Symbolic Reachability and Beyound

or how UPPAAL really works

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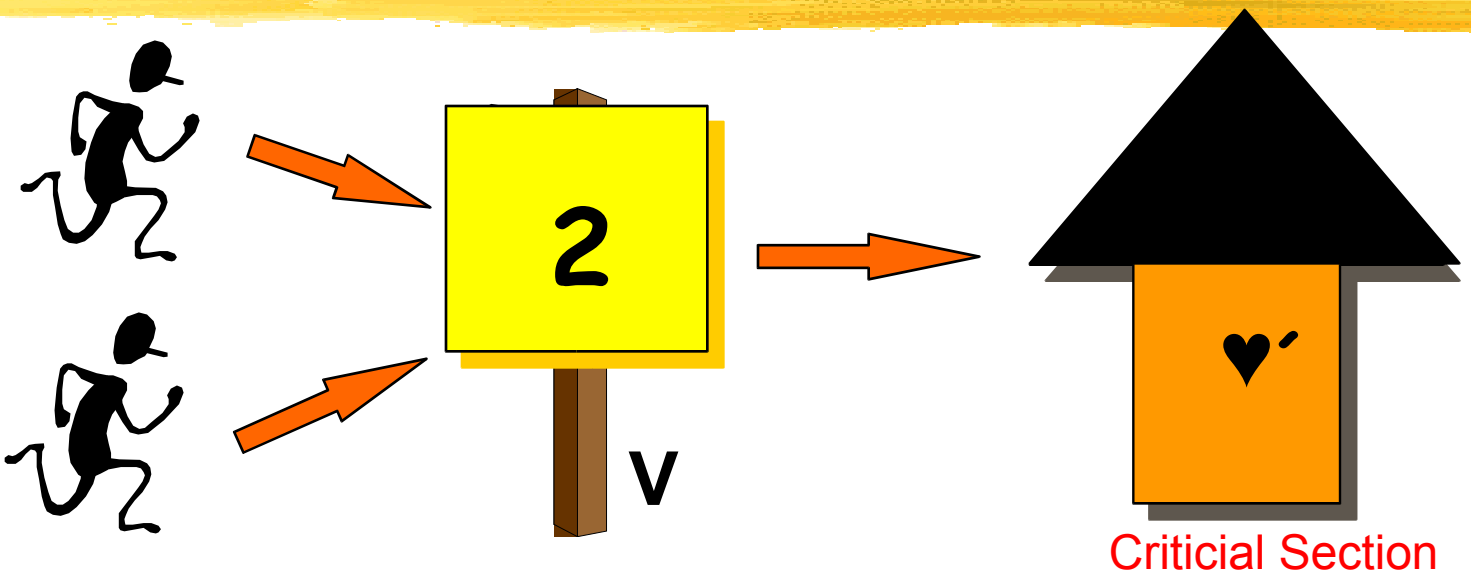
REGIONS

review

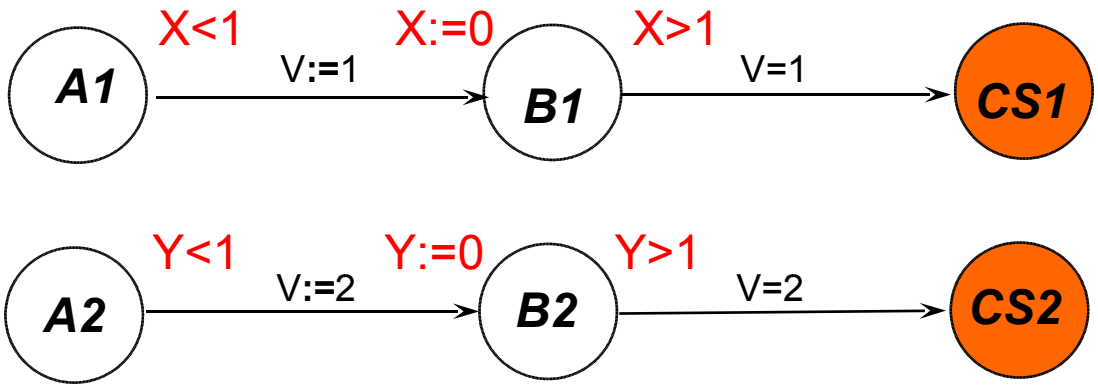
Fischer's Protocol

a simple real time mutual exclusion algorithm



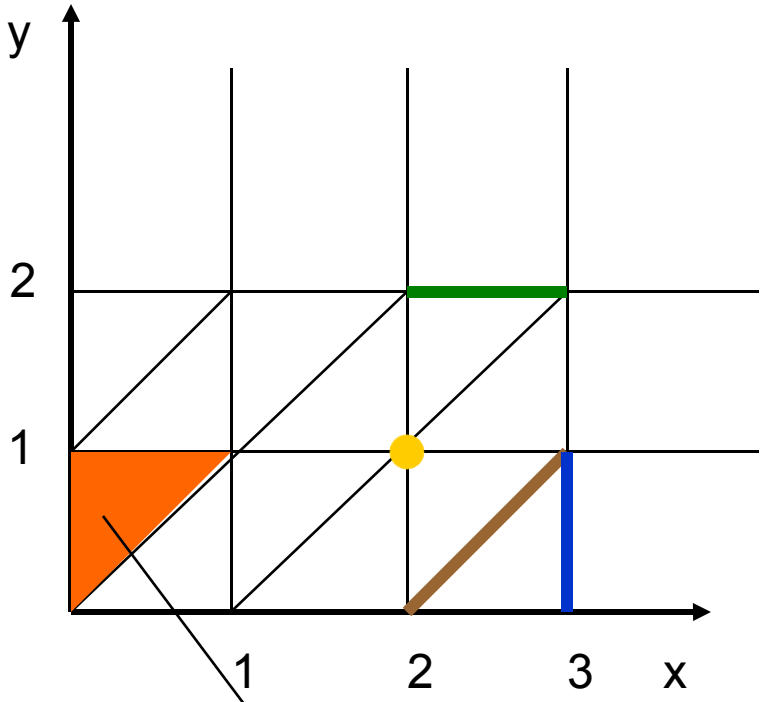
Init

V=1



Regions

Finite partitioning of state space



Definition

$w \approx w'$ iff w and w' satisfy the exact same conditions of the form

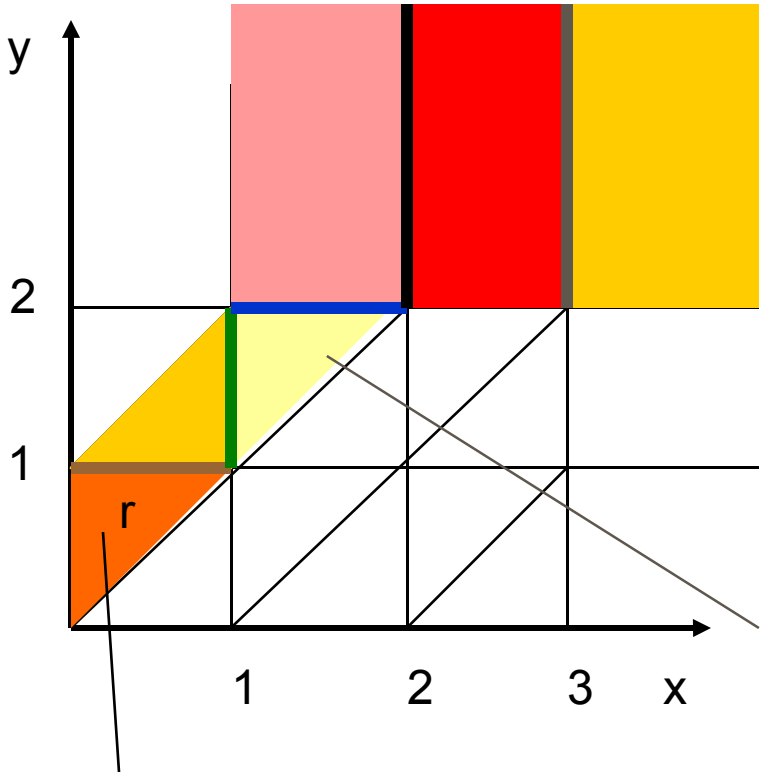
$$x_i \leq n \text{ and } x_i - x_j \leq n$$

where $n \leq \max$

An equivalence class (i.e. a *region*)
in fact there is only a *finite* number of regions!!

Regions

Finite partitioning of state space



Definition

$w \approx w'$ iff w and w' satisfy the exact same conditions of the form

$$x_i \leq n \text{ and } x_i - x_j \leq n$$

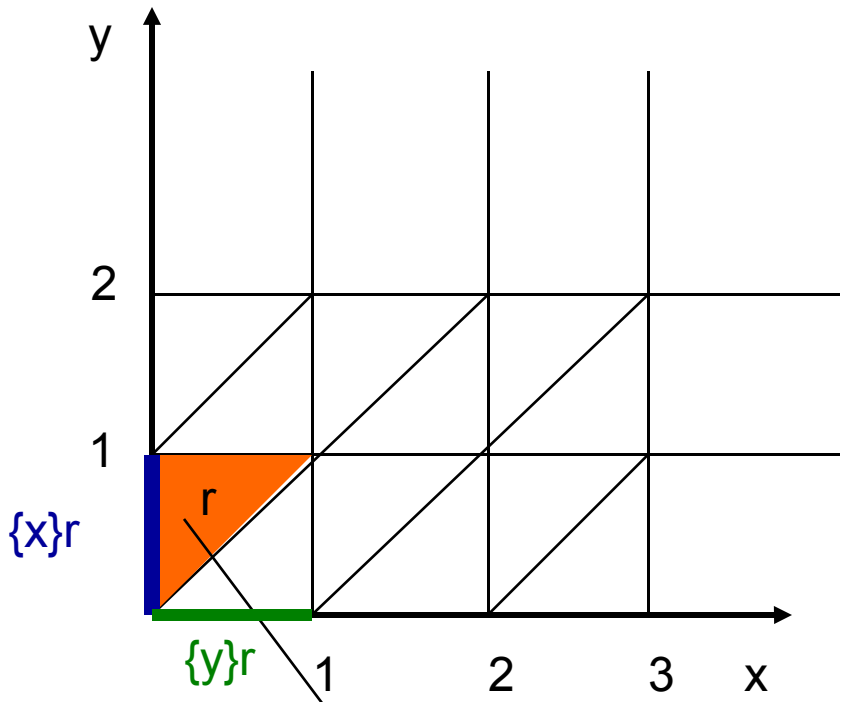
where $n \leq \max$

Successor regions, Succ(r)

An equivalence class (i.e. a *region*)

Regions

Finite partitioning of state space



Reset regions

An equivalence class (i.e. a *region*) r

Definition

$w \approx w'$ iff w and w' satisfy the exact same conditions of the form

$$x_i \le n \text{ and } x_i - x_j \le n$$

where $n \le \max$

THEOREM

Whenever $uv \approx u'v'$ then

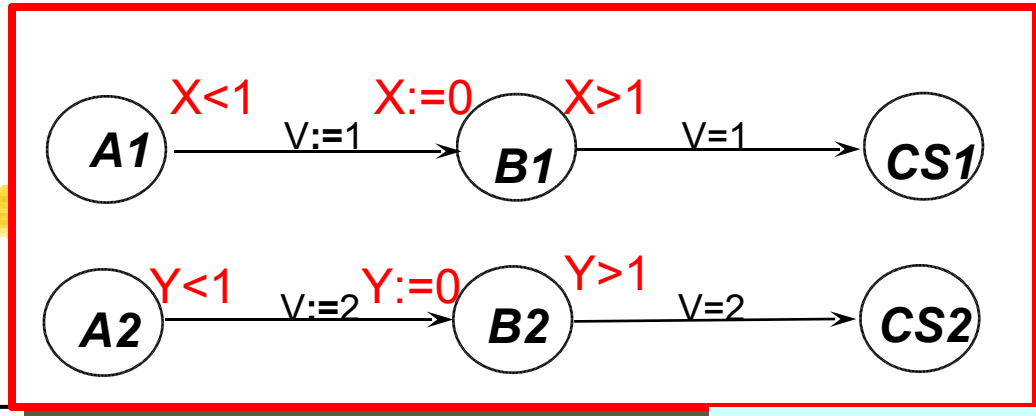
$$[(l,u),v] \text{ sat } \varphi$$

$$\Leftrightarrow$$

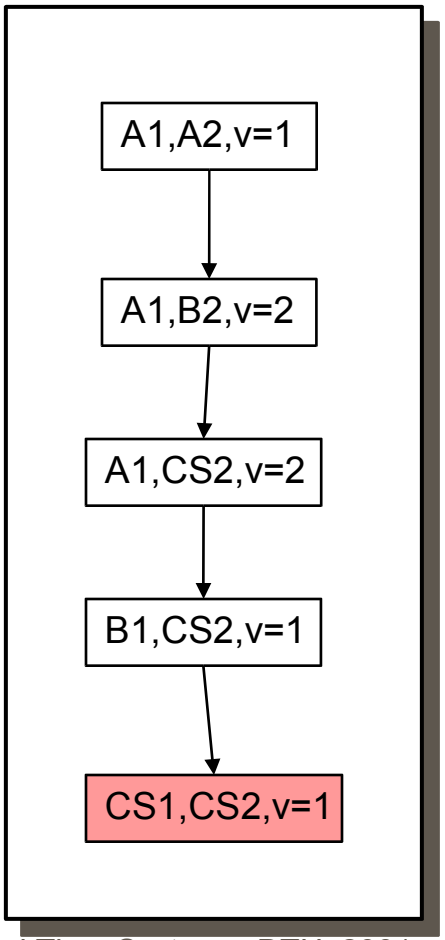
$$[(l,u'),v'] \text{ sat } \varphi$$

$$AG\left(\neg\left(CS_1 \wedge CS_2\right)\right)$$

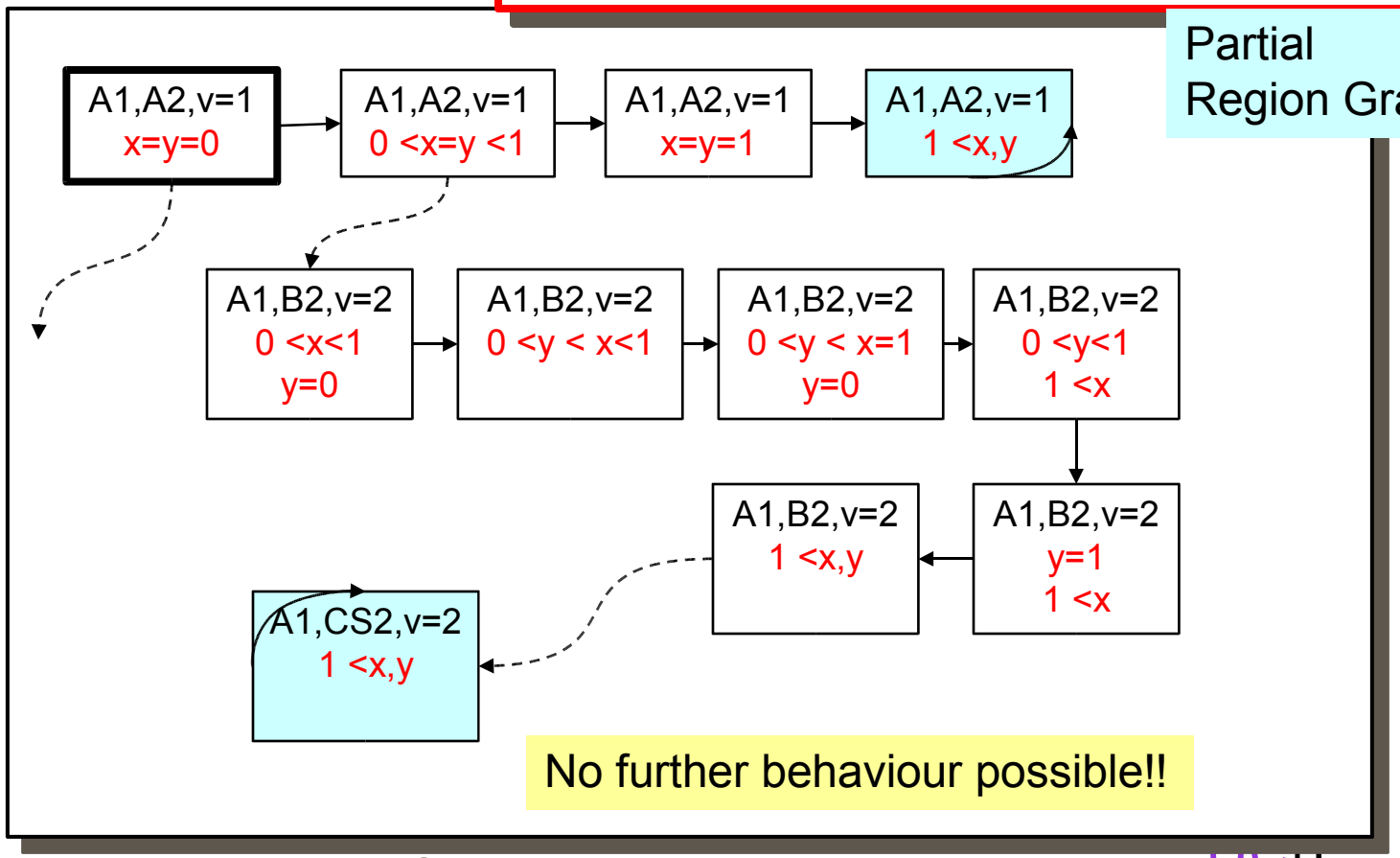
Fischers again



Untimed case



Timed case



Partial Region Graph

No further behaviour possible!!

Roughly speaking....

Model checking a timed automata against a TCTL-formula amounts to model checking its region graph against a CTL-formula

Problem to be solved

The worst-case time complexity of model checking TCTL*-formula* ϕ *over timed automaton* $\mathcal{A}$*, with the clock constraints of* ϕ *and* $\mathcal{A}$ *in* Ψ *is:*

$$\mathcal{O}\left(|\phi| \times (n! \times 2^n \times \prod_{x \in \Psi} c_x \times |L|^2)\right).$$

☺ (i) linear in the length of the formula ϕ

😐 (ii) exponential in the number of clocks in $\mathcal{A}$ and ϕ

☹ (iii) exponential in the maximal constants with which clocks are compared in $\mathcal{A}$ and ϕ.

Model Checking TCTL is PSPACE-hard

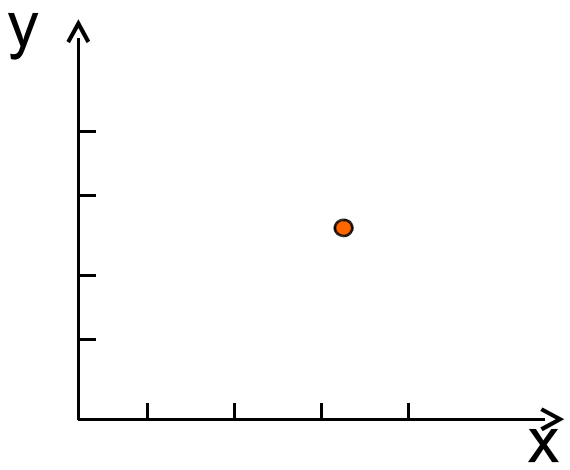
THE UPPAAL ENGINE

Reachability & Zones

Property and system dependent partitioning

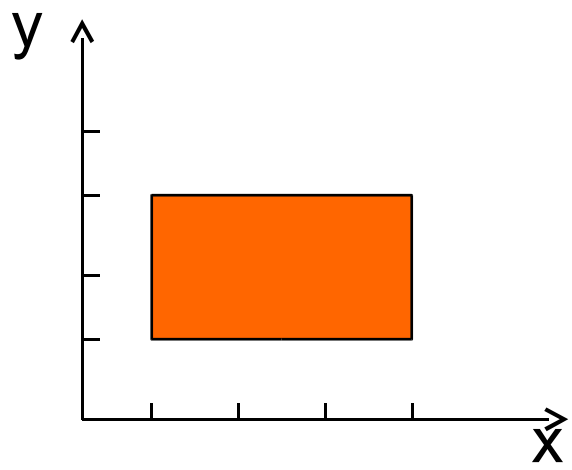
Zones
From infinite to finite

State
(n, x=3.2, y=2.5)

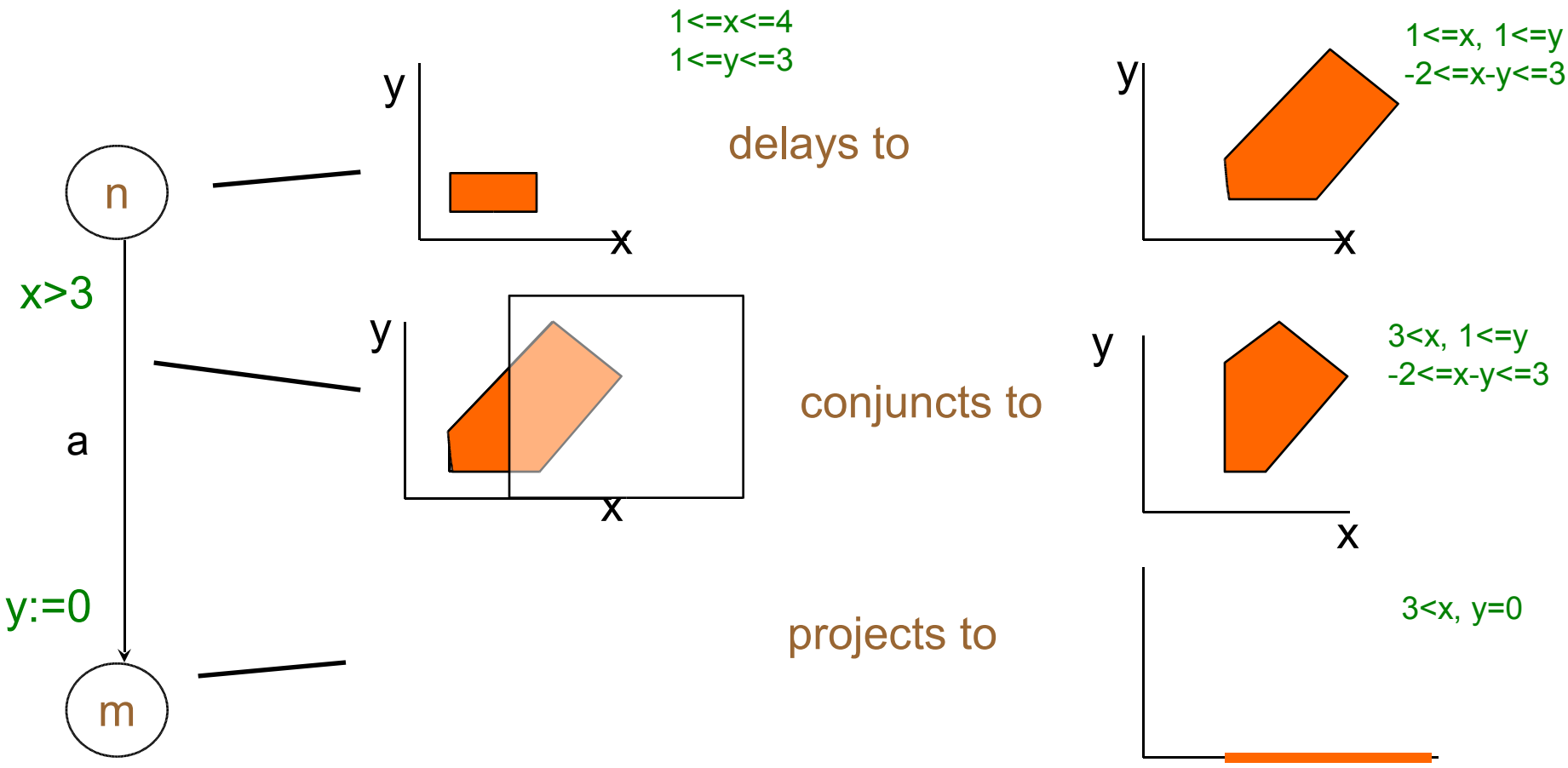


Symbolic state (set)
(n, $1 \le x \le 4, 1 \le y \le 3$)

Zone:
conjunction of
x-y<=n, x<=>n



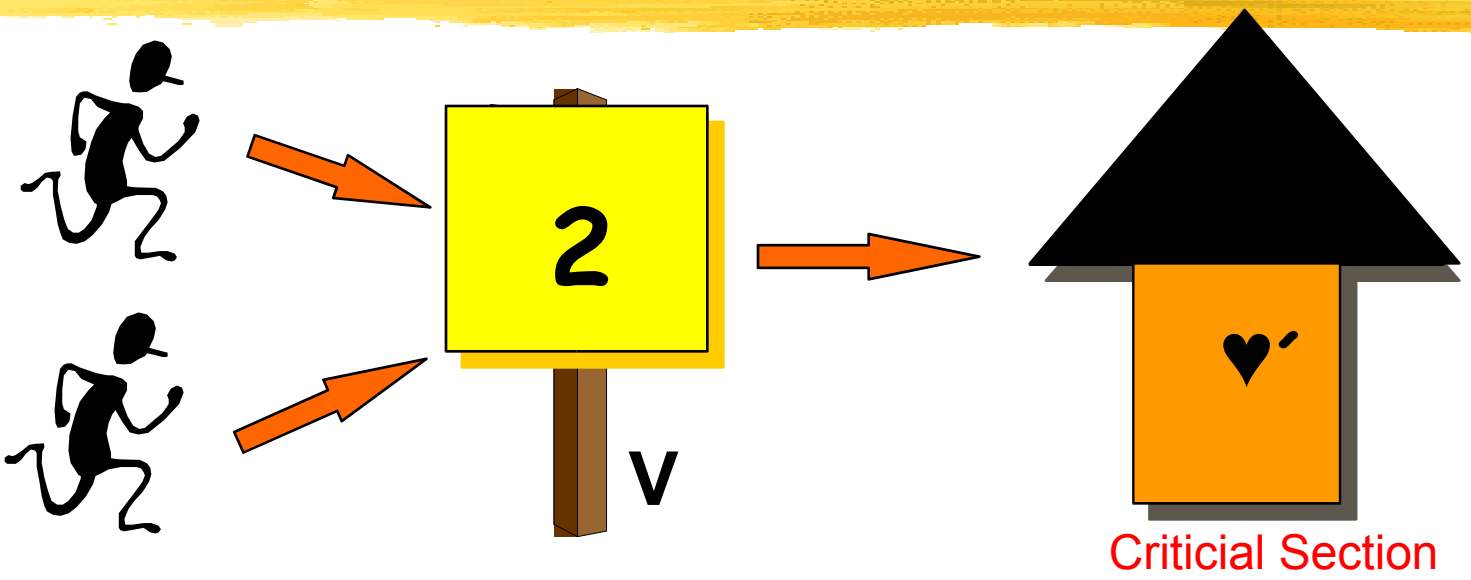
Symbolic Transitions

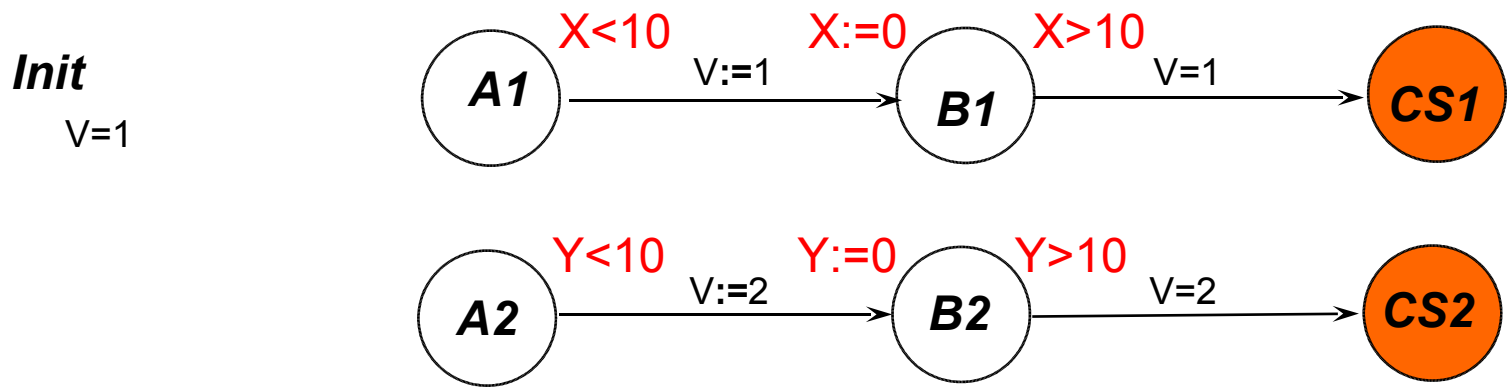


Thus (n,1<=x<=4,1<=y<=3) =a => (m,3<x, y=0)

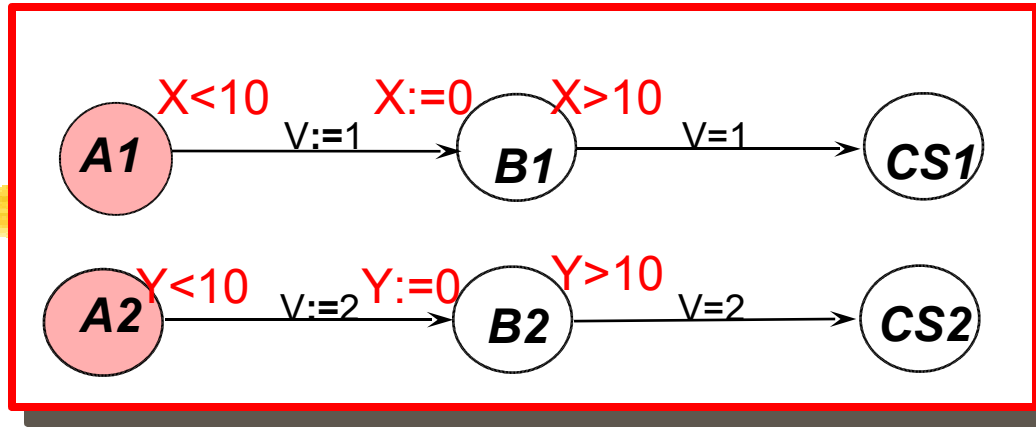
Fischer's Protocol

analysis using zones

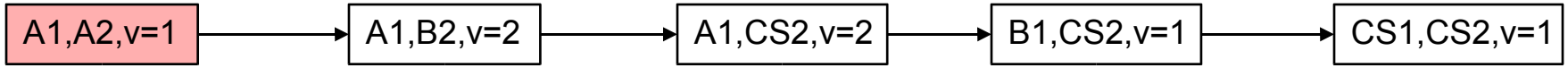




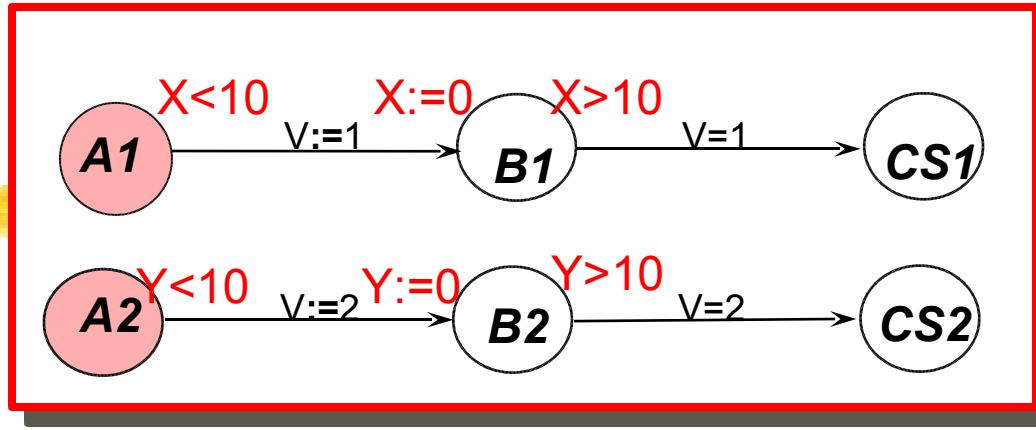
Fischers cont.



Untimed case



Fischers cont.



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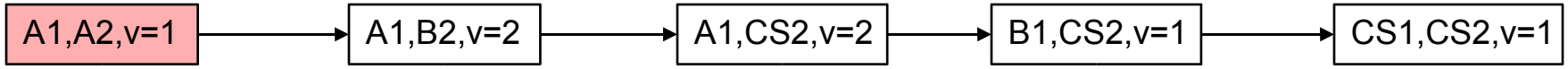
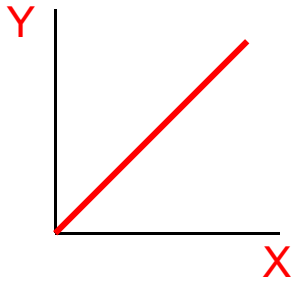
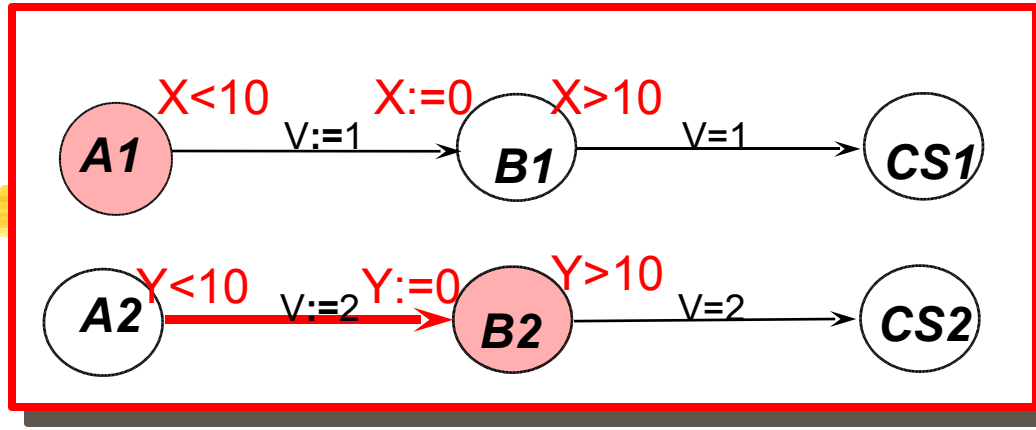


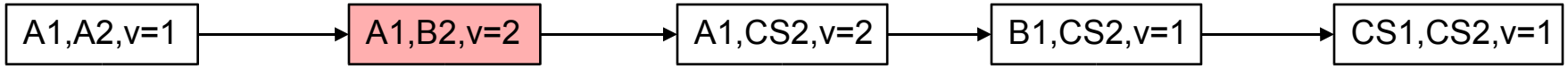
Taking time into account



Fischers cont.



Untimed case



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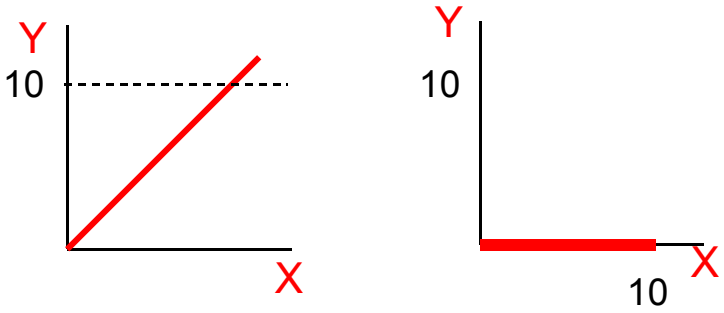

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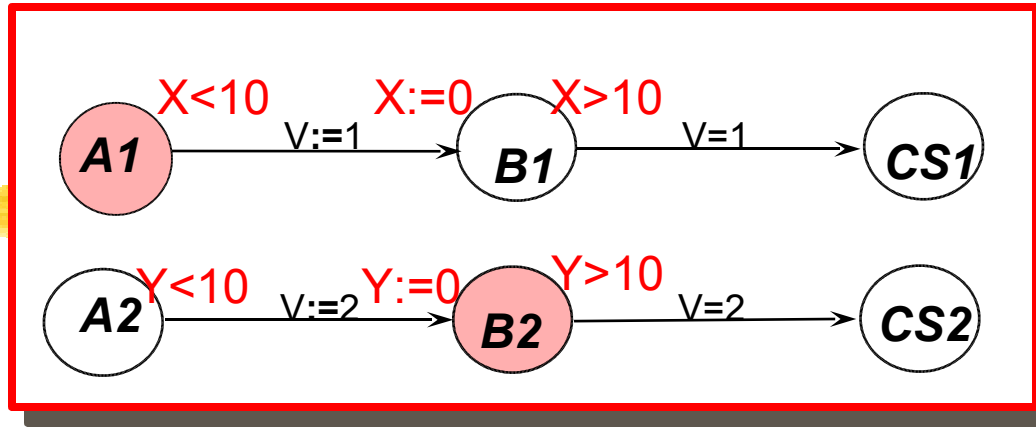

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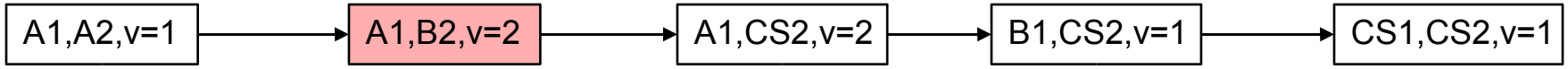

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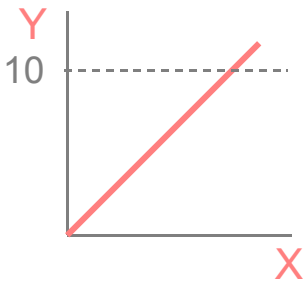

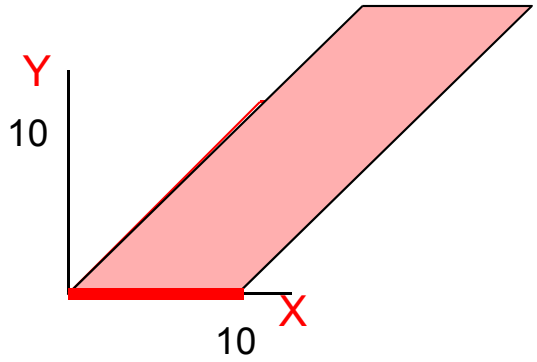

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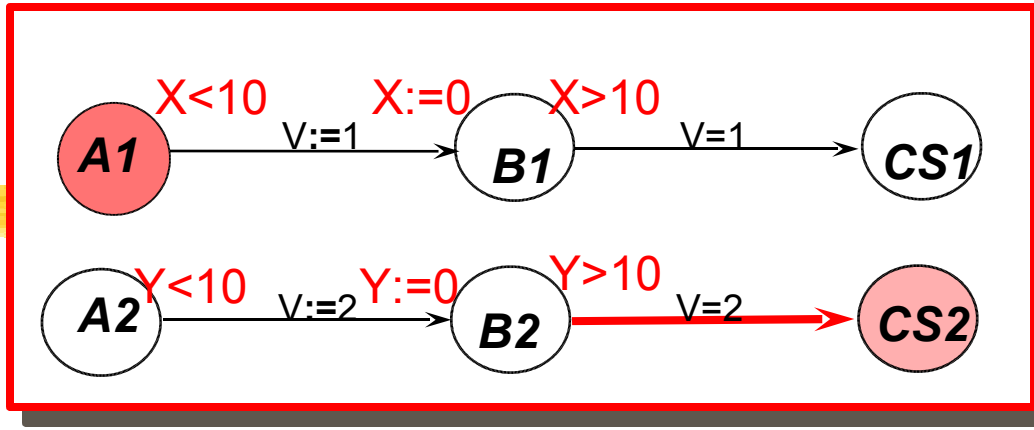
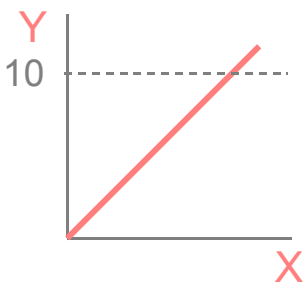
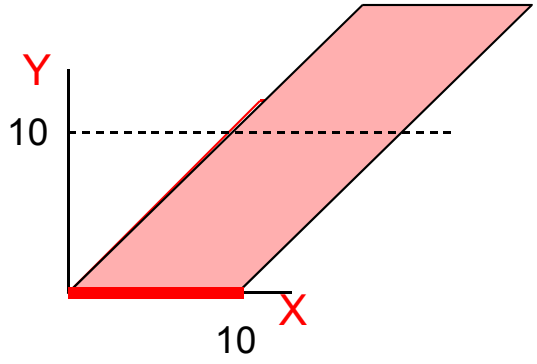


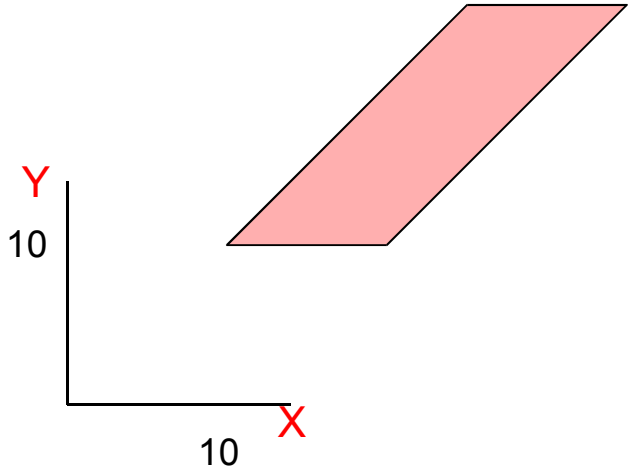
Untimed case

A1,A2,v=1 → A1,B2,v=2 → A1,CS2,v=2 → B1,CS2,v=1 → CS1,CS2,v=1

Taking time into account







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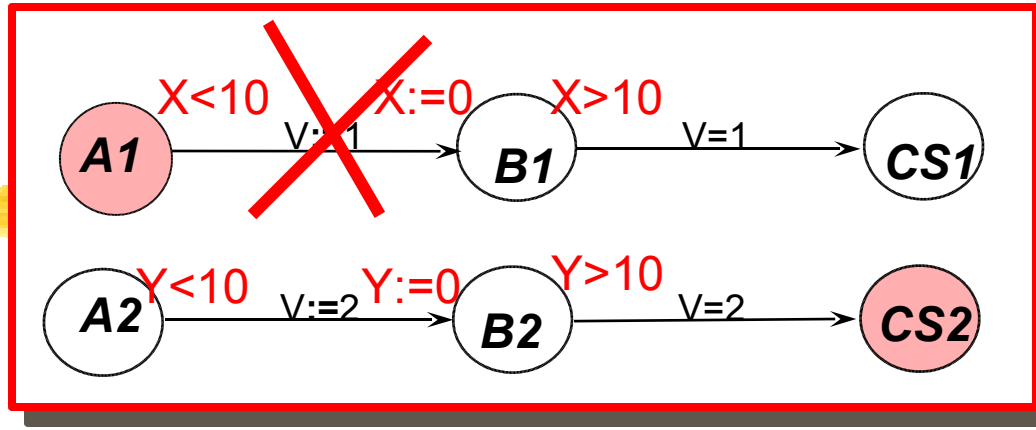

Untimed case

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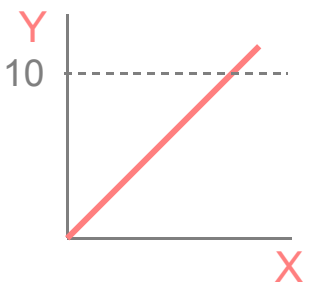
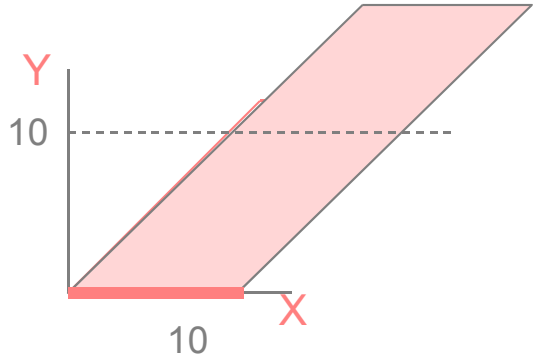
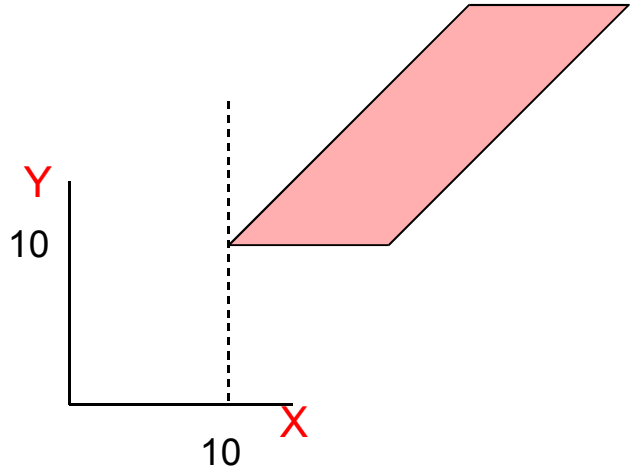
Taking time into account

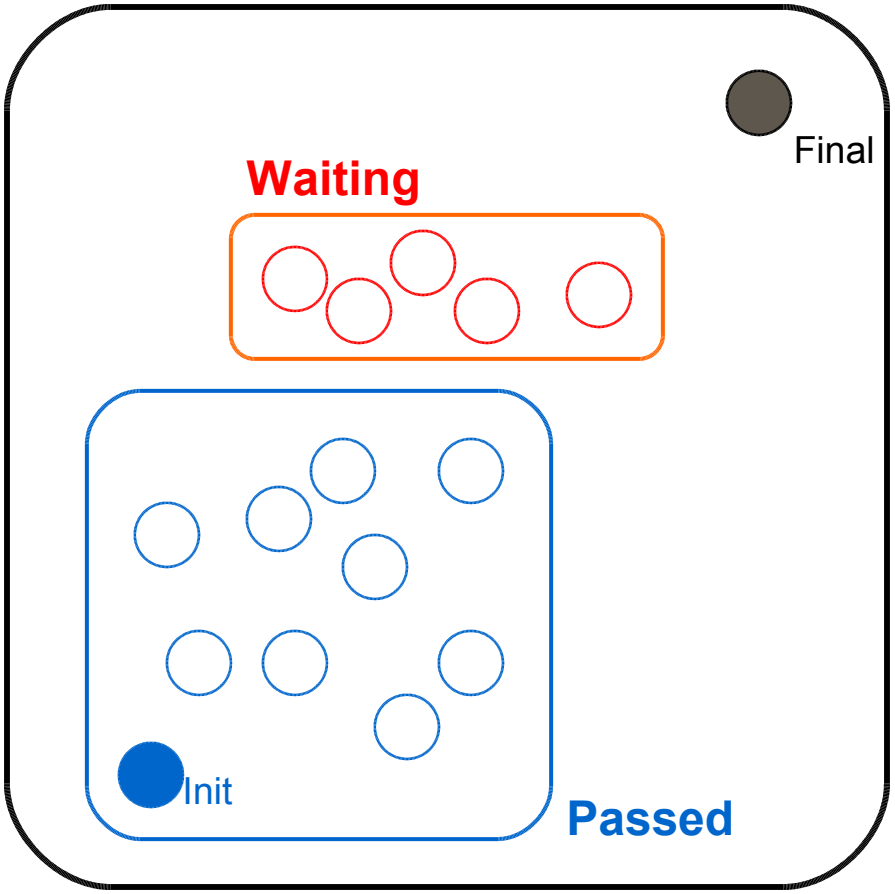






Forward Rechability

Init -> Final ?



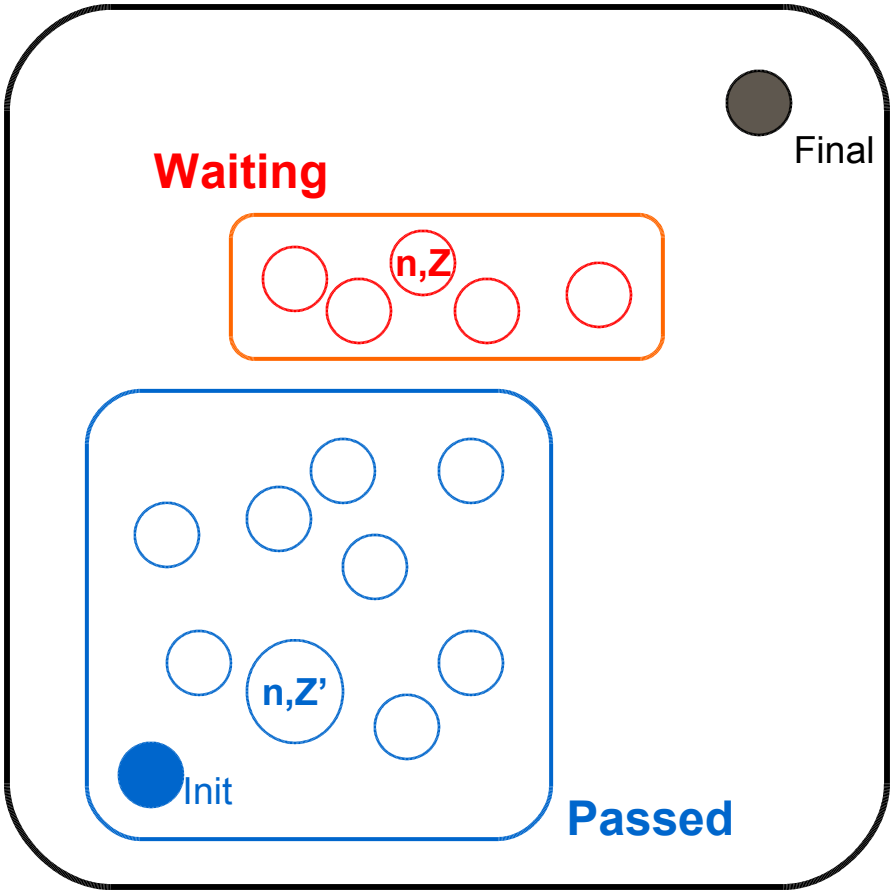
INITIAL **Passed** := Ø;
Waiting := {(n0,Z0)}

REPEAT

UNTIL **Waiting** = Ø
or
Final is in **Waiting**

Forward Rechability

Init -> Final ?



INITIAL **Passed** := Ø;
Waiting := {(n0,Z0)}

REPEAT
- pick (n,Z) in **Waiting**
- **if** for some Z' ¿ Z
(n,Z') in **Passed** **then** **STOP**

UNTIL **Waiting** = Ø
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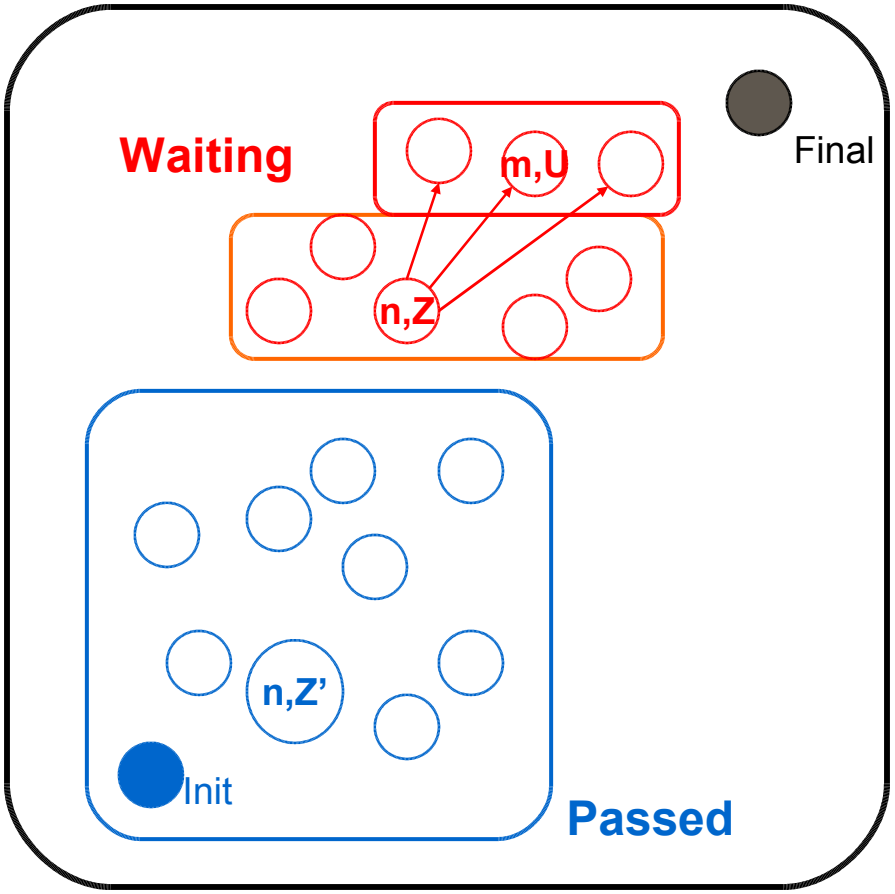
Forward Rechability

Init -> Final ?



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  { (m,U) : (n,Z) => (m,U) }
  to **Waiting**;

UNTIL **Waiting** = Ø
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Forward Rechability

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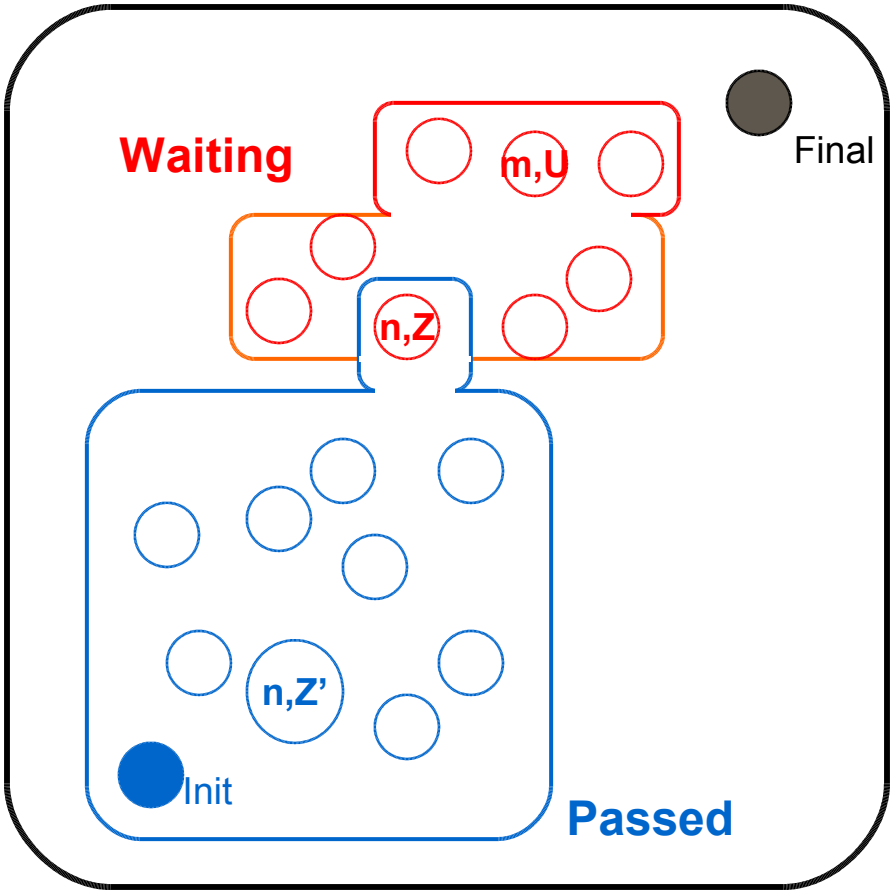

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  { (m,U) : (n,Z) => (m,U) }
  to **Waiting**;
  Add (n,Z) to **Passed**

UNTIL **Waiting** = Ø
or
Final is in **Waiting**

Canonical Dastructures for Zones

Difference Bounded Matrices

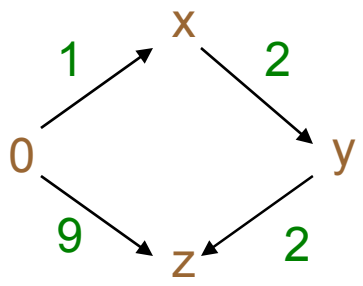
Bellman 1958, Dill 1989

Inclusion

D1

x<=1
y-x<=2
z-y<=2
z<=9

Graph

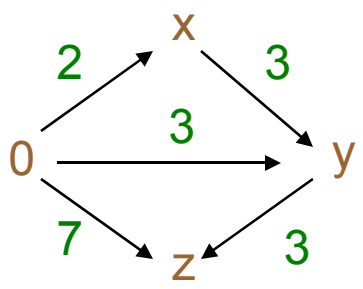


? ¿ ?

D2

x<=2
y-x<=3
y<=3
z-y<=3
z<=7

Graph



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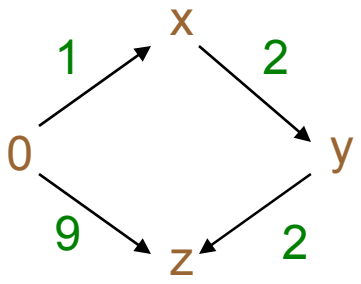
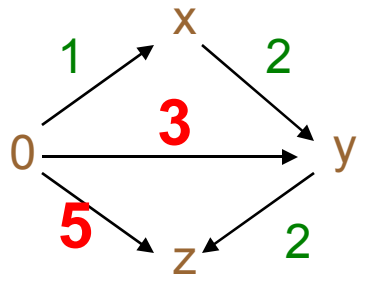


Shortest Path Closure



? ¿ ?

D2

x<=2
y-x<=3
y<=3
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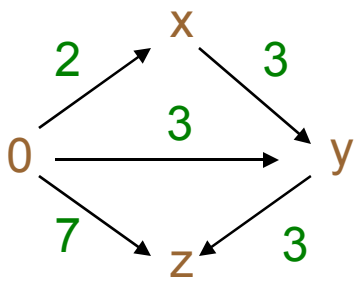
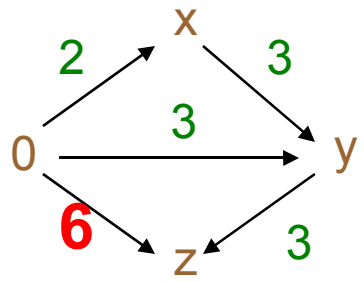
z<=7

Graph



Shortest Path Closure



Canonical Dastructures for Zones

Difference Bounded Matrices

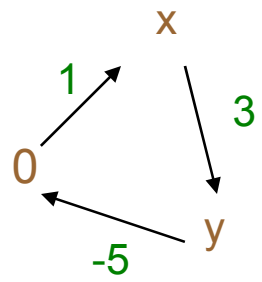
Bellman 1958, Dill 1989

Emptyness

D

x<=1
y<= -5
y-x<=3

Graph

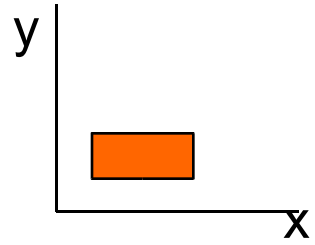


Negative Cycle
iff
empty solution set

Canonical Dastructures for Zones

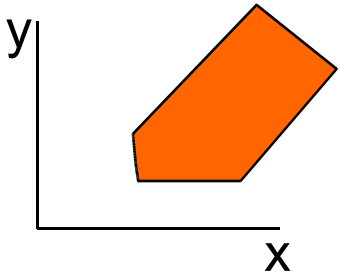
Difference Bounded Matrices

Future



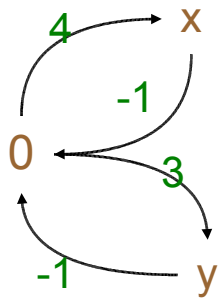
D

1<= x <=4
1<= y <=3

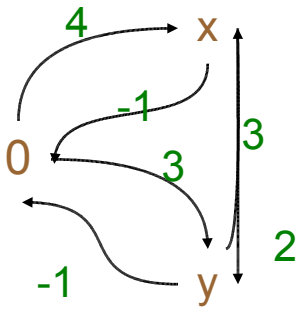


Future D

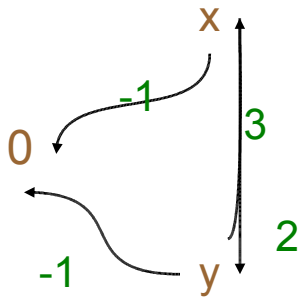
1<=x, 1<=y
-2<=x-y<=3



Shortest Path Closure



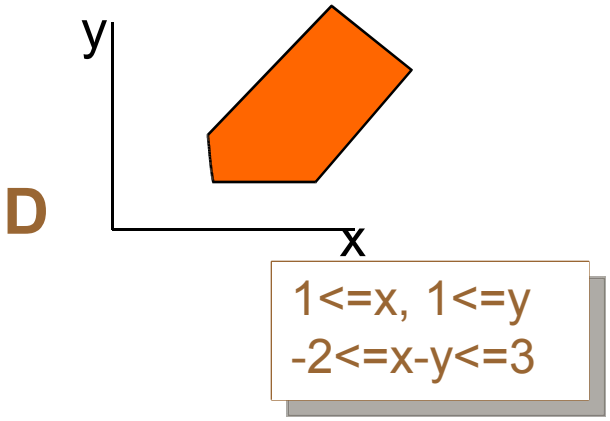
Remove upper bounds on clocks

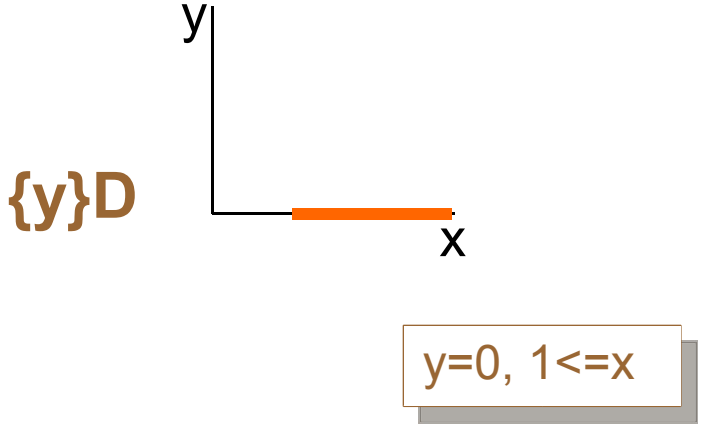


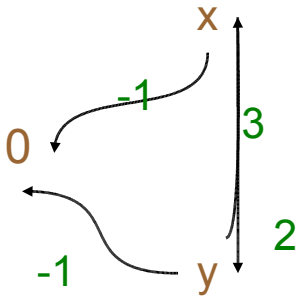
Canonical Dastructures for Zones

Difference Bounded Matrices

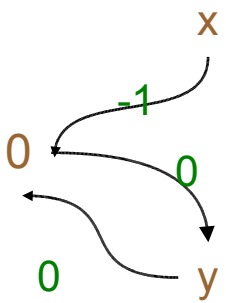
Reset







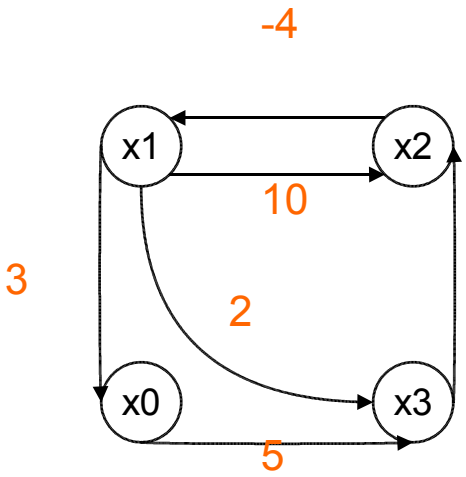
Remove all bounds involving y and set y to 0



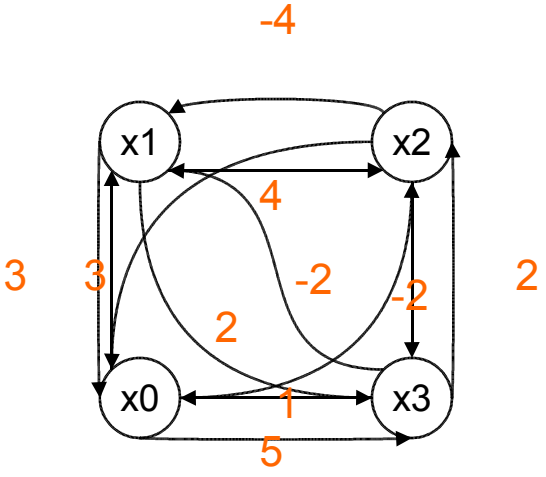
Improved Datastructures

Compact Datastructure for Zones

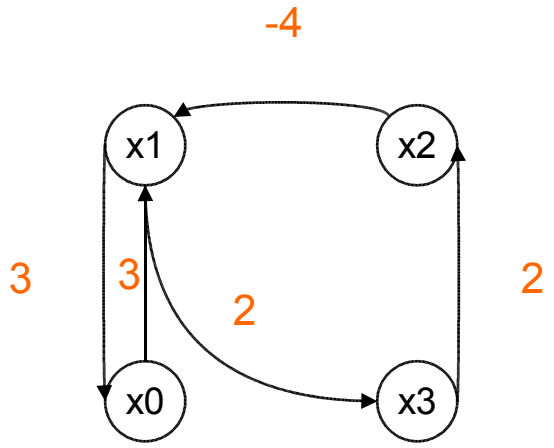
x1-x2<=4
x2-x1<=10
x3-x1<=2
x2-x3<=2
x0-x1<=3
x3-x0<=5



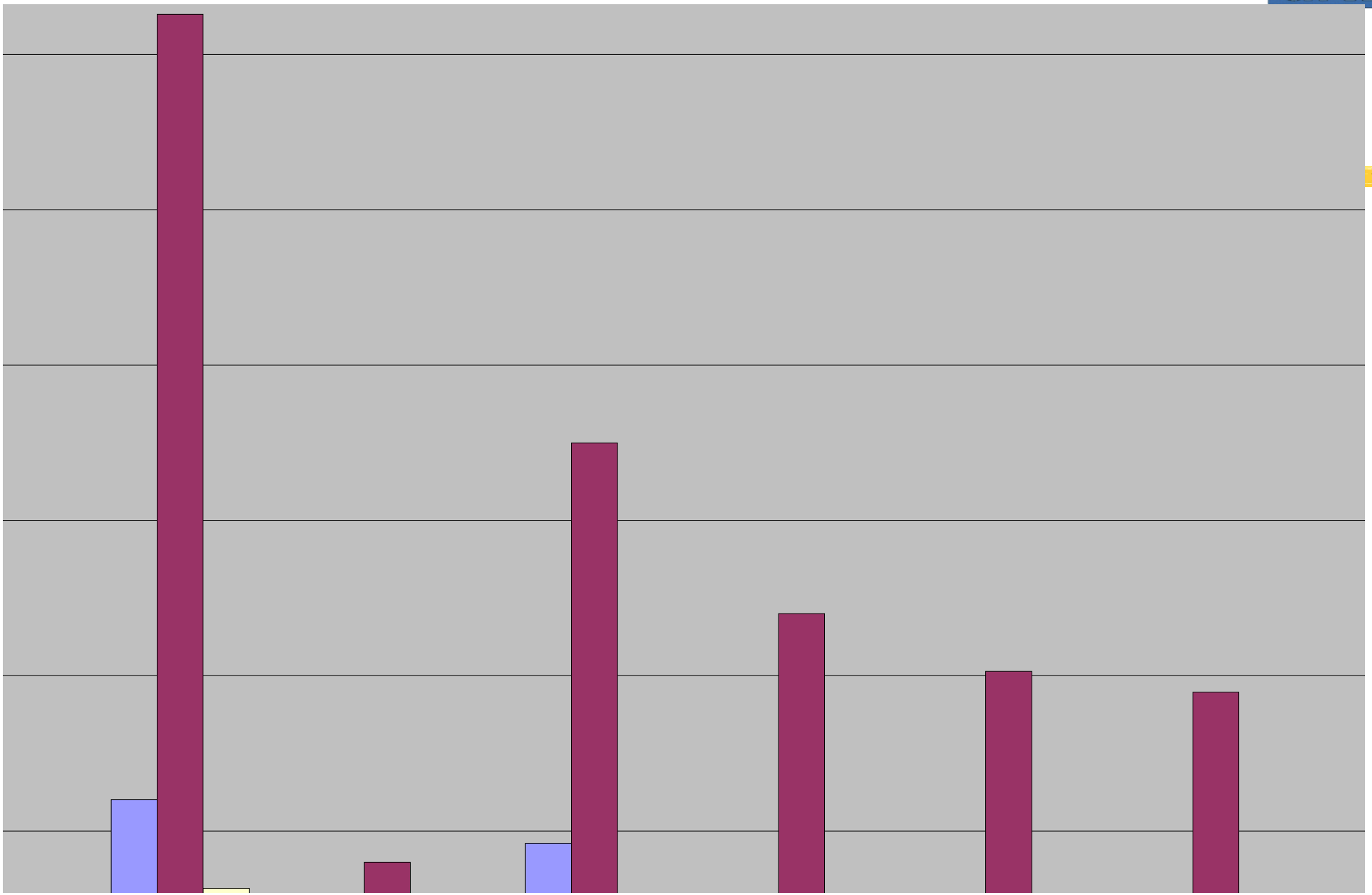
Shortest Path Closure O(n^3)

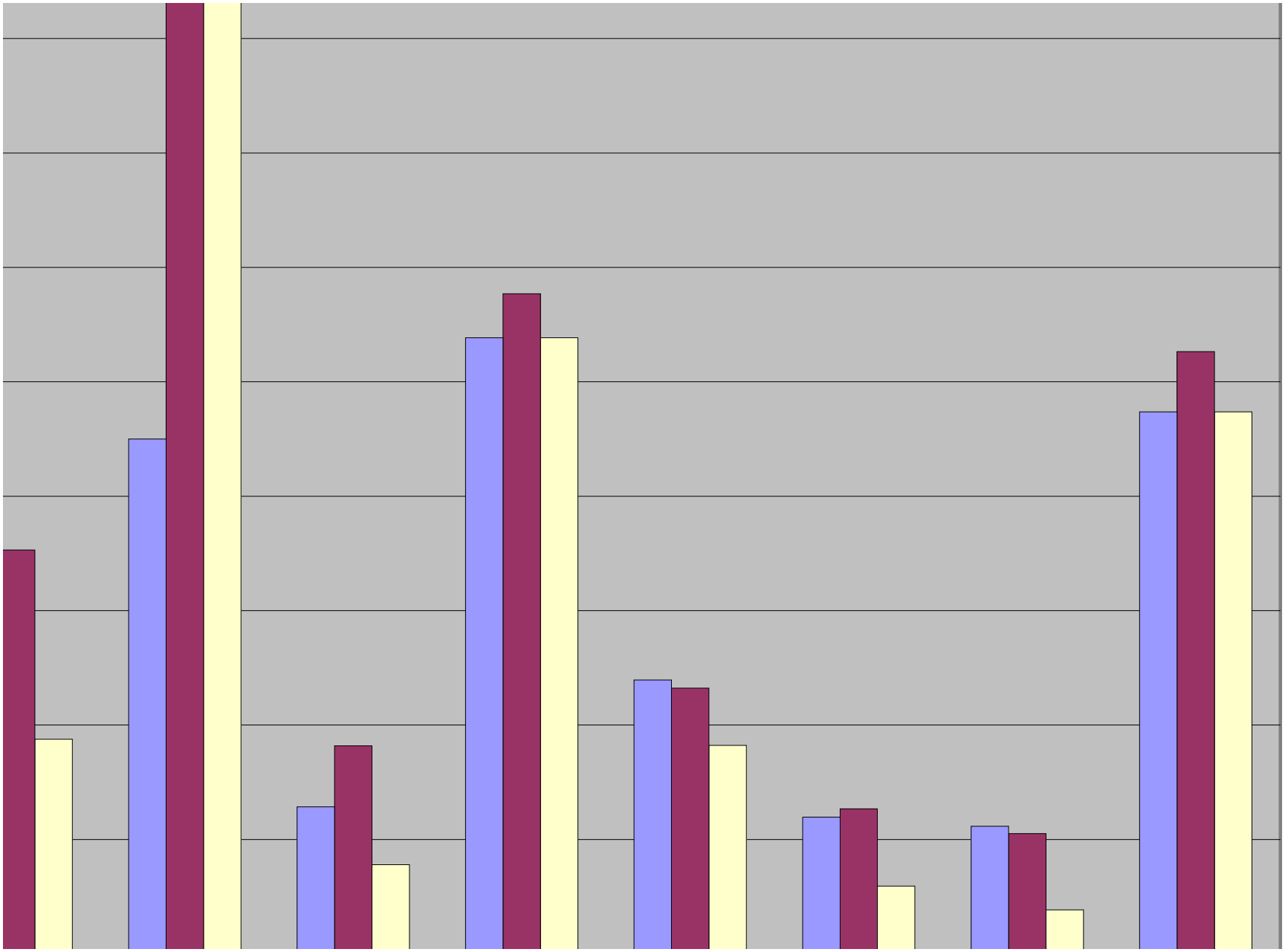


Shortest Path Reduction O(n^3)



Canonical wrt =
Space worst O(n^2)
practice O(n)





Beyond Reachability

-Bounded Liveness

-Abstraction & Simulation

Logical Formulas

Safety Properties:

F ::= A[ ] P | ——— Always P

E<> P ——— Possibly P

atomic properties

clock comparison

where P ::= Proc.l | x = n | v = n |

x<=n | x<n |

P and P | not P | P or P |

P imply P

Process Proc at location l

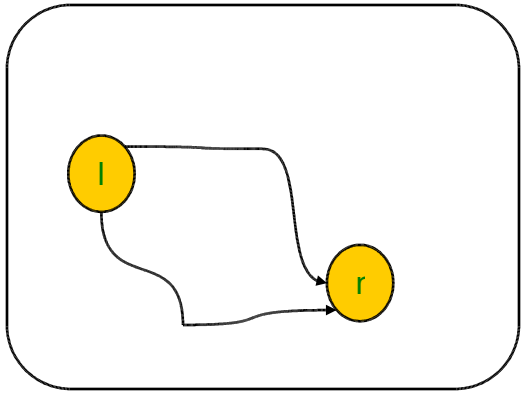
boolean combinations

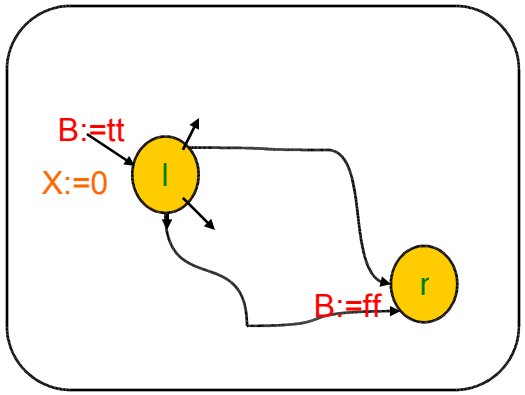
Beyound Safety

Decoration

$$\mathrm{AG}\,(a \Rightarrow \mathrm{AF}_{\leq t}\, b)$$

TACAS98a





Leadsto:
Whenever l is reached
then n is reached with t

Decoration
new clock **X**
boolean **B**

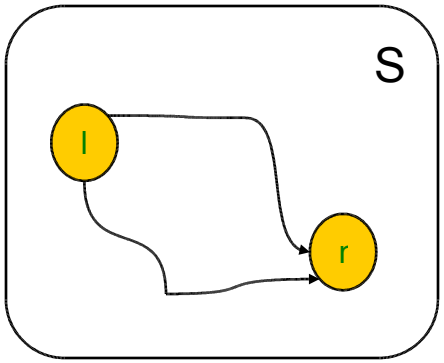
A[] (B implies x<=t)

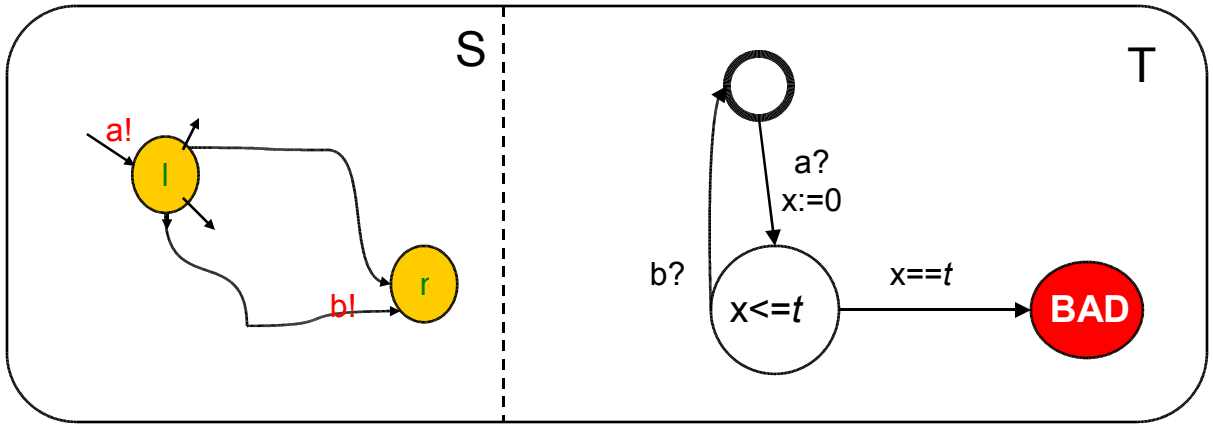
Beyond Safety

Test automata

$$AG\,(a \Rightarrow AF_{\leq t}\, b)$$

TACAS98b

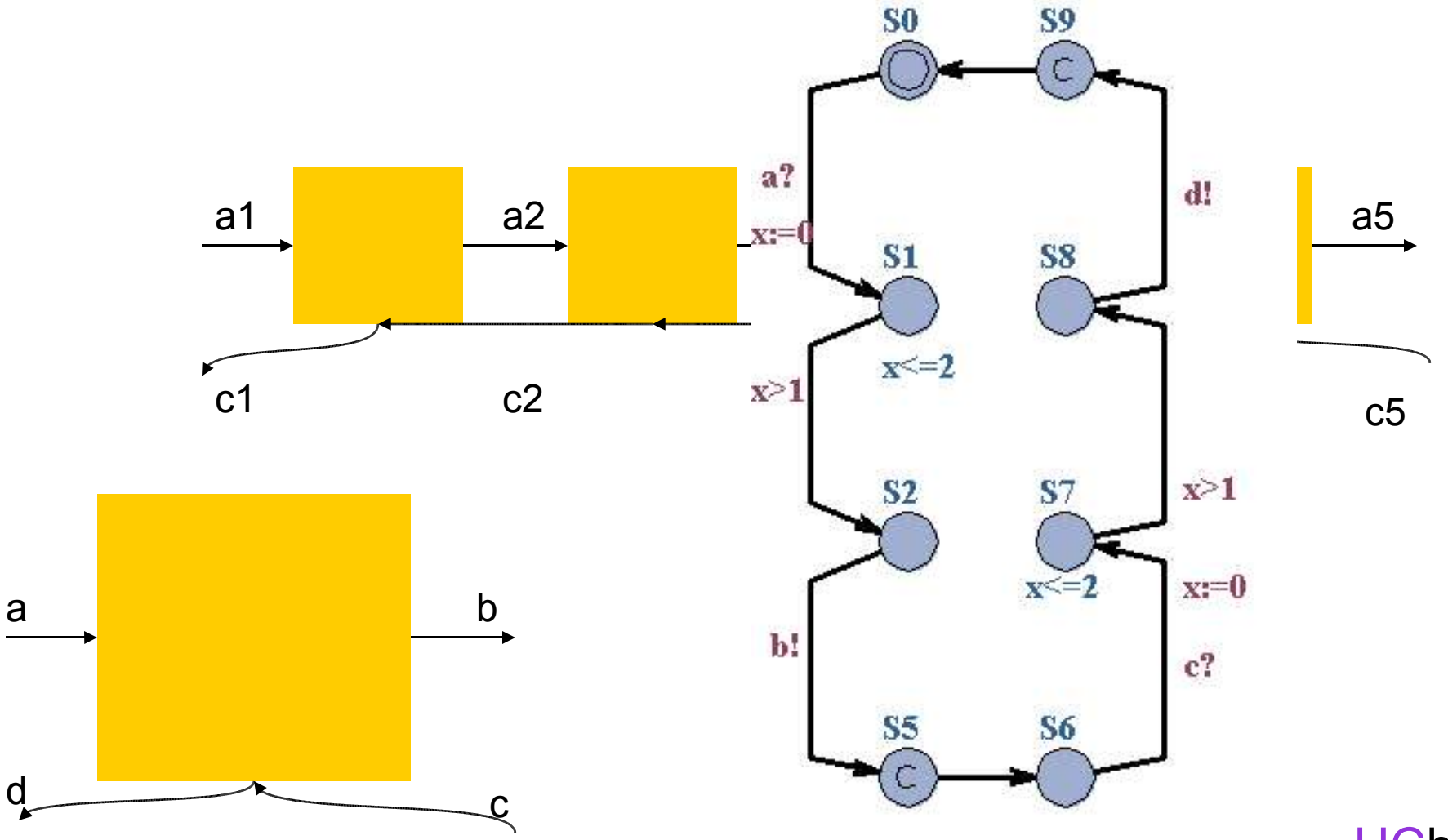




b urgent!

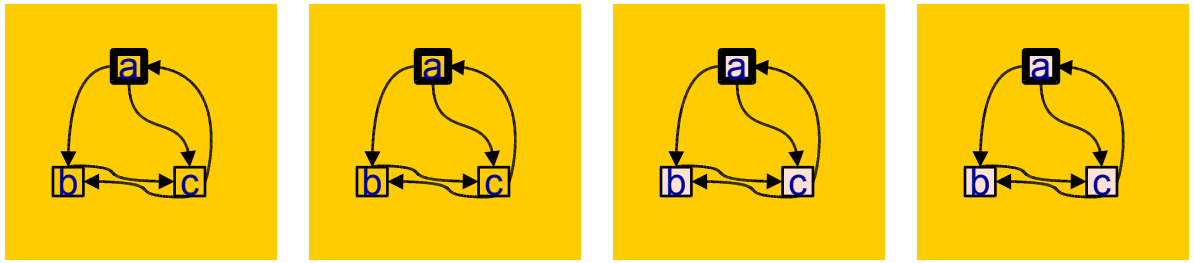
A[] (not T.BAD)

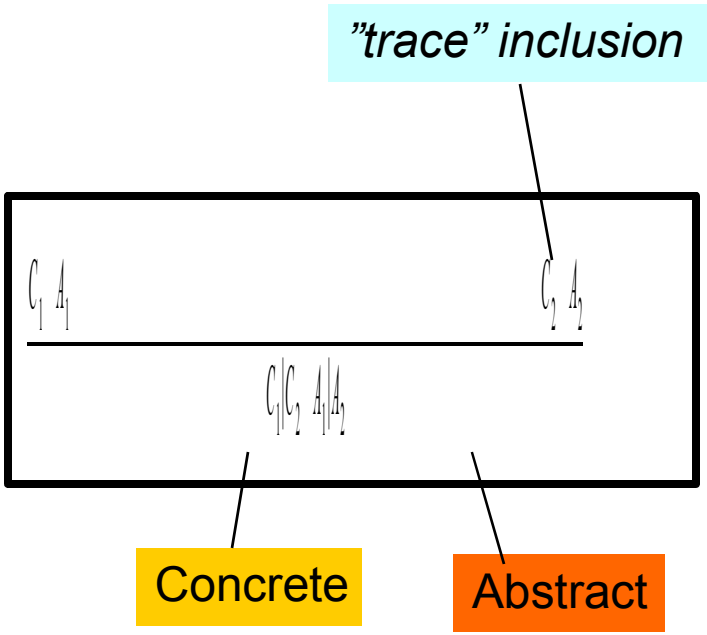
Example *bounded liveness*

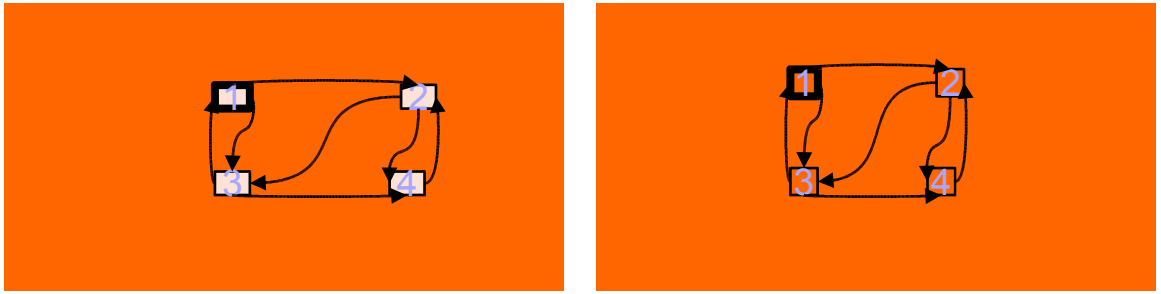


Abstraction & Compositionality

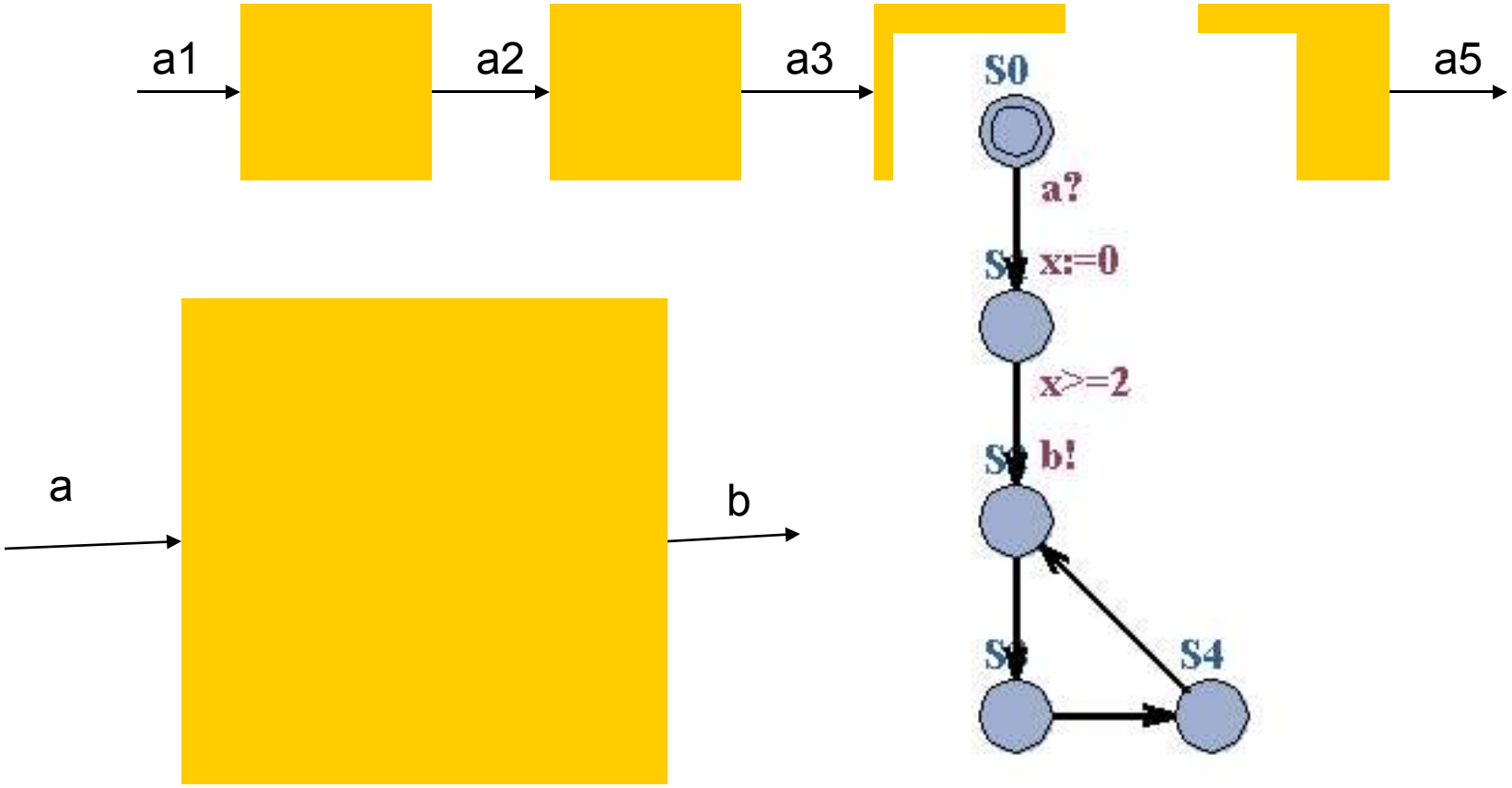
dealing w stateexplosion



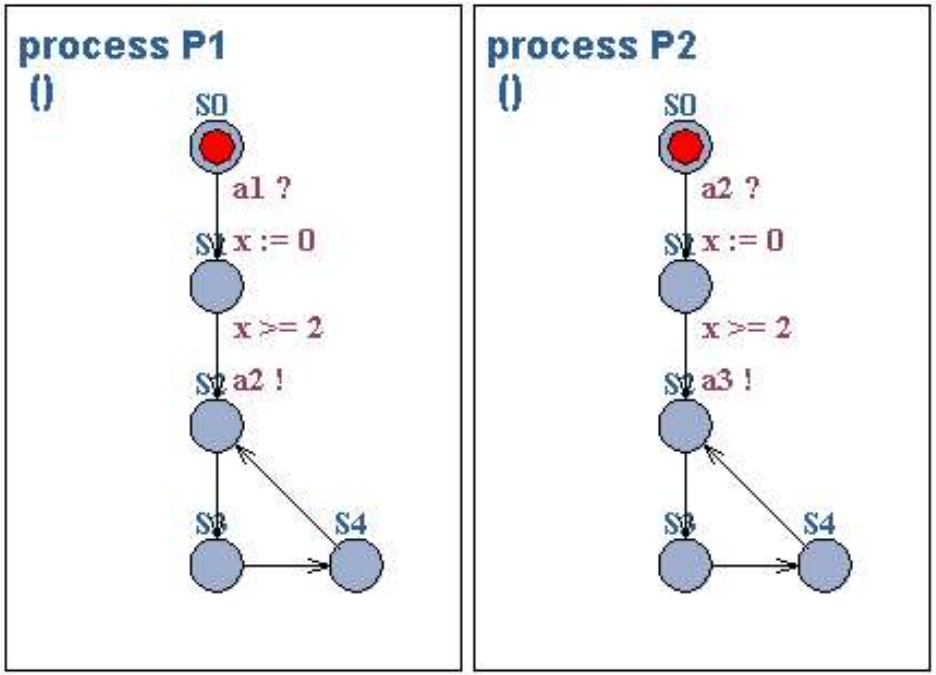




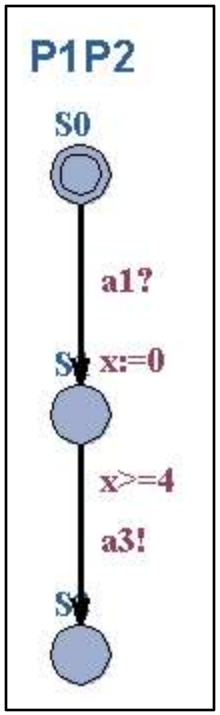
Abstraction *Example*



Example *Continued*

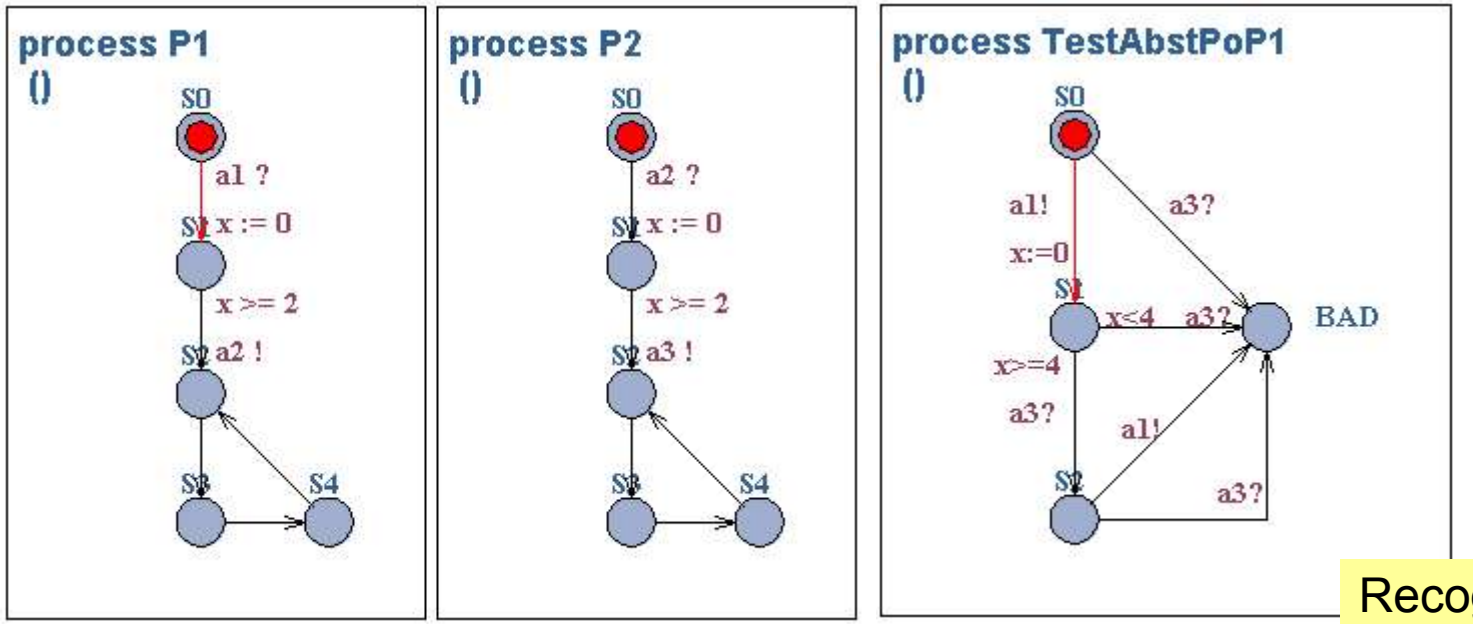


abstracted by



Proving abstractions

using reachability



Recognizes all the BAD computations of PoP1

A[] not TestAbstPoP1.BAD