Response Time Analysis for Arbitrary Deadlines

Example Calculation

Real-Time Software (TSW/SW5), Fall 2010

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1 Response Time Analysis

Under the assumption that *a process is allowed to complete before another instance is allowed to execute*, we can compute the worst case response time for systems with arbitrary deadlines using the formulae and method explained in [BW, Section 11.10.2]

The collective worst case response time for all overlapping releases in a given "window" can be computed as the limit (or fix point) of the following recurrence relation:

$$w_i^{n+1}(q) = B_i + (1+q)C_i + \sum_{j \in hp(i)} \left\lceil \frac{w_i^n(q)}{T_j} \right\rceil C_j \qquad (1)$$

where q denotes how many overlapping releases occur in the given window. Let $w_i(q)$ be the *limit* of the above recurrence relation:

$$w_i(q) = w_i^n(q) \qquad \text{with } n = \min\{m | w_i^m(q) = w_i^{m+1}(q)\}$$

The worst case response time for a single instance of process i in a window with q overlapping releases is then defined by:

$$R_i(q) = w_i(q) - qT_i$$

The value, $R_i(q)$, must be computed for all the values of q that are possible in the system. The values that q can take are bounded by the lowest q such that:

$$R_i(q) \leq T_i$$

Where the process completes before the next release.

The worst case response time for process i is now the maximum of the response times calculated for different values of q:

$$R_i = \max_{q \in \{0,1,\dots\}} R_i(q)$$

In the following we use these formulae to perform a response time analysis for a simple system.

2 Example System

The system consists of two processes with the following characteristics:

Id	Priority P	Period T	Deadline D	Cost C
P_1	1	10	20	5
P_2	2	20	20	10

In particular note that $D_1 > T_1$ and $D_2 = T_2$. To simplify calculations we further assume that the processes are *independent*, i.e., there will be no blocking and thus $B_1 = B_2 = 0$.

We first calculate the response time for P_1. Note that:

$$hp(1) = \{2\}$$

This allows us to simplify the recurrence relation for calculating $w_1^n(q)$:

$$\begin{aligned} w_1^{n+1}(q) &= B_1 + (1+q)C_1 + \sum_{j \in hp(1)} \left\lceil \frac{w_1^n(q)}{T_j} \right\rceil C_j \\ &= (1+q)C_1 + \left\lceil \frac{w_1^n(q)}{T_2} \right\rceil C_2 \\ &= (1+q) \cdot 5 + \left\lceil \frac{w_1^n(q)}{20} \right\rceil 10 \end{aligned} \tag{2}$$

We can now use the above, simplified, recurrence relation to find the number of releases of P_1 that gives rise to the maximum response time. Specifically we have to use equation (2) to calculate the worst case response time for an (almost) arbitrary number of overlapping releases (called q) of process P_1. Luckily we only need to continue until we encounter a q such that:

$$R_1(q) \leq T_1 \tag{3}$$

We start by first calculating the response time analysis in the case $q = 0$, i.e., where there is only one release of P_1 and thus no overlaps. The calculation proceed by iterating equation (2) until we reach a fix point:

$$\begin{aligned} w_1^0(0) &= (1+0) \cdot 5 + \left\lceil \frac{0}{20} \right\rceil 10 \\ &= 5 + 0 \\ &= 5 \\ w_1^1(0) &= (1+0) \cdot 5 + \left\lceil \frac{w_1^0(0)}{20} \right\rceil 10 \\ &= 5 + \left\lceil \frac{5}{20} \right\rceil 10 \\ &= 5 + 1 \cdot 10 \\ &= 15 \\ w_1^2(0) &= (1+0) \cdot 5 + \left\lceil \frac{w_1^1(0)}{20} \right\rceil 10 \\ &= 5 + \left\lceil \frac{15}{20} \right\rceil 10 \\ &= 5 + 1 \cdot 10 \\ &= 15 \\ &= w_1^1(0) \end{aligned}$$

Finally! We have reached the limit for $q = 0$ and therefore $w_1(0) = 15$. This leads to the following response time for one release of P_1:

$$\begin{aligned} R_1(0) &= w_1(0) - 0 \cdot T_1 \\ &= 15 - 0 \\ &= 15 \\ &> T_1 \end{aligned}$$

We note that $R_1(0) = 15 > 10 = T_1$ and therefore, by equation (3), we have to continue and repeat the above calculations for $q = 1$:

$$\begin{aligned}
w_1^0(1) &= (1+1)\cdot 5 + \left\lceil \tfrac{0}{20} \right\rceil 10 \\
&= 10 + 0 \\
&= 10 \\
w_1^1(1) &= (1+1)\cdot 5 + \left\lceil \tfrac{w_1^0(1)}{20} \right\rceil 10 \\
&= 10 + \left\lceil \tfrac{10}{20} \right\rceil 10 \\
&= 10 + 1 \cdot 10 \\
&= 20 \\
w_1^2(1) &= (1+1)\cdot 5 + \left\lceil \tfrac{w_1^1(1)}{20} \right\rceil 10 \\
&= 10 + \left\lceil \tfrac{20}{20} \right\rceil 10 \\
&= 10 + 1 \cdot 10 \\
&= 20 \\
&= w_1^1(1)
\end{aligned}$$

With a fix point reached for $q = 1$, and thus $w_1(1) = 20$, we can compute the maximum response time for the case where there are two overlapping releases of P_1:

$$\begin{aligned}
R_1(1) &= w_1(1) - 1 \cdot T_1 \\
&= 20 - 1 \cdot 10 \\
&= 10 \\
&= T_1
\end{aligned}$$

Since $R_1(1) = T_1$ the two overlapping releases of P_1 are now able to finish before a third release of P_1. It is therefore not necessary to carry out further calculations for $q > 1$.

We are now in a position to calculate the maximum response time for process P_1 by taking the largest response time calculated for various q:

$$\begin{aligned}
R_1 &= \max_{q \in \{0,1\}} R_1(q) \\
&= \max\{15, 10\} \\
&= 15
\end{aligned}$$

In this case it turns out that the worst case response time for P_1 is attained when there is only *one* release of P_1.

Similar calculations must now be carried out for process P_2. Again we note that:

$$hp(2) = \{\}$$

which leads to the following simplification of equation (1):

$$\begin{aligned}
w_2^{n+1}(q) &= B_2 + (1+q)C_2 + \textstyle\sum_{j \in hp(2)} \left\lceil \tfrac{w_2^n(q)}{T_j} \right\rceil C_j \\
&= (1+q)C_2 \\
&= (1+q) \cdot 10
\end{aligned} \tag{4}$$

Using equation (4) we calculate $w_2(0)$:

$$\begin{aligned}
w_2^0(0) &= (1+0) \cdot 10 \\
&= 10 \\
w_2^1(0) &= (1+0) \cdot 10 \\
&= 10 \\
&= w_2^0(0)
\end{aligned}$$

Thus $w_2(0) = 10$ and therefore the worst case response time for P_2 with no overlapping releases is given as follows:

$$\begin{aligned} R_2(0) &= w_2(0) - 0 \cdot T_2 \\ &= 10 \\ &< T_2 \end{aligned}$$

Since $R_2(0) \leq T_2$ it is not necssary to compute $R_2(q)$ for $q > 0$. The worst case response time for process P_2 then follows trivially:

$$\begin{aligned} R_2 &= \max_{q \in \{0\}} R_2(q) \\ &= \max\{10\} \\ &= 10 \end{aligned}$$

All that now remains is to verify that the system is indeed schedulable:

$$R_1 = 15 \leq 20 = D_1 \quad \text{and} \quad R_2 = 10 \leq 20 = D_2$$

Exercise 1. Redo the above calculation(s) assuming process P_2 is *blocked* for 15 time units.

Exercise 2. Perform a response time analysis on a system with the following characteristics:

Id	**Priority** P	**Period** T	**Deadline** D	**Cost** C
P_1	1	15	20	5
P_2	2	20	20	5
P_3	3	30	30	5

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