp, S, i+1)
17:         add p to S_i
18:       break
19:   if p does not belong to any S_i then
20:     append {p} to S
21: return S
```

Specifically, the skyline order $\mathcal{S}$ is initialized to be empty (line 1). For each point p in P, it is compared to each point q in each S_i sequentially from S_1 to the current S_n (lines 2–16). If p is dominated by q, the algorithm breaks from the loop on the current S_i and skips to the next subset in the skyline order (lines 6–7). If p dominates q, q will be moved into a temporary list S_{tmp}, which contains all those points that come from the current S_i and are dominated by p (lines 9–10). If p is found to be a qualified skyline point for the current S_i (line 11), necessary actions are to be taken to ensure the skyline order is updated correctly, followed by breaking from the loop on $\mathcal{S}$ (line 18). If S_i is empty, which indicates that p dominates all

points previously in S_i, $\{p\}$ will be used as the new S_i and S_{tmp} will be inserted into $\mathcal{S}$ after the new S_i (lines 12–13). Otherwise, the procedure AdjustSkyOrd will be called to adjust the subsets after S_i in the skyline order (lines 15–16) in case S_{tmp} is not empty, and p will be added into S_i (line 17). If p does not belong to any S_i in $\mathcal{S}$, $\{p\}$ will be appended to $\mathcal{S}$ (lines 19–20).

The pseudo code of AdjustSkyOrd is given in Algorithm 2. It requires three parameters: the temporary list S_{tmp} containing points excluded from the current skyline subset in the scanning, the current Skyline order $\mathcal{S}$, and the next subset index in the scanning. It loops on each subset S_i in $\mathcal{S}$ starting from S_{next}. Each point p in S_i is moved to a temporary list $temp$ if it is not dominated by any point in S_{tmp} (lines 3–5). If S_i has not been changed, indicated by an empty temporary list $temp$, S_{tmp} is added to $\mathcal{S}$ immediately before S_i, and the algorithm returns (lines 6–7). A non-empty $temp$ is merged into S_{tmp} (line 8). If S_i becomes empty after all its points have been checked, S_{tmp} replaces it in $\mathcal{S}$, and the algorithm returns (lines 9–10). Otherwise, S_i and S_{tmp} are swapped, and the loop continues to the next subset in $\mathcal{S}$ (line 11). If S_{tmp} is not empty when the loop is over, it is appended to the skyline order (lines 12–13).

Algorithm 2 AdjustSkyOrd(Temporary list S_{tmp}, Current Skyline order $\mathcal{S}$, Next subset index $next$)

```
1:  for each subset S_i from S_next to S_n in S do
2:      temp = ∅
3:      for each point p in S_i do
4:          if ∄q ∈ S_tmp s.t. q ≺ p then
5:              move p from S_i to temp
6:      if temp == ∅ then
7:          add S_tmp to S immediately before S_i; return
8:      merge temp to S_tmp
9:      if S_i == ∅ then
10:         replace S_i in S with S_tmp; return
11:     swap S_i and S_tmp
12: if S_tmp ≠ ∅ then
13:     append S_tmp to S
```

Taking Figure 2 as an example, let the processing order of all hotels be I, E, F, A, D, H, G, C, B. Table 3 lists the resulting skyline order after each hotel is processed. The * symbol indicates that the current p triggers a call of the AdjustSkyOrd algorithm.

When E is processed, it is merged with I to form the only skyline order subset because they are incomparable to each other. As F dominates I, this causes $S_{tmp} = \{\mathrm{I}\}$ that becomes the new tail in the skyline order after the AdjustSkyOrd algorithm is called. When G is processed, I is again moved out to the new tail.

Finally, when C is processed, the situation is more complex. The $S_{tmp} = \{\mathrm{E}, \mathrm{F}\}$ is the input to the AdjustSkyOrd algorithm, in which S_{tmp} is compared with the skyline order subsets $S_2 = \{\mathrm{G}, \mathrm{H}\}$ and $S_3 = \{\mathrm{I}\}$ in turn. In the first iteration of the for-loop, G is merged with $\{\mathrm{E}, \mathrm{F}\}$, as G is dominated by neither E nor F. Then $S_{tmp} = \{\mathrm{E}, \mathrm{F}, \mathrm{G}\}$ is swapped with the reduced $S_2 = \{\mathrm{H}\}$. In the second iteration, $S_{tmp} = \{\mathrm{H}\}$ is expanded to become $\{\mathrm{H}, \mathrm{I}\}$ and replaces $S_3 = \{\}$ as the new S_3.

4.2.2 Improvements

The basic algorithm calls the procedure AdjustSkyOrd when a new point p is found to dominate some old points that form the temporary list S_{tmp} passed to AdjustSkyOrd. We can eliminate the cost of this by pre-sorting the data set P. As we assume that smaller values are preferred in skyline computation, we pre-sort P in the non-descending order of the sum of a point's dimension values. As a result, a point p is not able to dominate any point q that has been processed before it [13]. The relevant improved pseudo code is described

Table 3: Example Steps of Algorithm 1

Current p	**Resulting Skyline Order**
I	$\langle\{\mathrm{I}\}\rangle$
E	$\langle\{\mathrm{E},\mathrm{I}\}\rangle$
F*	$\langle\{\mathrm{E},\mathrm{F}\},\{\mathrm{I}\}\rangle$
A	$\langle\{\mathrm{A},\mathrm{E},\mathrm{F}\},\{\mathrm{I}\}\rangle$
D	$\langle\{\mathrm{A},\mathrm{D},\mathrm{E},\mathrm{F}\},\{\mathrm{I}\}\rangle$
H	$\langle\{\mathrm{A},\mathrm{D},\mathrm{E},\mathrm{F}\},\{\mathrm{H},\mathrm{I}\}\rangle$
G*	$\langle\{\mathrm{A},\mathrm{D},\mathrm{E},\mathrm{F}\},\{\mathrm{G},\mathrm{H}\},\{\mathrm{I}\}\rangle$
C*	$\langle\{\mathrm{A},\mathrm{C},\mathrm{D}\},\{\mathrm{E},\mathrm{F},\mathrm{G}\},\{\mathrm{H},\mathrm{I}\}\rangle$
B	$\langle\{\mathrm{A},\mathrm{B},\mathrm{C},\mathrm{D}\},\{\mathrm{E},\mathrm{F},\mathrm{G}\},\{\mathrm{H},\mathrm{I}\}\rangle$

in Algorithm 3. Here, we have no list S_{tmp}, and neither do we call the procedure AdjustSkyOrd.

Algorithm 3 SkylineOrderPreSort (sorted data set P)

```
1:  S = ⟨⟩
2:  for each point p in P do
3:    for each S_i from S_1 to S_n in S do
4:      isSky=TRUE
5:      for each point q in S_i do
6:        if q ≺ p then
7:          isSky=FALSE; break
8:      if isSky then
9:        add p to S_i; break
10:   if p does not belong to any S_i then
11:     append {p} to S
12: return S
```

A further improvement is to carry out a binary search for the loop on all existing subsets in the skyline order, instead of the sequential scan in line 3 in Algorithm 3. Essentially, the loop on all skyline order subsets finds the subset S_x that satisfies: (a) all points in S_x are incomparable to p; (b) x is the minimum index among all such subsets satisfying (a). A binary search starts with $low = 1$ and $high = n$, the skyline order length. At each loop step, we check the dominance relationship between the current point p and the subset S_{mid} where $mid = \lfloor low + high \rfloor/2$.

Due to P being sorted, only two possibilities exist: p is dominated by some point(s) in S_{mid} or p is incomparable to all points in S_{mid}. For the former, we set $low = mid + 1$ and continue the binary search. For the latter, we look one subset backwards, checking the relationship between p and the subset S_{mid-1}. If p is dominated by some point(s) in S_{mid-1}, S_{mid} is exactly what we need and the binary search ends. Otherwise, i.e., points in S_{mid-1} are also incomparable to p, we set $high = mid-1$ and continue the binary search. When the binary search terminates, either the expected subset S_x is found and p is inserted into S_x, or the tail of the skyline order is reached and p will constitute a new singleton tail. Note that p cannot be a new singleton head as it cannot dominate any point before it when P is sorted as described above.

We note that the backward check above is necessary to ensure the correctness of the binary search based algorithm. The binary search here is different from an exact match binary search, which returns as soon as the expected value is found. The incomparable relationship here is not equivalent to an exact match. Rather, we need to find the minimum (instead of any) index x such that any point in the subset S_x is incomparable to p.

Referring to Figure 2, let the current skyline order be $\langle\{\mathrm{C}\},\{\mathrm{F}\},\{\mathrm{I}\}\rangle$ and B be the p being processed. For simplicity we ignore the other points. Note that B should be inserted into $\{\mathrm{C}\}$ to maintain a correct

skyline order, as $\{\mathrm{C}\}$ is the first skyline order subset and C is comparable with B. However, if we do not employ a backward check, B will be inserted into $\{\mathrm{F}\}$ instead, since we have $low = 1$ and $high = 3$, which yields $mid = 2$.

The binary search idea can also be used in the basic algorithm in Section 4.2.1, but the improvement will not be significant. The binary search method does not lower the possibility that a new point dominates an old one when it is integrated into the basic algorithm on a unsorted data set. This means that the frequent and time-consuming call of AdjustSkyOrd is not avoided at all. Therefore, we do not apply the binary search to the basic algorithm.

4.2.3 Analyses

We briefly analyze the proposed skyline order computation algorithms, by regarding the dominance comparisons between two points as the characteristic operation. For the sake of presentation simplicity, we do not count the presorting time cost when analyzing the SkyOrdPreSort algorithm and its variant with binary search.

For the SkylineOrderScan algorithm, the best case is that each point p from P forms a new first skyline order subset in $\mathcal{S}$, without invoking the procedure AdjustSkyOrd. In this case, only one dominance comparison is needed, involving p and the single point in the current S_1 in $\mathcal{S}$. As a result, the total number of dominance comparisons, processing the whole data set P, is $N - 1$.

The worst case, however, involves cascading adjustments caused by a call of AdjustSkyOrd. Suppose we are processing the j'th point p from P, and the cardinalities of S_{tmp} and S_{i+1} to S_n are $c_0, c_{i+1}, \ldots, c_n$, respectively, before AdjustSkyOrd is called in line 16 of Algorithm 1. Note that $c_0 + \sum_{k=i+1}^{n} c_k \leq j - 2$ as we are processing the j'th point p from P and S_i cannot be empty if AdjustSkyOrd is called (lines 12–16 in Algorithm 1).

A total cascading adjustment chain in AdjustSkyOrd (Algorithm 2) maintains all but one point in every current skyline subset S_i. This is because if $temp$ is empty, there will be no more cascading adjustment (lines 6–7 in Algorithm 2). Those points are swapped into S_{tmp} (line 11 in Algorithm 2) and then compared with all points in S_{i+1}. This happens until S_n is reached. Considering that all points in the input S_{tmp} are compared with the initial S_{next}, the total number of dominance comparisons is $c_0 \cdot c_{i+1} + (c_{i+1} - 1) \cdot c_{i+2} + \ldots + (c_{n-1} - 1) \cdot c_n = c_0 \cdot c_{i+1} + \sum_{k=i+1}^{n-1} c_k \cdot c_{k+1} - \sum_{k=i+2}^{n} c_k \leq (c_0 + \sum_{k=i+1}^{n} c_k)^2/2 = (j-2)^2/2$. In the worst case, processing each j'th ($j \geq 2$) point p from P incurs this cost. As a result, the number of dominance comparisons contributed by AdjustSkyOrd is bounded by $\sum_{j=2}^{N}(j-2)^2/2 = (\sum_{k=1}^{N-2} k^2)/2 = (N-2) \cdot (N-1) \cdot (2N-3)/12$, yielding a worst case complexity of $\mathcal{O}(N^3)$. This is also the upper bound of the worst case cost of the SkylineOrderScan algorithm, as it cannot incur more dominance comparisons without calling AdjustSkyOrd.

For the SkyOrdPreSort algorithm without binary search, the best case is the same as that of SkylineOrderScan, i.e., $N - 1$ dominance comparisons. Its worst case happens if each point p from P is compared with every point in every skyline subset from the first to the last in $\mathcal{S}$. The worst case leads to a total of $N(N-1)/2$ dominance comparisons.

For the SkyOrdPreSort variant with binary search, the best case is that every point from P forms a singleton skyline order subset, and all points already in $\mathcal{S}$ are compared with the newly encountered i'th point p from P in a binary search fashion. This requires a total of $\sum_{i=1}^{N} \log_2 i = \log_2(N!) = \mathcal{O}(N \log N)$ dominance comparisons. The worst case is that all points from P form only one skyline order subset, which causes each point p from P to be compared with all points encountered before it. As a result, the total number of dominance comparisons in the worst case is $N(N-1)/2$.

A summarization of cost analysis is given in Table 4.

Table 4: Summary on Algorithm Complexity

Algorithm	Best Case	Worst Case
SkylineOrderScan	$\mathcal{O}(N)$	$\mathcal{O}(N^3)$
AdjustSkyOrd	$\mathcal{O}(N)$	$\mathcal{O}(N^2)$
SkylineOrderPreSort	$\mathcal{O}(N \log N)$	$\mathcal{O}(N^2)$

4.3 Maintenance of Skyline Order Upon Updates

Updates on a point set P can consequently change its computed skyline order $\mathcal{S}_P$ $(S_1, S_2, \ldots, S_n)$. In this section, we discuss how to maintain the skyline order upon such updates in an incremental way, instead of re-computing the whole skyline order totally from scratch.

When a new point p is inserted to P, a loop starts from S_1 to find the subset p belongs to. It works in the same way as the counterpart in Algorithm 1 (lines 3–16). When a point p is deleted from P, the subset S_l to which p belongs is found by a sequential scan on $\mathcal{S}_P$. Then, Algorithm 2 will be called with $S_l \setminus p$ being the temporary list and $l + 1$ being the next subset index. This is because the deletion of p can only affect those subsets after S_l, according to the skyline order properties. If S_l happens to be the last one in $\mathcal{S}_P$, nothing will be done further.

When a point p in P is modified, a simple way is to treat it as a deletion followed by insertion, taking corresponding actions presented above. There may exist complex but more efficient ways to deal with this problem. One option is to compare p and its updated counterpart p'. If p dominates p', p' will definitely not belong to any subset before S_l to which p belongs, according to the skyline order properties. As a result, the insertion can be simplified without checking those subsets. As the point modification problem itself is not directly relevant to our targeted size constrained skyline queries, we will not discuss such alternatives in this paper but leave them for possible future work.

5 Processing Size Constrained Skyline Queries with Skyline Order

We next show how skyline order can be used to process size constrained skyline queries. We first give a query definition, then present several algorithms that compute the query.

5.1 Definition

Based on skyline order, we are able to define a new size constrained skyline query that combines pointwise ranking and set-wide maximization to return interesting points according to both approaches. The definition is given as follows.

Definition 5.1 (**Skyline Order Based Size Constrained Skyline Queries**) *Let a set P of d-dimensional points with skyline order $\mathcal{S} = \langle S_1, S_2, \ldots, S_n \rangle$ be given. The* skyline order based size constrained skyline query $Q_k^{soscs}(P)$ *retrieves the set S_{soscs} defined as follows:*

$$S_{soscs} = (\bigcup_{i=1}^{l} S_i) \cup S'_{l+1}$$

$$\textit{where } l \textit{ is defined such that } \sum_{i=1}^{l} |S_i| \leq k < \sum_{i=1}^{l+1} |S_i|,$$

$$\textit{and } S'_{l+1} \subset S_{l+1} \textit{ such that } |S_{soscs}| = k.$$

Here, S'_{l+1} is selected from S_{l+1} using some set-wide maximization approach presented in Section 3.2. The query returns consecutive skyline order subsets from the original skyline until k points have been returned.

5.2 Algorithms

We present two algorithms that process a size constrained skyline query using the concept of skyline order. One takes advantage of a pre-computed skyline order, whereas the other processes a given query without a pre-computed skyline order.

5.2.1 Algorithm With Pre-Computed Skyline Order

If the skyline order $\mathcal{S}$ of the data set P has been computed before a size constrained skyline query $Q_k^{scs}(P)$ is issued, $\mathcal{S}$ can be used to facilitate the query processing. Following Definition 5.1, our algorithm uses a simple loop on all pre-computed skyline order subsets while maintaining a count of the points seen. The relevant pseudo code is described in Algorithm 4. First, the result set S is initialized to be empty, and a

Algorithm 4 SCSQuerySkyOrdPre (Skyline order $\mathcal{S}$, number of points to retrieve k)

```
1: S = ∅; cnt = k
2: for each subset S_i from S_1 to S_n in S do
3:    if |S_i| == cnt then
4:       S = S ∪ S_i; break
5:    if |S_i| < cnt then
6:       S = S ∪ S_i; cnt = cnt − |S_i|
7:    else
8:       S = S ∪ rSKY(S_i, cnt); break
9: return S
```

count variable cnt is set to k (line 1). Then each subset S_i in $\mathcal{S}$ is checked sequentially. If the cardinality of S_i equals the current count in cnt, S_i is merged into S, and the loop stops (lines 3–4). If the cardinality of S_i is smaller than cnt, S_i is merged into S and the loop continues with an updated cnt (lines 5–6). Otherwise, rSKY is called to select the last cnt points from S_i, and the loop stops (line 8).

The set-wide maximization approaches discussed in Section 3.2 can be used for implementing rSKY. In Section 6.3, we will experimentally compare our estimation based heuristic (Section 3.2.2) with the count based approach.

As a remark, skyline orders are pre-computed (and maintained) with the same reasoning as are materialized views [8, 23]. They can help efficiently solve arbitrary size constraints posed by future skyline queries.

5.2.2 Algorithm Without Pre-Computed Skyline Order

If no precomputed skyline order $\mathcal{S}$ of the data set P is available, query $Q_k^{scs}(P)$ can be processed as described in Algorithm 5, which accepts a sorted data set P as described in Section 4.2.2 and accomplishes the query processing in two phases. In the first phase, it follows the same logic as the improved skyline order computation in Algorithm 3. But only a partial skyline order $\mathcal{S}$ is now maintained. As soon as there are enough points in $\mathcal{S}$, the extra subsets on the tail are removed from $\mathcal{S}$, any point p belonging to those subsets are also ignored (lines 10–12). A new singleton tail is created only when $\mathcal{S}$ still has less than k points in total (lines 13–14). By keeping a partial skyline order only, considerable computation cost is saved. In

the second phase, SCSQuerySkyOrdPre (Algorithm 4) is called to pick up k points from the partial skyline order $\mathcal{S}$ (line 15).

Algorithm 5 SCSQuerySkyOrd (sorted data set P, number of points to retrieve k)

```
1: S = ⟨⟩
2: for each point p in P do
3:    for each S_i from S_1 to S_n in S do
4:       isSky=TRUE
5:       for each point q in S_i do
6:          if q ≺ p then
7:             isSky=FALSE; break
8:       if isSky then
9:          add p to S_i; break
10:      else
11:         if Σ_{1≤j≤i} |S_j| ≥ k then
12:            remove all subsets after S_i in S; break
13:   if (!isSky AND Σ_{S_i∈S} |S_i| < k) then
14:      append {p} to S
15: return SCSQuerySkyOrdPre(S, k)
```

Following the same line of reasoning as covered in as Section 4.2.2, the sequential scan on all skyline order subsets (line 3 in Algorithm 5) can also be replaced by a binary search. We will experimentally study this improvement in Section 6.

6 Experimental Studies

This section presents results of extensive experimental studies on both synthetic and real data sets. One part of experiments reveals the skyline orders of different data sets; the other compares different approaches to the size constrained skyline queries.

6.1 Experimental Settings

We generate both independent and anti-correlated synthetic data sets. We vary their cardinality from 100K to 1,000K, with 1K = 1024, and their dimensionality from 2 to 20. Anti-correlated data sets are generated according to a method introduced in previous work [7]. The value domain for each dimension is $[0, 1]$.

All parameters and their settings are listed in Table 5. All code is written in Java and run on a Windows XP PC with a 2.8GHz Intel Pentium D CPU and 1GB RAM.

Table 5: Parameters Used in Experiments

Parameter	**Setting**
Data set distribution	Indep. (IN), Anti-Correl. (AC)
Data set cardinality	100K, 200K, ..., 1000K
Data set dimensionality	2, 3, 4, 5, 10, 15, 20
k	50, 100, 150, ..., 500

6.2 Experiments on Skyline Order

6.2.1 Skyline Order Distribution

We first fix the data set dimensionality to 2 and 5, and vary the cardinality from 100K to 1,000K. The resulting skyline order lengths, i.e., the number of subsets a skyline order contains, are listed in Table 6. The sizes of the first two skyline order subsets of each data set are given in parentheses. We can see that for both data distributions, larger cardinality leads to higher skyline order. However, an anti-correlated data set has lower skyline order than its independent counterpart. This is because an anti-correlated data set has more skyline points than an independent one, when they are of the same cardinality.

Table 6: Skyline Order Length vs. Cardinality

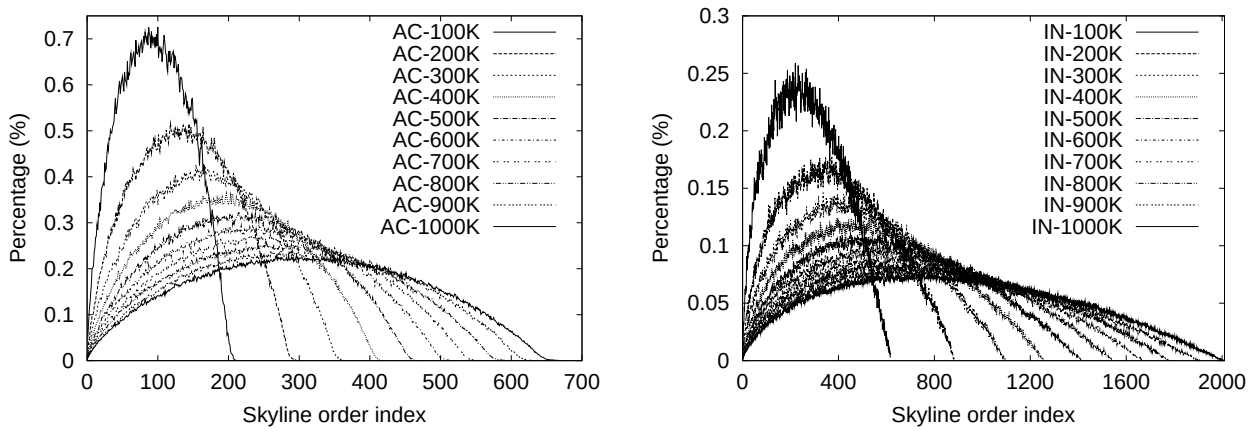
Card.	**Anti-Corre.**	**Indep.**
100K, 2D	210 (52, 82)	617 (8, 18)
5D	18 (15493, 29209)	23 (1003, 2684)
200K, 2D	297 (46, 80)	886 (9, 17)
5D	19 (21143, 45826)	27 (1108, 3308)
300K, 2D	363 (48, 107)	1096 (15, 19)
5D	21 (26037, 59842)	28 (1413, 4347)
400K, 2D	418 (47, 88)	1257 (14, 16)
5D	23 (29200, 70356)	30 (1561, 4691)
500K, 2D	467 (53, 81)	1412 (10, 14)
5D	23 (31589, 79498)	32 (1512, 4649)
600K, 2D	518 (51, 73)	1546 (9, 15)
5D	24 (34680, 89413)	32 (1646, 5164)
700K, 2D	556 (45, 79)	1668 (14, 22)
5D	26 (36377, 94982)	34 (1886, 5853)
800K, 2D	593 (64, 110)	1778 (15, 22)
5D	26 (38394, 102850)	35 (1876, 6300)
900K, 2D	633 (61, 88)	1909 (9, 15)
5D	27 (39767, 108147)	37 (1727, 5888)
1000K, 2D	671 (67, 98)	2008 (13, 24)
5D	27 (41991, 114960)	38 (1956, 6010)

For anti-correlated data sets, the skyline order distributions are shown in Figure 3(a). The x-axis is the skyline order subset index, and the y-axis shows the percentage of data points falling into the corresponding skyline order subset. For each data set, its skyline order approximates a normal distribution with respect to the subset indexes. Intuitively, the mean appears around the middle subset index, while the standard deviation becomes larger when the cardinality increases.

For independent data sets, the skyline order distributions are shown in Figure 3(b), from which we can see that the trends are similar to those for anti-correlated data sets above. However, because independent data sets have longer skyline orders, we also see more and larger zigzag fluctuations here.

Next we fix the data set cardinality to 100K and vary the dimensionality from 3 to 5, 10, 15, and 20. The resulting skyline order length are listed in Table 7. For both data distributions, the dimensionality has an apparent impact on the skyline order length: as the dimensionality increases, the skyline order get significantly lower. This is attributed to the "curse of dimensionality" that also affects skyline queries: the more dimensions a data set has, the higher the percentage of its points enter its skyline. Further, the difference between anti-correlated data sets and independent data sets is also apparent here.

The impact of data set dimensionality on the skyline order distributions is reported in Figure 4. For 3- to 5-dimensional data sets, the skyline order still approximate a normal distribution with respect to subset



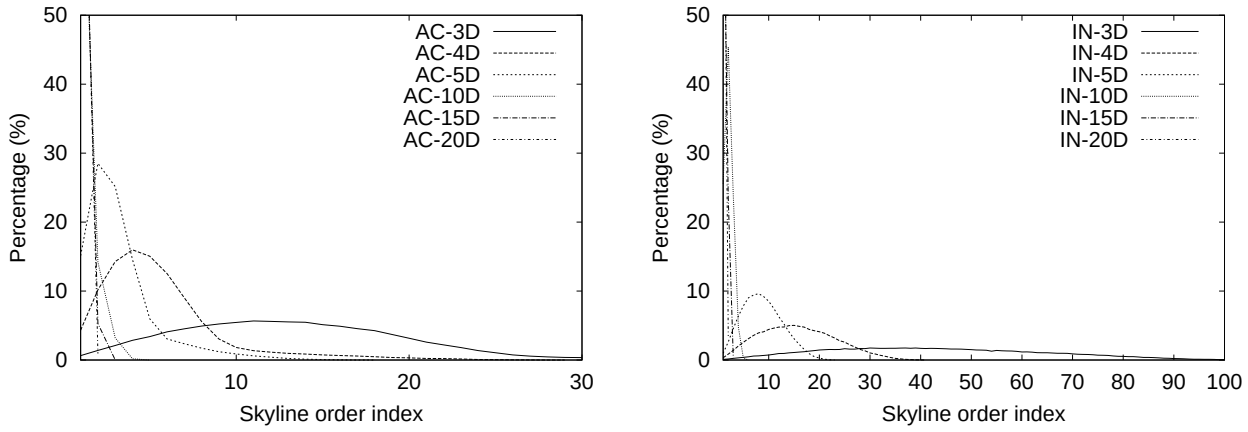
(a) AC Data Sets (b) IN Data Sets

Figure 3: Skyline Order Distribution vs. Cardinality

Table 7: Skyline Order Length vs. Dimensionality

Dim. \ Distrib.	**Anti-Corre.**	**Indep.**
2D	210 (52, 82)	617 (8, 18)
3D	66 (653, 1420)	105 (85, 155)
4D	29 (4432, 10414)	41 (338, 868)
5D	18 (15493, 29209)	23 (1003, 2684)
10D	5 (84393, 14531)	6 (26207, 46478)
15D	3 (96972, 5417)	3 (77920, 24095)
20D	2 (101438, 962)	2 (98826, 3574)

indexes. However, the most left parts of 4- and 5-dimensional anti-correlated data sets curves are "cut off", because they both have large-sized original skylines caused by the special data distribution. For 10-, 15- and 20-dimensional data sets, the approximate normal skyline order distribution is no longer visible. This is apparently because their skyline orders are too short, with as few as 2 to 6 subsets only as reported in Table 7.

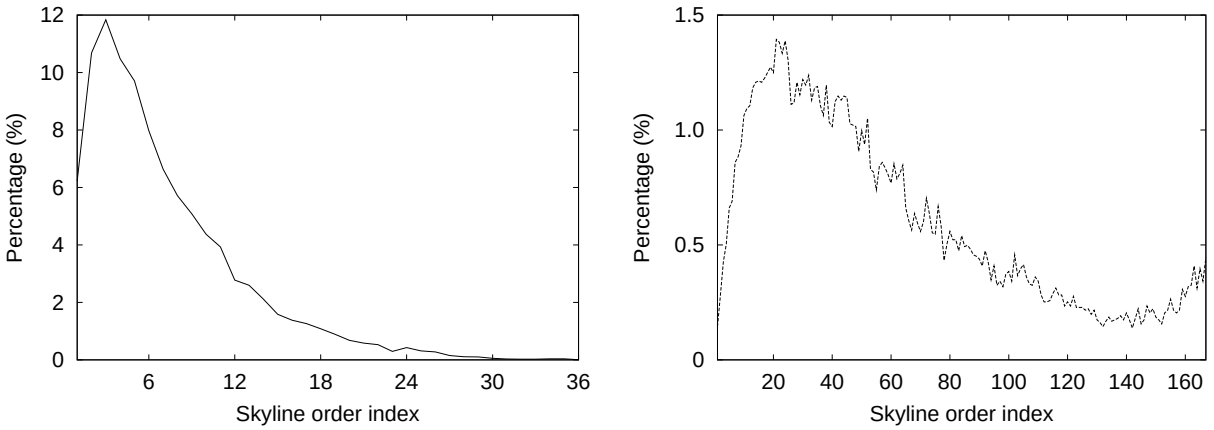


(a) AC Data Sets (b) IN Data Sets

Figure 4: Skyline Order Distribution vs. Dimensionality

We further investigate into the skyline order of a real-life data set. We use the NBA players season

statistics from 1949 to 2003[2], which has 16,644 17-attribute records and approximates a correlated data distribution. All records are normalized to the space $[0, 1]^{17}$. We first compute the skyline order by taking all 17 attributes into account. The percentage each subset occupies is plotted in Figure 5(a), where we get 36 skyline order subsets totally. Then, we pick three most important attributes, namely number of points, number of rebounds and number of assists from all 17 ones, and compute the skyline order of all records in terms of these 3 attributes. As a result, we get a total of 167 skyline order subsets. The percentage each subset occupies is plotted in Figure 5(b).



(a) Full Attributes (b) Partial Attributes

Figure 5: Skyline Order Distribution of NBA Data Set

For both results, the size of skyline order subset grows as the index increases, reaching its largest at some point, and then it goes down as index continues increasing. Although the curves here do not approximate normal distributions, we still have the "up-then-down" trends which have presented on the synthetic data sets above. Note the curve of full attributes is more smooth than that of partial attributes. This is due to the shortened skyline order length caused by a much higher dimensionality.

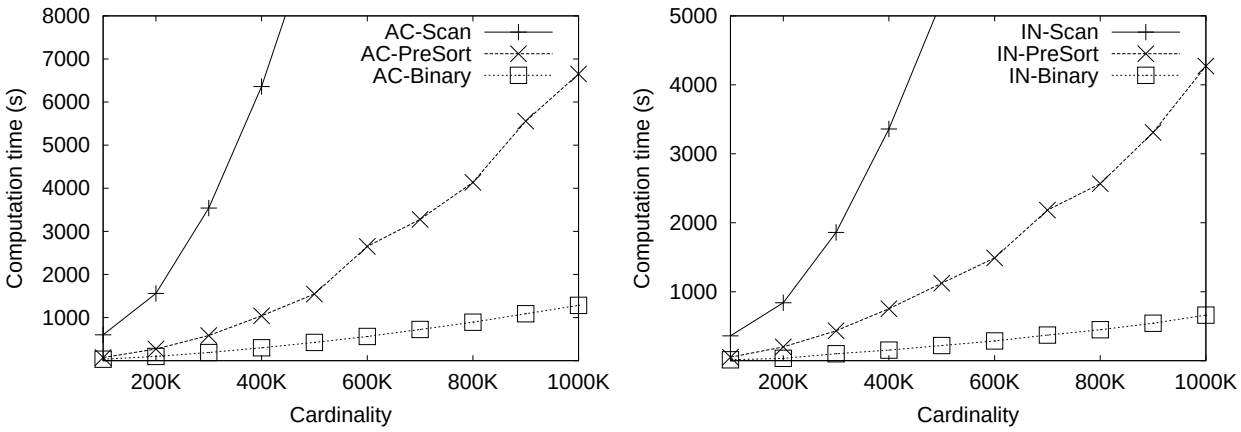
6.2.2 Skyline Order Computation Cost

We compare three skyline order computation approaches: the basic scan approach (Algorithm 1), the improved approach with pre-sorting (Algorithm 3), and the one with binary search in addition to pre-sorting. We consider the total computation time cost for each approach. As we are focused on the efficiency of these approaches themselves, the time for pre-sorting is excluded from the results.

The effect of data set cardinality on the computation time cost is reported in Figure 6. For both distributions, we fix the dimensionality to 2. It can be seen that as cardinality increases, the basic scan approach deteriorates dramatically, the binary search approach only slowly incurs higher cost at a low pace, and the pre-sorting approach remains between the two. The pre-sorting approach performs better than the basic scan approach because the pre-sorting of a data set avoids the expensive cascading adjustment (the calling of AdjustSkyOrd in Algorithm 2), which happens when a current point p dominates some points scanned before it.

The binary search approach improves the performance further, as it does not sequentially scan all skyline order subsets. In addition, the performance improvement between the basic scan approach and the pre-sorting approach is much more significant than the one between the latter and the binary search approach. This difference shows that AdjustSkyOrd is the most time-consuming part of the basic scan approach. This

[2]http://databasebasketball.com

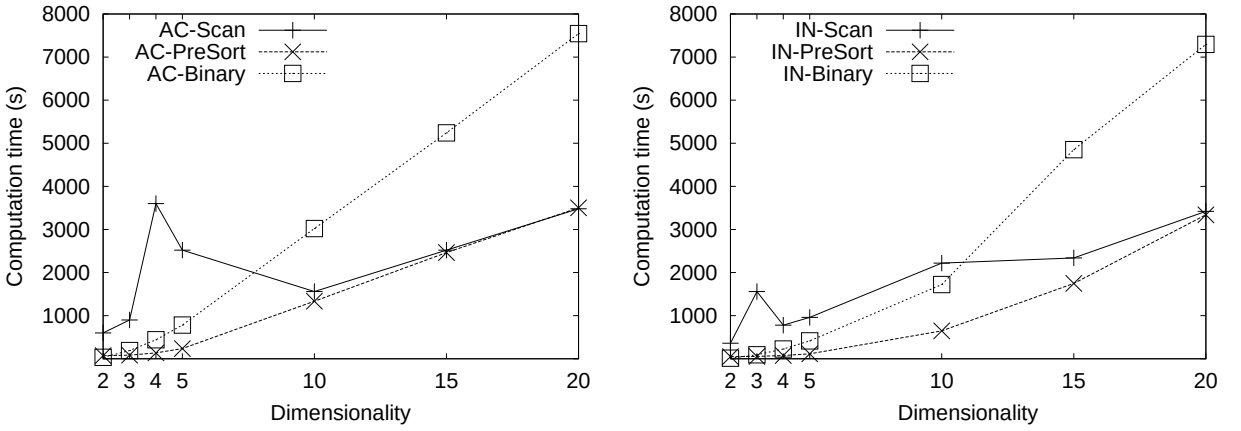


(a) AC Data Sets (b) IN Data Sets

Figure 6: Computation Cost vs. Cardinality

also indicates that applying the binary search idea directly to the basic scan approach is not attractive, as it has no chance to outperform the pre-sorting approach.

The effects of data set dimensionality are reported in Figure 7. Here we fix the cardinality to 100K, and have two important observations. First, the binary search approach is no longer the best when the dimensionality exceeds 2; rather, it becomes the worst when the dimensionality is higher than 10. This is because the skyline order length of a given data set shrinks as the dimensionality increases (refer to Table 7). Shorter skyline orders disfavor binary search, which consequently performs poorly with considerable extra costs for worst-case inputs.



(a) AC Data Sets (b) IN Data Sets

Figure 7: Computation Cost vs. Dimensionality

Second, as the dimensionality increases (higher than 10 for anti-correlated data sets and 15 for independent data sets), the basic scan approach becomes comparable to the pre-sort approach. For dimensionality 10 to 20, the very short skyline order lengthes (seen from Table 7) provide very limited room for optimizations that would otherwise benefit the pre-sort approach.

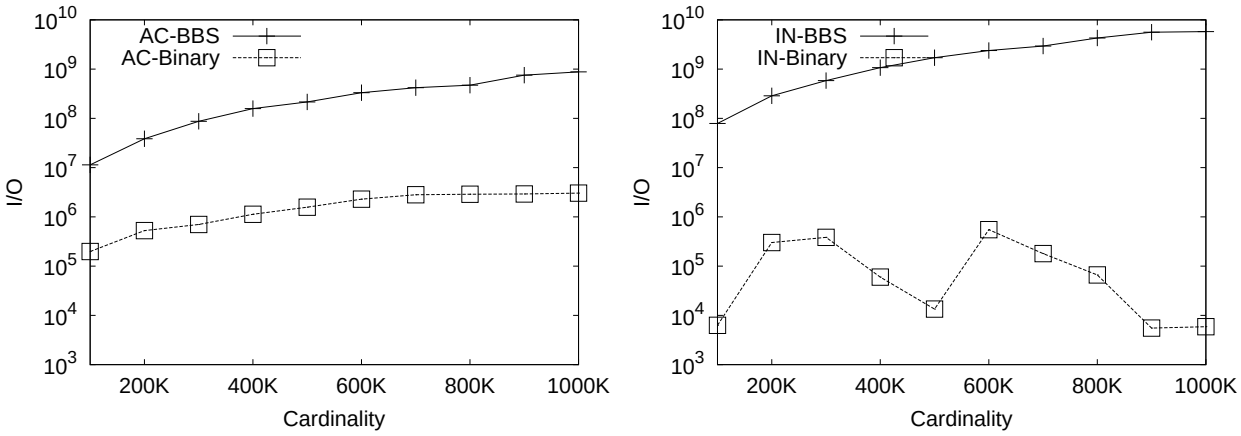
6.2.3 Adapting to Disk-Resident Data

So far we have assumed that both input data and skyline order subsets are kept in the main memory. If the data is too large to fit in memory, skyline order computation algorithms requires appropriate adaptations to work properly. We adapt the binary search approach since it is the most efficient in most cases.

In particular, all data points are loaded into memory in different batches. For each batch, a binary search is executed. A skyline order subset is initially created in memory, and may be output to the disk later to get free memory blocks. When a subset needed for comparison is not in memory, it is loaded from disk. Small skyline order subsets are allowed to share a memory block, whereas a large one may use multiple blocks with the last one accepting insertions of points. When a dirty memory block is to be eliminated, modified data will be output to the disk to the corresponding skyline order subset(s). When all points are processed, all dirty memory blocks will be output.

We also adapt the IO-optimal BBS algorithm [26] to compute skyline orders as follows. In addition to the main min-heap for all R-tree entries containing skyline points, we use an extra min-heap to contain all those entries pruned. When the skyline (also the first subset of the skyline order) is computed for the input data set, the extra heap is upgraded as the main heap and the BBS algorithm is executed to obtain the second skyline order subset, with the old main heap being the extra heap for pruned entries. This upgrading and processing is repeated until all data points are processed, indicated by an empty extra min-heap.

We use an LRU buffer whose size is 1% that of the input data set. For the adapted binary search approach, one buffer block is used for loading the input data, and others for skyline order subsets. The R-tree page size in BBS is set to 4K. The results of IO costs are reported in Figures 8 and 9. The inferiority of the adapted BBS approach, especially when the data are large, is attributed to the disadvantage that all pruned R-tree entries in the extra heap are accessed again subsequently, which renders the approach to have no real effective pruning.



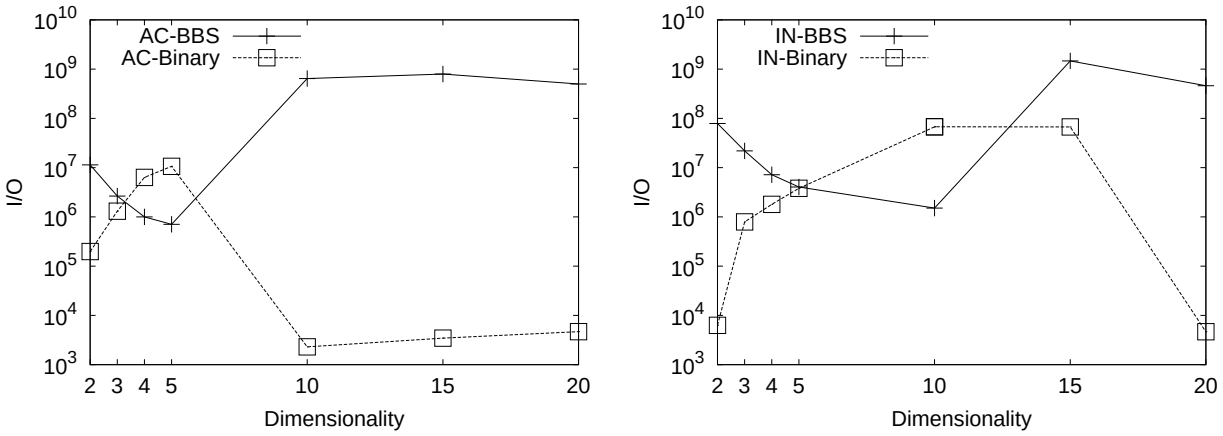
(a) AC Data Sets

(b) IN Data Sets

Figure 8: IO Cost vs. Cardinality

6.3 Experiments on Queries

We proceed to compare different approaches that can be used for answering size constrained skyline queries. From the ranking based approaches, we choose the subspace skyline frequency approach [10] (SSF) as a representative for comparison. The algorithm parameters in SSF are set as: $\epsilon = 0.2$ and $\delta = 0.05$. Readers are referred to [10] for detailed explanations on these parameters.



(a) AC Data Sets (b) IN Data Sets

Figure 9: IO Cost vs. Dimensionality

We also compare different skyline orders based query processing approaches: the algorithm with pre-computed skyline order (Algorithm 4 in Section 5.2.1), the algorithm without pre-computed skyline order (Algorithm 5 in Section 5.2.2), and the binary search algorithm (an improvement of Algorithm 5 in Section 5.2.2).

We combine each of the algorithms with different skyline reduction algorithms to return exact number of result points. We compare our volume of dominating region (VDR) estimation based approach (Section 3.2.2) with the top-k representative skyline points approach [25]. For the former, we use the heuristic that maximizes the sum of VDR values of candidate points. For the latter, we implement a bitmap based greedy algorithm, where each candidate point has a long bitmap indicating all points dominated by it. Note that the greedy algorithm does not return the optimal result for the top-k representative problem, which actually is NP-hard for data sets with dimensionality 3 or higher [25].

All approaches, apart from SSF, and their abbreviations to be used in the performance result graphs are listed in Table 8. We proceed to compare the response times of these approaches. Although SSF and the proposed techniques differ with respect to definition and output, comparing their response time is meaningful because they aim to meet similar user needs.

Table 8: Description of Approaches

	Set-Wide Maximization	
SCS query algorithm	*Count* (C [25])	*Estimation* (E)
Pre-computation based (P)	PCM	PEM
No pre-computation (O)	OCM	OEM
Binary search (B)	BCM	BEM

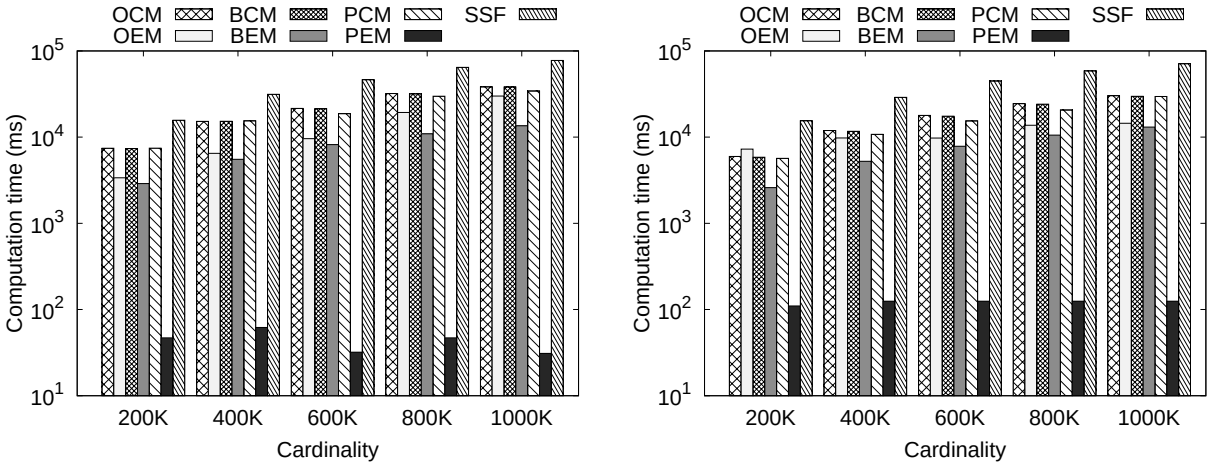
E: Estimation of VDR (Sec. 3.2.2)
P: Algorithm SCSQuerySkyOrdPre (Sec. 5.2.1)
O: Algorithm SCSQuerySkyOrd (Sec. 5.2.2)
B: Improvement on O (Sec. 5.2.2)

6.3.1 Effect of Cardinality

In this batch of experiments, we fix the data set dimensionality at 2 and k at 100, and we study the effects of the cardinality on the different approaches. We vary the cardinality from 100K to 1,000K; the resulting

query computation times are reported in Figure 10. For clarity, we only plot a portion of the results here. The omitted results exhibit similar trends.

For both data distributions, several observations are noteworthy. First, pre-computation based approaches usually outperform their corresponding counterparts without pre-computation: PCM is better than both OCM and BCM, and PEM is better than both OEM and BEM. This shows that pre-computed skyline orders save query processing time.



(a) AC Data Sets

(b) IN Data Sets

Figure 10: Query Cost vs. Cardinality

Second, when reducing skyline size, the VDR estimation based approach is better than the count of dominated points based approach: OEM is better than OCM, BEM is better than BCM, and PEM is better than PCM. The estimation based approach is much faster because it does not explore the global dominance relationship to count, but makes estimates based on the given candidate point only. Because of the dual merits of pre-computation and estimation, the PEM approach has a significant advantage over its competitors.

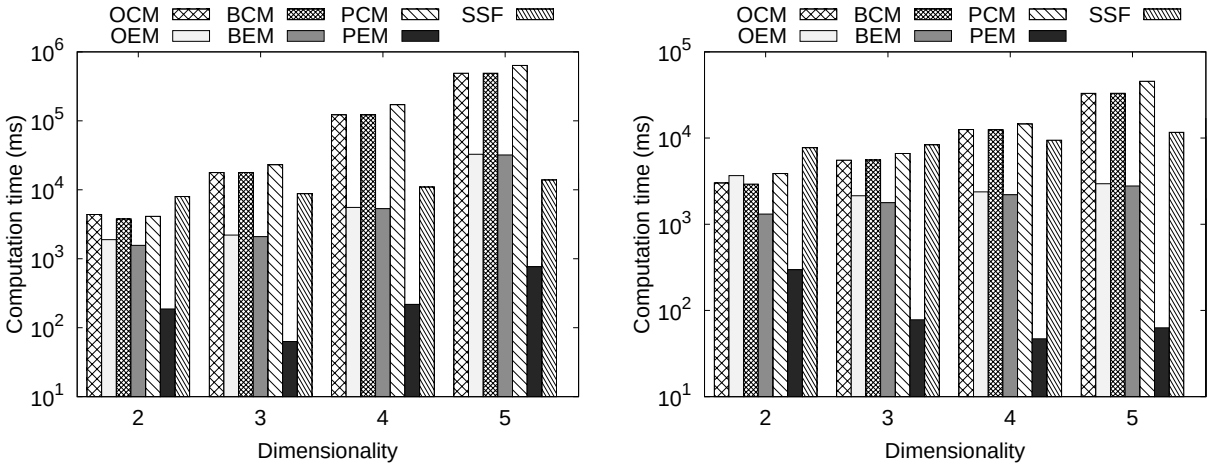
Third, the total ranking approach SSF is the worst among all approaches, and it degrades noticeably as the cardinality increases. The total pointwise ranking of SSF is very time-consuming, and larger cardinalities inevitably worsen the situation.

Fourth, PEM performs better on anti-correlated data sets than on independent ones. For an anti-correlated data set, its starting skyline order subsets contain more points than those of an independent data set. This difference makes PEM explore fewer skyline order subsets than the former. However, PCM does not have this advantage. The benefit of pre-computation of PCM is counteracted by its time-consuming count based skyline reduction approach.

6.3.2 Effect of Dimensionality

Next, we fix the data cardinality at 100K and k at 100, and then vary the dimensionality from 2 to 5. The results are reported in Figure 11.

As the dimensionality grows, most approaches deteriorate markedly, especially OCM, BCM, and PCM. The count based reduction incorporated by these approaches requires more time to check for dominance when dimensionality increases. In contrast, the estimation based approaches OEM and BEM are more steady. The cost of PEM even decreases in some cases: from 2D to 3D on anti-correlated data sets; from 2D to 4D on independent data sets. These surprising improvements are attributed to skyline order subset size variations. Referring to Table 7, PEM needs to explore 2 subsets for 2D data set, but only one for other data sets. This helps reduce query processing time on one hand. On the other hand, 4D and 5D data sets contain



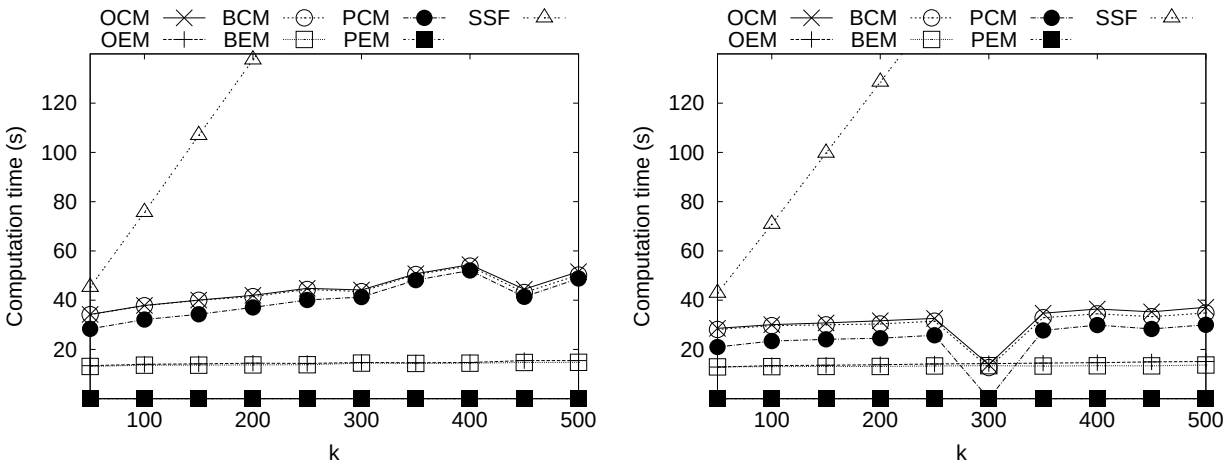
(a) AC Data Sets (b) IN Data Sets

Figure 11: Query Cost vs. Dimensionality

much more points than k in their first skyline order subsets, meaning that considerable time is needed to reduce points. As a result, the overall costs on 4D and 5D data sets increase. Similar reasoning applies to independent data sets.

6.3.3 Effect of k

In the next experiment, we fix the data cardinality at 1,000K and the dimensionality at 2, and we then vary k from 50 to 500. Figure 12 reports the relevant results.



(a) AC Data Sets (b) IN Data Sets

Figure 12: Query Cost vs. k

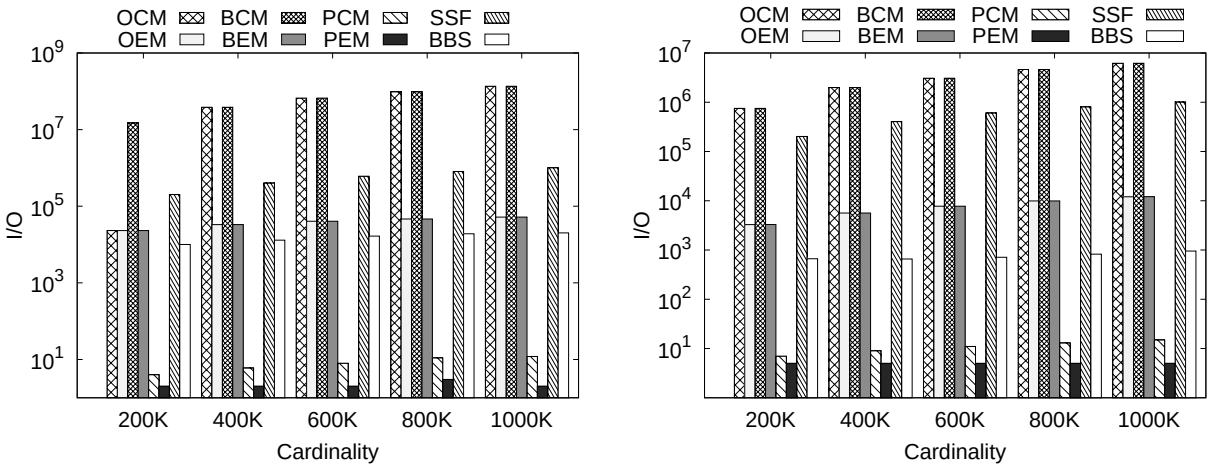
It is again seen that pre-computation is beneficial and that VDR estimation is better than the count based approach. For the approaches incorporating count based reduction (the $*$CM approaches), the costs usually increase for larger k. As k increases, more skyline order subsets are explored. For those subsets, the numbers of candidate points increase as the subset indexes increase. Hence, the $*$CM approaches slow down because their greedy heuristic reduction gets larger inputs. In contrast, the approaches with estimation based reduction (the $*$EM approaches) are considerably more steady because VDR estimation is computationally quite fast even for larger numbers of candidate points.

It can be seen that the $*$CM approaches exhibit "drops": at $k = 450$ for the anti-correlated data set, and at $k = 300$ for the independent data set. The independent data set of 1,000K points has exactly 300 points in its first eight skyline order subsets. As a result, for the case $k = 300$ no skyline reduction occurs for any of the $*$CM and the $*$EM approaches. Therefore, the cost of any $*$CM approach is exactly the same as that of its $*$EM counterpart. The anti-correlated data set of 1,000K points has a total of 448 points in its first four skyline order subsets. As a result, for $k = 450$ any of the $*$CM approaches need to choose only 2 additional points from the fifth skyline order subset with 169 points, which is quite easy for the greedy heuristic employed by the $*$CM approaches.

6.3.4 Performance on Disk-Resident Data

In this section, we investigate the query performance on disk-resident data. We adapt the algorithms in the similar way as in Section 6.2.3. We use an LRU buffer with size of 1% data size. We also adapt the IO-optimal BBS algorithm [26] to process queries. In particular, skyline order subsets are computed on-the-fly one by one using the adapted BBS version with two min-heaps, as detailed in Section 6.2.3. After a skyline order subset is computed, the BBS processing stops if all current subsets totally contain k or more points. For the latter, the VDR estimation 3.2.2 is used to reduce the result to exact k points. The R-tree page is set to 4K for the BBS algorithm.

The results on IO costs are reported in Figures 13 to 15. The default settings are given in each title. It is seen that our PEM approach is always the best in terms of query IO; PCM approach is the second best for most cases.



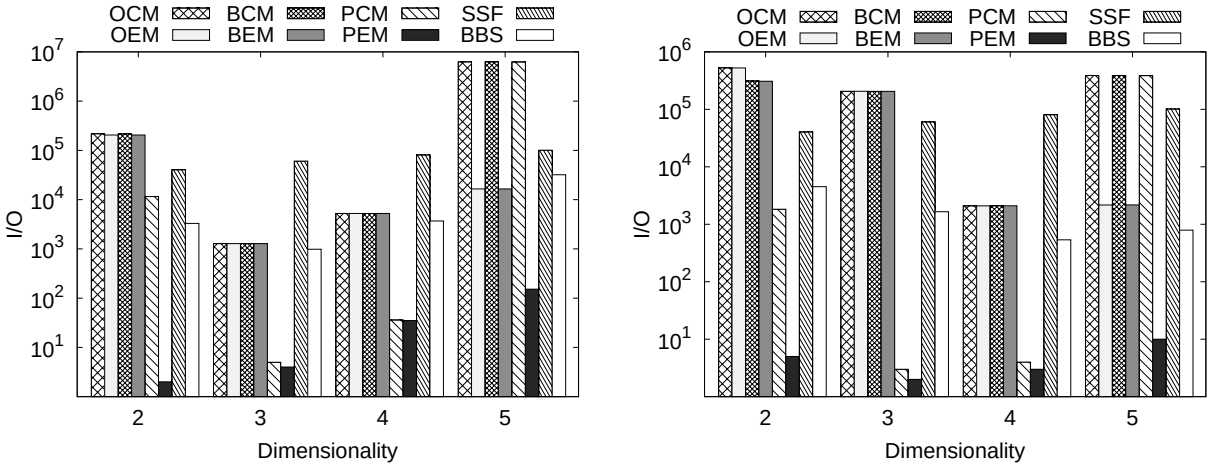
(a) AC Data Sets (b) IN Data Sets

Figure 13: IO Cost vs. Cardinality ($d = 5, k = 100$)

6.3.5 Interpretation of Query Results

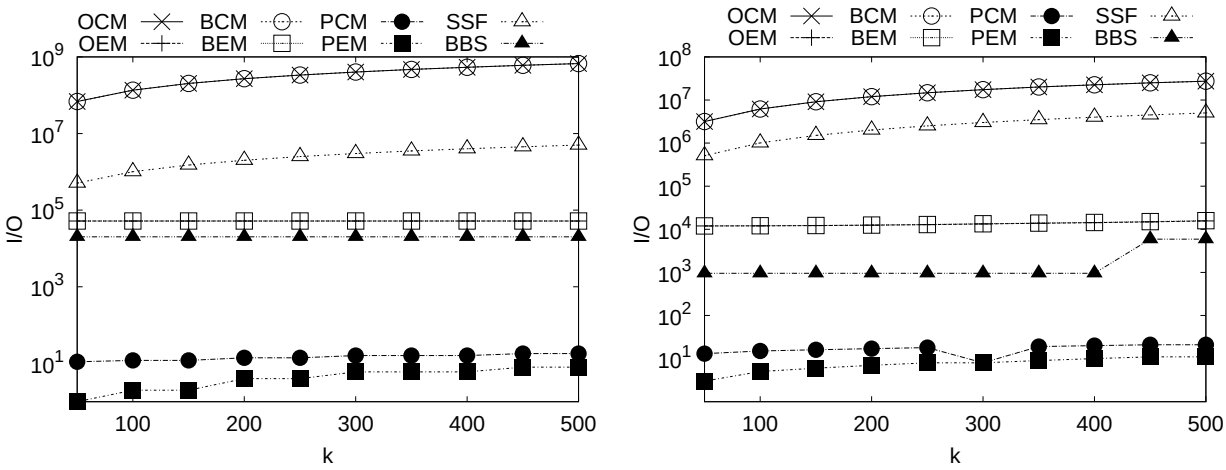
Note that all the $*$EM approaches produce the same set of points given the same input; so do all $*$CM approaches. Therefore, we compare the results of PEM and PCM with those of alternatives.

As mentioned in Section 3.3, pointwise ranking is prone to ties and thus to selecting points with similar features, while results with diverse points are usually more desirable to users issuing skyline queries. Therefore, we evaluate the diversity of the query results computed by the different approaches based on the variance among the points returned from synthetic data sets. Let R be the set of points returned as the query result. Its variance $V(R)$ is defined as the average Euclidean distance between points in R and the mean of R, i.e., $V(R) = \frac{1}{k}\sum_{p \in R} \|p - \overline{R}\|^2$, where k is $|R|$.



(a) AC Data Sets (b) IN Data Sets

Figure 14: IO Cost vs. Dimensionality ($|P| = 100$K, $k = 100$)



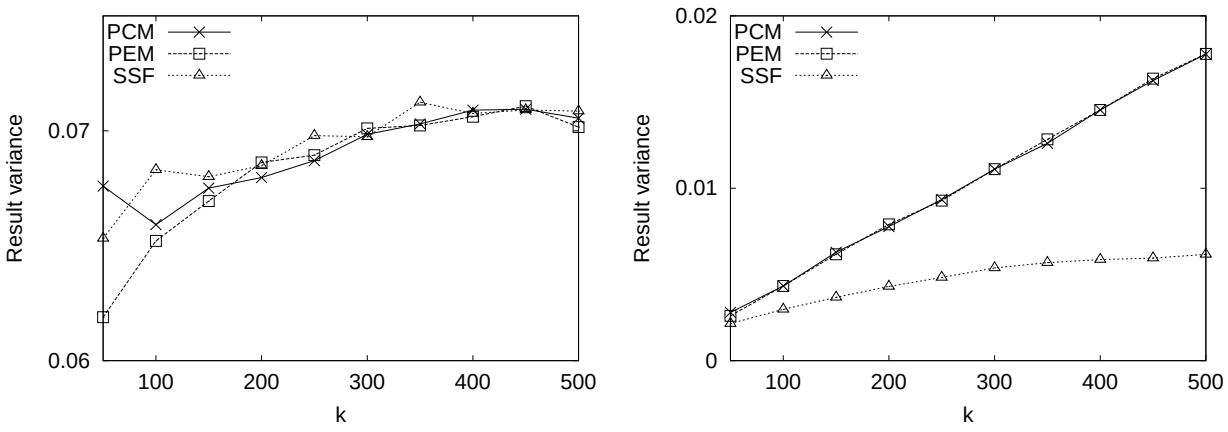
(a) AC Data Sets (b) IN Data Sets

Figure 15: IO Cost vs. k ($|P| = 1000$K, $d = 5$)

We fix the synthetic data cardinality at 1,000K and the dimensionality at 5. The results obtained from synthetic data sets are shown in Figure 16. For the anti-correlated data set, all approaches are quite close to each with in terms of the result variance. Whereas for the independent data set, our skyline order concept helps achieve result variance due to its combined advantages; SSF fails to do so due to its pointwise ranking nature.

We proceed to compare the query results obtained from the NBA data set [3] which has 19,112 17-attribute records and generally follows a correlated data distribution. Following [30], we select four most important attributes to query: games played, points, rebounds and assists. All records are normalized to the space $[0, 1]^4$. The query results for k=5 are listed in Table 9. PEM returns impressive result by identifying the peak seasons of the legend player Wilt Chamberlain. PCM outputs query results with larger variance, which also overlap with those returned by the top-k dominating approach. Finally, SSF fails to identify star players as it faces too many ties in ranking.

[3]NBA Statistics v2.0: http://databasebasketball.com.



(a) AC Data Sets

(b) IN Data Sets

Figure 16: Result Variance vs. k

Table 9: Comparison of Query Results

k	PCM	PEM	Top-k	SSF
1	Wilt Chamberlain (1967)	Wilt Chamberlain (1965)	Wilt Chamberlain (1967)	John Abramovic (1946)
2	Julius Erving (1974)	Wilt Chamberlain (1963)	Billy Cunningham (1972)	Chet Aubuchon (1946)
3	Julius Erving (1971)	Oscar Robertson (1961)	Kevin Garnett (2002)	Norm Baker (1946)
4	Oscar Robertson (1961)	Wilt Chamberlain (1966)	Julius Erving (1974)	Herschel Baltimore (1946)
5	Isiah Thomas (1983)	Wilt Chamberlain (1967)	Kareem Abdul-Jabbar (1975)	John Barr (1946)

7 Conclusion

Skyline queries are able to find interesting data points in multi-dimensional data sets. However, existing approaches to skyline queries offer little control on their result cardinalities. Most previous work has been focused on how to reduce skyline result cardinalities. In this paper, we aim to resolve arbitrary size constraints on skyline queries, without any a priori assumptions about the relationship between the expected number of points and the skyline cardinality.

The paper describes how typical existing techniques are not well equipped to contend with arbitrary size constraints. Then the concept of skyline order is proposed where a data set is partitioned using the skyline dominance relationship. The skyline ordering provides an order among different partitions, but still reserves room for different local optimizations within each partition.

Efficient algorithms are developed for computing skyline orders and resolving arbitrary size constraints using skyline order. Extensive empirical studies show that skyline order presents a flexible framework for efficient and scalable resolution of arbitrary size constraints on skyline queries.

References

[1] W.-T. Balke, J. X. Zheng, and U. Guentzer. Approaching the dfficient frontier: cooperative database retrieval using high-Dimensional skylines. In *Proc. DASFAA*, pp. 410–421, 2005.

[2] W.-T. Balke, U. Guentzer, C. Lofi. Eliciting matters - controlling skyline sizes by incremental integration of user preferences. In *Proc. DASFAA*, pp. 551–562, 2007.

[3] J. L. Bentley, K. L. Clarkson, and D. B. Levine. Fast linear expected-time algorithms for computing maxima and convex hulls. In *Proc. SODA*, pp. 179–187, 1990.

[4] I. Bartolini, P. Ciaccia, and M. Patella. Efficient sort-based skyline evaluation. *TODS*, 33(4):1–49, 2008.

[5] J. L. Bentley, H. T. Kung, M. Schkolnick, and C. D. Thompson. On the average number of maxima in a set of vectors and applications. *JACM*, 25(4):536–543, 1978.

[6] H. Blunck and J. Vahrenhold. In-place algorithms for computing (layers of) maxima. In *Proc. SWAT*, pp. 363–374, 2006.

[7] S. Borzonyi, D. Kossmann, and K. Stocker. The skyline operator. In *Proc. ICDE*, pp. 421–430, 2001.

[8] J. A. Blakeley, P. Larson, and F. W. Tompa. Efficiently Updating Materialized Views. In *Proc. SIGMOD*, pp. 61–71, 1986.

[9] C.-Y. Chan, H. Jagadish, K.-L. Tan, A. K. Tung, and Z. Zhang. Finding k-dominant skylines in high dimensional space. In *Proc. SIGMOD*, pp. 503–514, 2006.

[10] C.-Y. Chan, H. Jagadish, K.-L. Tan, A. K. Tung, and Z. Zhang. On high dimensional skylines. In *Proc. EDBT*, pp. 478–495, 2006.

[11] Y.-C. Chang, L. Bergman, V. Castelli, C.-S. Li, M.-L. Lo, and J. R. Smith. The onion technique: Indexing for linear optimizatoin queries. In *Proc. SIGMOD*, pp. 391–402, 2000.

[12] J. Chomicki. Preference formulas in relational queries. *TODS*, 28(4):427–466, 2003.

[13] J. Chomicki, P. Godfrey, J. Gryz, and D. Liang. Skyline with presorting. In *Proc. ICDE*, pp. 717–719, 2003.

[14] B. Cui, H. Lu, Q. Xu, L. Chen, Y. Dai, and Y. Zhou. Parallel Distributed Processing of Constrained Skyline Queries by Filtering. In *Proc. ICDE*, pp. 546–555, 2008.

[15] P. Godfrey, R. Shipley, and J. Gryz. Maximal vector computation in large data sets. In *Proc. VLDB*, pp. 229–240, 2005.

[16] G. Hjaltason and H. Samet. Distance browsing in spatial database. *ACM TODS*, 24(2):265–318, 1999.

[17] Z. Huang, C. S. Jensen, H. Lu, and B. C. Ooi. Skyline queries against mobile lightweight devices in MANETs. In *Proc. ICDE*, page 66, 2006.

[18] W. Jin, M. Ester, and J. Han. Efficient processing of ranked queries with sweeping selection. In *Proc. PKDD*, pp. 527–535, 2005.

[19] W. Jin, J. Han, and M. Ester. Mining thick skylines over large databases. In *Proc. PKDD*, pp. 255–266, 2004.

[20] V. Koltun and C. H. Papadimitriou. Approximately dominating representatives. In *Proc. ICDT*, pp. 204–214, 2005.

[21] D. Kossmann, F. Ramsak, and S. Rost. Shooting stars in the sky: An online algorithm for skyline queries. In *Proc. VLDB*, pp. 275–286, 2002.

[22] H. T. Kung, F. Luccio, and F. P. Preparata. On finding the maxima of a set of vectors. *JACM*, 22(4):469–476, 1975.

[23] P. Larson, H. Z. Yang. Computing Queries from Derived Relations. In *Proc. VLDB*, pp. 259–269, 1985.

[24] J. Lee, G. You, and S. Hwang. Telescope: Zooming to InterestingSkylines. In *Proc. DASFAA*, pp. 539–550, 2007.

[25] X. Lin, Y. Yuan, Q. Zhang, and Y. Zhang. Selecting stars: The k most representative skyline operator. In *Proc. ICDE*, pp. 86–95, 2007.

[26] D. Papadias, Y. Tao, G. Fu, and B. Seeger. An optimal and progressive algorithm for skyline queries. In *Proc. SIGMOD*, pp. 467–478, 2003.

[27] D. Papadias, Y. Tao, G. Fu, and B. Seeger. Progressive skyline computation in database systems. *ACM TODS*, 30(1):41–82, 2005.

[28] K. L. Tan, P. K. Eng, and B. C. Ooi. Efficient progressive skyline computation. In *Proc. VLDB*, pp. 301–310, 2001.

[29] A. Vlachou, C. Doulkeridis, K. Nørvåg, and M. Vazirgiannis. Skyline-based Peer-to-Peer Top-k Query Processing. In *ICDE*, pp. 1421–1423, 2008.

[30] M. L. Yiu and N. Mamoulis. Efficient processing of top-k dominating queries on multi-dimensional data. In *Proc. VLDB*, pp. 483–494, 2007.

[31] Z. Zhang, X. Guo, H. Lu, A. K. Tung, and N. Wang. Discovering strong skyline points in high dimensional spaces. In *Proc. CIKM*, pp. 247–248, 2005.

[32] Shiming Zhang, N. Mamoulis, and D. W. Cheung Scalable skyline computation using object-based space partitioning. In *Proc. SIGMOD*, pp. 483–494, 2009.