# Towards Time-Discounted Influence Maximization

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## ABSTRACT

The classical influence maximization (IM) problem in social networks does not distinguish between whether a campaign gets viral in a week or in a year. From the practical standpoint, however, campaigns for a new technology or an upcoming movie must be spread as quickly as possible, otherwise they will be obsolete. To this end, we formulate and investigate the novel problem of maximizing the time-discounted influence spread in a social network, that is, the campaigner is interested in both "*when*" and "*how likely*" a user would be influenced. In particular, we assume that the campaigner has a utility function which monotonically decreases with the time required for a user to get influenced, since the activation of the seed nodes. The problem that we solve in this paper is to maximize the expected aggregated value of this utility function over all network users. This is a novel and relevant problem that, surprisingly, has not been studied before.

Time-discounted influence maximization (TDIM), being a generalization of the classical IM, still remains $\mathbf{NP}$-hard. However, our main contribution is to prove the sub-modularity of the objective function for *any* monotonically decreasing function of time, under a variety of influence cascading models, e.g., the independent cascade, linear threshold, and maximum influence arborescence models, thereby designing approximate algorithms with theoretical performance guarantees. We also illustrate that the existing optimization techniques (e.g., CELF) for influence maximization are more efficient over TDIM. Our experimental results demonstrate the effectiveness of our solutions over several baselines including the classical influence maximization algorithms.

## 1. INTRODUCTION

In influence maximization (IM), whenever a social network user buys a product or endorses an action (e.g., sharing photos, re-tweeting hash tags), she is viewed as being influenced or activated. The classical influence maximization problem [5, 10] identifies the top-$k$ seed users in a social network such that the expected number of influenced users in the network, starting from those seed users and following some influence cascading model, is maximized. The budget $k$ on the seed-set size usually depends on how many initial users the campaigner can directly influence to buy her product by advertisements, giving free samples, and discounted prices.



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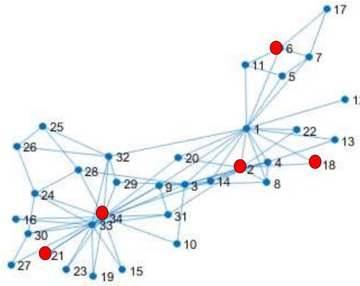
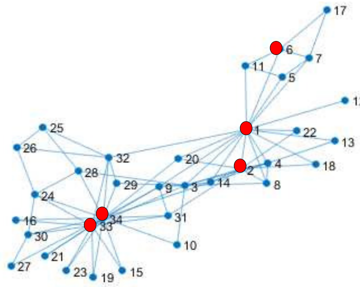



(a) IM




(b) TDIM

Figure 1: Top-5 Seeds (in Bold), *Zachary's Karate Club* Dataset

In their seminal paper [10], Kempe et al. left the time required for influence maximization unconstrained. However, in most cases, the campaigner is also interested in when a user would be influenced – the sooner, the better. For example, multiple competing companies launch comparable products around the same time (e.g., Nintendo's Wii vs. Sony's Playstation vs. Microsoft's X-Box; Microsoft's Surface vs. Apple's iPad vs. Samsung Note 3) [12]. Thus, a campaigner would like to inform the maximum number of people about her latest products as quickly as possible. In the domain of movies, the success of a film depends on public interests, and with the general attention span being far shorter, most films stay in theatres only in their first week or two [1].

In the aforementioned temporal context of information diffusion, we design a novel influence maximization problem, called the time-discounted influence maximization (TDIM). The campaigner provides a utility function $U\big(\delta t(u,S)\big)$ that monotonically decreases with the time $\delta t(u,S)$ required to influence a user $u$, since the activation of the seed set $S$. As information diffusion is a probabilistic process, we aim at finding the optimal seed set that maximizes the campaigner's aggregated expected utility considering all users.

[1] http://www.boxofficemojo.com/yearly/chart/?yr=2015

TDIM is a non-trivial problem because $U$ can be *any* arbitrary monotonically decreasing function; and therefore, the top-$k$ seed set could be very different than those for the classical influence maximization. We illustrate this with the well-known Zachary's karate club dataset [15], which is a karate club social network of 34 members, documenting 78 pairwise links between these members who interacted outside the club from 1970 to 1972. Edge probabilities are assigned in proportion to the number of interactions, and we consider the IC model of influence cascade. In Figure 1, we show the top-5 seed nodes both for IM and TDIM, with a utility function exponentially decreasing over time. As one may observe, the seed nodes are more centrally located (i.e., closeness centrality) in case of TDIM, which facilitates a rapid dissemination of information to a large number of users. Clearly, this is the problem of interest for most practical scenarios.

**Related Research and Novelty of Our Work.** The temporal aspect of information diffusion has been explored in statistical physics [9]. Prior works [2, 6] considered influence maximization within a given time deadline, i.e., a finite time window must be considered. Our work TDIM is different because the campaigner assigns a utility score based on when a node gets influenced – the earlier, the better. This is often more useful than prior works where every node activation is treated equally if that happens within the specified time window. In fact, TDIM is a generalization of these prior works, because the time deadline can be simulated by properly selecting the utility function. A few other works [7, 13] aimed at finding the seed set such that a predefined coverage is achieved in the minimum time. Recently, Chang et. al. studied influence sustainability [1], that is, to maximize the count of those time steps when more than a threshold number of new users get activated. Clearly, the notion of influence sustainability is also different from ours.

# 2. PRELIMINARIES

A social network $\mathcal{G} = (V, E, P)$ consists of a set of $n$ nodes $V$, $E \subseteq V \times V$ is a set of $e$ directed edges, and $P : E \rightarrow (0, 1)$ is a probability function that assigns a probability to each edge. The probability $p_{uv}$ on a directed edge $(u, v) \in E$ represents the probability that node $v$ adopts a product due to the influence of node $u$, because $u$ adopted that product before. When $v$ adopts the product, it automatically becomes eligible to influence its neighbors. We shall discuss various influence cascading models in Section 2.1.

We denote by $S$ the seed set for the campaigner. As influence cascade is a probabilistic process, let us denote by $\Delta T(u, S)$ the random variable corresponding to the time required for influencing a user $u$, since the activation of the seed set $S$. Let $\delta t(u, S)$ be a realization of $\Delta T(u, S)$. Next, we assume that the campaigner has a utility function $U\big(\delta t(u, S)\big)$, where $U$ can be *any* monotonically decreasing function of $\delta t$. We are now ready to define our problem.

PROBLEM 1 (TDIM). *Given a network $\mathcal{G} = (V, E, P)$, a budget on the seed set size $k$, and the campaigner's utility function $U$ which decreases monotonically with time, find the seed set $S$ of size $k$ such that the campaigner's expected aggregated utility considering all network users is maximized. Formally,*

$$\arg\max_{S} \sum_{u \in V} \mathbb{E}[U(\Delta T(u, S))] \quad \textit{such that} \quad |S| = k \qquad (1)$$

We denote by $\mathbb{E}$ the expectation in the above equation. TDIM, being a generalization of the influence maximization (IM) problem, is also $\mathbf{NP}$-hard. Therefore, the question that remains is whether our new objective function in Equation 1 still retains some of the other properties, such as monotonicity and sub-modularity, of the classical IM. We shall discuss them in the context of three widely-used influence diffusion models – independent cascade (IC) [10], linear threshold (LT) [10], and maximum influence arborescence (MIA) [3], which are introduced below.

## 2.1 Influence Diffusion Models

In all our models, the campaign spreads from an initially active set of seed nodes in discrete steps. By following [7, 10], we consider one time step as one attempt of a node to influence its users.

**Independent Cascade Model.** In the IC model, when some node $u$ first becomes active at step $t$, it gets a single chance to activate each of its currently inactive out-neighbors $v$; it succeeds with probability $p_{u,v}$. If $u$ succeeds, then $v$ will become active at step $t + 1$. Whether or not $u$ succeeds at step $t$, it cannot make any further attempts in the subsequent rounds. If a node $v$ has incoming edges from multiple newly activated nodes, their attempts are sequenced in an arbitrary order. Also, each node can be activated only once and it stays active until the end. The campaigning process runs until no more activations are possible.

**Linear Threshold Model.** In the LT model, each node $v$ has an activation threshold $\theta_v$, selected uniformly from $(0, 1)$. In addition, there is a constraint that the sum of the probabilities of all incoming edges for every node must be at most 1. If the sum of the probabilities of the incoming edges from all active nodes is greater than or equal to the activation threshold of an inactive node, then the node gets activated in the next round. Each node can only be activated once and stays active until the end.

**Maximum Influence Arborescence.** This model assumes that the influence from the seed nodes propagates only via the *maximum influence paths*. A path from a source to a destination node is called the maximum influence path if this has the highest probability compared to all other paths between the same pair of nodes. Ties are broken in a predetermined and consistent way, such that the maximum influence path between a pair of nodes is always unique.

## 2.2 Properties of TDIM

Let us denote $F(S) = \sum_{u \in V} \mathbb{E}[U(\Delta T(u, S))]$. A function $F$ is sub-modular if it satisfies the following. $F(S_1 \cup \{v\}) - F(S_1) \geq F(S_2 \cup \{v\}) - F(S_2)$, for all elements $v$ and all pairs of sets $S_1 \subseteq S_2$. We prove sub-modularity and monotonicity of our objective function $F(S)$ below.

THEOREM 1. TDIM *objective function is sub-modular under the* IC *model.*

PROOF. Our proof follows by constructing an equivalent view of the IC model with the notion of *possible worlds* [10]. Each possible world is a certain instance of the uncertain graph, and obtained by independent sampling of the edges. Every possible world $X$ is associated with a probability of existence $prob(X)$. Let us consider the campaigner's aggregated utility in the possible world $X$, i.e., $F_X(S) = \sum_{u \in V} U(\delta t_X(u, S))$. The following holds.

$$F(S) = \sum_{\text{all possible world } X} [F_X(S) \times prob(X)] \qquad (2)$$

Our proof follows by showing that $F_X(S)$ is sub-modular. As the non-negative linear combination of sub-modular functions is also sub-modular, then $F(S)$ would be sub-modular. However, in an analogous argument, $F_X(S)$ is the summation of $U(\delta t_X(u, S))$ for all $u \in V$. Therefore, we show the sub-modularity of $U(\delta t_X(u, S))$ in order to illustrate the sub-modularity of $F(S)$. In other words, for all $S_1 \subseteq S_2$, we prove the following.

$$U(\delta t_X(u, S_1 \cup \{v\})) - U(\delta t_X(u, S_1)) \geq U(\delta t_X(u, S_2 \cup \{v\})) - U(\delta t_X(u, S_2)) \qquad (3)$$

Note that $X$ is a deterministic graph and the IC model follows discrete time steps. Therefore, $\delta t_X(u, S)$ can be measured as the shortest path distance in $X$ from any seed node in $S$ to $u$. Formally, $\delta t_X(u, S) = \min_{s \in S} l_X(s, u)$, where $l_X(s, u)$ is the shortest path distance from $s$ to $u$ in $X$. Now consider two distinct cases.

**Case 1.** $\delta t_X(u, S_2) > \delta t_X(u, \{v\})$. It immediately follows that $\delta t_X(u, S_1) > \delta t_X(u, \{v\})$. This is because $S_1 \subseteq S_2$. Hence, the left-hand and right-hand sides of Equation 3 become $U(\delta t_X(u, \{v\})) - U(\delta t_X(u, S_1))$ and $U(\delta t_X(u, \{v\})) - U(\delta t_X(u, S_2))$, respectively. Next, one can verify that $\delta t_X(u, S_1) \geq \delta t_X(u, S_2)$, and $U(\delta t_X(u, S_1)) \leq U(\delta t_X(u, S_2))$. This is because $U$ is monotonically decreasing. Therefore, Equation 3 holds.

**Case 2.** $\delta t_X(u, S_2) \leq \delta t_X(u, \{v\})$. In this case, the right-hand side of Equation 3 becomes zero. However, the left-hand side is always no less than zero, because $\delta t_X(u, S_1 \cup \{v\}) \leq \delta t_X(u, S_1)$. Since $U$ is monotonically decreasing, $U(\delta t_X(u, S_1 \cup \{v\})) \geq U(\delta t_X(u, S_1))$. Therefore, Equation 3 also holds in this case. This completes the proof. □

THEOREM 2. TDIM *objective function is sub-modular under the* LT *model.*

PROOF. The proof follows by considering the *live-edge* model, which is shown to be equivalent to the LT model in [10]. In the live-edge model, each node $v$ picks at most one of its incoming edges at random, that is, it selects the incoming edge from $u$ with probability $p_{u,v}$, and it does not select any incoming edge with probability $1 - \sum_{u \in V} p_{u,v}$. Let $X$ be one possible world with probability $Prob(X)$ under the live-edge model, and $F_X(S)$ be the campaigner's aggregated utility in $X$. Analogous to the proof of Theorem 1, one can show that $F_X(S)$ is sub-modular. Now, our objective function $F(S)$ can be expressed as follows.

$$F(S) = \sum_{\text{all possible world } X} [F_X(S) \times prob(X)] \tag{4}$$

As the non-negative linear combination of sub-modular functions is sub-modular, $F(S)$ is sub-modular. □

It is easy to verify that $F(S)$ is monotonic under the IC and the LT model. Below, we prove its monotonicity and sub-modularity under the MIA model, which are not immediately realized.

THEOREM 3. TDIM *objective function is monotonic under the* MIA *model.*

PROOF. Let us denote by $F_u(S) = \mathbb{E}[U(\Delta T(u, S))]$, and therefore, $F(S) = \sum_{u \in V} F_u$. We will show that $F_u(S)$ increases monotonically with $S$, thereby proving the monotonicity of $F(S)$.

We consider the uncertain sub-graph with the maximum influence paths from $S$ to $u$. We denote by $X$ one possible world of this uncertain sub-graph, having probability $prob(X)$, and shortest-path length of $\delta t_X(u, S)$ from $S$ to $u$. Therefore, we get:

$$F_u(S) = \sum_{\text{all possible world } X} [U(\delta t_X(u, S)) \times prob(X)] \tag{5}$$

Next, consider a node $v \notin S$. With the possible worlds for the seed set $S$, we show how one can derive the possible worlds for the seed set $S \cup \{v\}$. Let us denote by $E_1$ the set of edges that are in the maximum influence path from $v$ to $u$, but not in the maximum influence paths from $S$ to $u$. For a specific possible world $X$ (corresponding to seed set $S$), if we start sampling the edges in $E_1$, we get a set of new possible worlds $Y_1, Y_2, \ldots, Y_r$, where $r = 2^{|E_1|}$, such that $prob(X) = prob(Y_1) + prob(Y_2) + \ldots + prob(Y_r)$. Note that if we initially started with two different possible worlds $X_1 \neq X_2$, all the new possible worlds generated from $X_1$ and $X_2$ will be pairwise independent. In fact, the set of new possible worlds generated as above will correspond to all the possible worlds for the seed set $S \cup \{v\}$. Next, we observe that if $Y$ is a new possible world generated from $X$, $\delta t_X(u, S) \geq \delta t_Y(u, S \cup \{v\})$. Therefore, $U$ being monotonically decreasing, $U(\delta t_X(u, S)) \leq U(\delta t_Y(u, S \cup \{v\}))$. Finally, by applying Equation 5 on the new possible worlds, which correspond to the seed set $S \cup \{v\}$, we get $F_u(S) \leq F_u(S \cup \{v\})$. Hence, the theorem. □

THEOREM 4. TDIM *objective function is sub-modular under the* MIA *model.*

PROOF. We first show that $F_u(S) = \mathbb{E}[U(\Delta T(u, S))]$ is sub-modular under the MIA model. Since $F(S) = \sum_{u \in V} F_u$, the sub-modularity of $F(S)$ will automatically hold.

Consider any two seed sets $S_1$, $S_2$ such that $S_1 \subseteq S_2$, and any other node $v$. Let us consider the uncertain sub-graph consisting of the maximum influence paths from $S_2 \cup \{v\}$ to $u$. Let $X$ be one possible world of this uncertain sub-graph, with probability $prob(X)$. Then, for any node set $S \subseteq S_2 \cup \{v\}$, we have:

$$F_u(S) = \sum_{\text{all possible world } X} [U(\delta t_X(u, S)) \times prob(X)] \tag{6}$$

By following an argument similar to the proof of Theorem 1, one can show that:

$$\begin{aligned} &U(\delta t_X(u, S_1 \cup \{v\})) - U(\delta t_X(u, S_1)) \\ &\geq U(\delta t_X(u, S_2 \cup \{v\})) - U(\delta t_X(u, S_2)) \end{aligned} \tag{7}$$

By combining Equations 6 and 7, we get the following.

$$F_u(S_1 \cup \{v\}) - F_u(S_1) \geq F_u(S_2 \cup \{v\}) - F_u(S_2) \tag{8}$$

Therefore, $F_u(S)$ is sub-modular. Hence, the result follows. □

Finally, given a specific seed set, computing the campaigner's expected aggregated utility is $\#\mathbf{P}$-hard for IC, LT, and MIA models. This directly follows from the fact that TDIM is a generalization of the IM, and computation of the expected spread, given a seed set, is $\#\mathbf{P}$-hard under IC, LT, and MIA models [3, 7].

# 3. ALGORITHMS

As the TDIM objective function is non-negative, monotone, and sub-modular, an iterative hill-climbing approach that greedily maximizes the marginal gain at every iteration provides a solution with an approximation guarantee $(1 - \frac{1}{e}) \approx 0.63$ of the optimal solution [10]. Here, $e$ is the base of the natural logarithm. We describe our greedy algorithm below.

**Greedy Algorithm.** We perform $k$ iterations to identify the top-$k$ seed nodes. At each iteration, we add the seed node $s^*$ to $S_1$ that maximizes the marginal gain in the campaigner's expected aggregated utility. Here, $S_1$ denotes the partial seed set which was already computed in previous iterations. Formally,

$$s^* = \arg\max_{s \in V \setminus S_1} [F(S_1 \cup \{s\}) - F(S_1)] \tag{9}$$

We denote by $F(S)$ the campaigner's expected aggregated utility for the seed set $S$. As stated earlier, unfortunately there is no efficient way to compute $F(S)$ for a given $S$. Therefore, we employ Monte Carlo (MC) sampling to estimate it. Thus, in essence, our greedy algorithm produces a solution within $(1 - \frac{1}{e} - \epsilon)$ of the optimal influence spread, where $\epsilon$ depends on the accuracy of the MC estimate for expected utility given a seed set.

Table 1: Graph Dataset Characteristics

| Dataset | # Node | # Edge | Edge Prob: Mean, SD, Quartiles |
|---|---|---|---|
| *DBLP* | 684 911 | 4 569 982 | 0.008±0.007, {0.005, 0.005, 0.010} |
| *NetHEPT* | 15 235 | 62 776 | 0.010±0.000, {0.010, 0.010, 0.010} |

**Time Complexity.** The time complexity of each iteration of our greedy algorithm is $\mathcal{O}(nK(n+e))$, where $K$ is the number of MC samples to get a good estimate. Since, we require $k$ iterations, the overall complexity of our greedy algorithm is $\mathcal{O}(nkK(n+e))$.

**Optimization Techniques.** The greedy algorithm, as described above, is not very efficient because at every iteration, it recomputes the marginal gains for all remaining nodes. Leskovec et. al. earlier proposed the CELF optimization [11] based on the idea of sub-modularity. The main intuition is that the marginal gain provided by a node in the current iteration cannot be better than the marginal gain provided by the node in previous iterations. Hence, we may not require to compute the marginal gains for all remaining nodes in every iteration. As the TDIM objective function is also sub-modular, we apply the CELF optimization.

# 4. EXPERIMENTS

**Datasets.** We involve two real-world datasets, *DBLP* [8] and *NetHEPT* [4], each representing a directed uncertain graph (Table 1). For *DBLP*, as in [8], the edge probabilities are proportional to an exponential cdf of mean $\mu = 10$ to the number of collaborations, i.e., if two authors collaborated $c$ times, the corresponding edge probability is proportional to: $1 - \exp^{-c/10}$. The *NetHEPT* dataset was used in [4] for the influence-maximization task with constant edge probabilities $(0.01)$.

**Campaigner's Utility Function.** We consider three utility functions that decays differently over time.

*Exponential*: $U(\delta t) = \alpha^{\delta t}$
*Multiplicative*: $U(\delta t) = \frac{1}{\delta t+1}$
*Linear*: $U(\delta t) = \max\{1 - \beta \times \delta t, 0\}$

In our experiments, we set $\alpha = 0.9$ and $\beta = 0.2$. We selected the above functions in a way such that the campaigner's utility per user varies on a scale from 0 to 1.

**Comparing Methods.** We compare our TDIM greedy algorithm with two baselines. First, we compare against the classical IM greedy algorithm. Second, as the utility function monotonically decreases with time, we consider a variation of the classical IM, that selects seed nodes only considering the influence within its 1-hop. We refer to our two baselines as IM and IM1, respectively. In all cases, we apply the CELF optimization [11] for efficiency.

The code is implemented in C++ and the experiments were performed on a single core of a 100GB, 2.40GHz Xeon server. In all our experiments, the number of MC samples is fixed as $K = 1000$ [8]. Due to limitation of space, we only demonstrate our results with IC model and top-$k = 50$ seed nodes.

**Effectiveness and Efficiency.** As illustrated in Table 2, TDIM usually outperforms both the baselines in terms of the campaigner's expected utility. This is because the TDIM greedy algorithm directly optimizes the campaigner's utility at every iteration. We recall that our designed utility functions assign a utility per user in the scale from 0 to 1. Therefore, the improvement by TDIM over baselines would be even higher if we consider utility functions having larger ranges. We found that IM1 often outperforms IM, which is due to the monotonically decreasing nature of our utility functions.

Table 3 presents the seed set finding times. As it is expected, IM1 requires the least amount of time because it computes influence only within 1-hop of every node. However, we also found that TDIM is always faster than classical IM, which indicates that the CELF optimization is more effective for TDIM. As demonstrated in Figure 1, the TDIM seed nodes are more centrally located based on closeness centrality. Therefore, if a node (e.g., a peripheral node) is not close to a large number of other nodes in the network, that node will never be considered as one of the top candidates in any iteration of TDIM greedy algorithm. However, such a node, if reachable to many other nodes in the graph, could still be considered as a top candidate in the subsequent iterations of IM greedy method. Thus, the number of re-computations required for TDIM is usually smaller than that for IM. This makes the CELF optimization more effective in case of TDIM. We also demonstrate this fact in our experimental results. This is evident from the less number of recomputations required for TDIM, as shown in Table 3.

Table 2: Effectiveness of Top-50 Seed Nodes

| | Utility Function | Expected Utility IM | IM1 | TDIM |
|---|---|---|---|---|
| *NetHEPT* | Linear | 235.74 | 935.31 | **968.31** |
| | Multiplicative | 397.66 | 654.61 | **681.32** |
| | Exponential | 1205.09 | 1558.60 | **1672.34** |
| *DBLP* | Linear | 230.22 | **255.62** | 254.51 |
| | Multiplicative | 165.68 | 179.72 | **181.47** |
| | Exponential | 303.17 | 322.44 | **323.75** |

Table 3: Efficiency of Finding Top-50 Seed Nodes

| | Utility Function | Seeds Finding Time (sec) | | | # Recomputation | | |
|---|---|---|---|---|---|---|---|
| | | IM | IM1 | TDIM | IM | IM1 | TDIM |
| *NetHEPT* | Linear | 13252.50 | 74.07 | 4037.49 | 15744 | 332 | 3261 |
| | Multiplicative | | | 7942.96 | | | 8850 |
| | Exponential | | | 12705.30 | | | 12141 |
| *DBLP* | Linear | 6138.41 | 6006.54 | 5949.23 | 1476 | 178 | 379 |
| | Multiplicative | | | 5589.80 | | | 425 |
| | Exponential | | | 6131.49 | | | 754 |

# 5. CONCLUSIONS

We formulate and study the novel problem of maximizing a campaigner's time-discounted utility via influence maximization. We show that the problem is sub-modular and monotonic under various influence diffusion models, and propose approximate algorithms with theoretical performance guarantees. We also demonstrate that the existing optimization techniques for the classical influence maximization could be more effective in our current setting. In future work, we shall consider the problem with more advanced optimization techniques, e.g., TIM [14] that was employed for the classical influence maximization.

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